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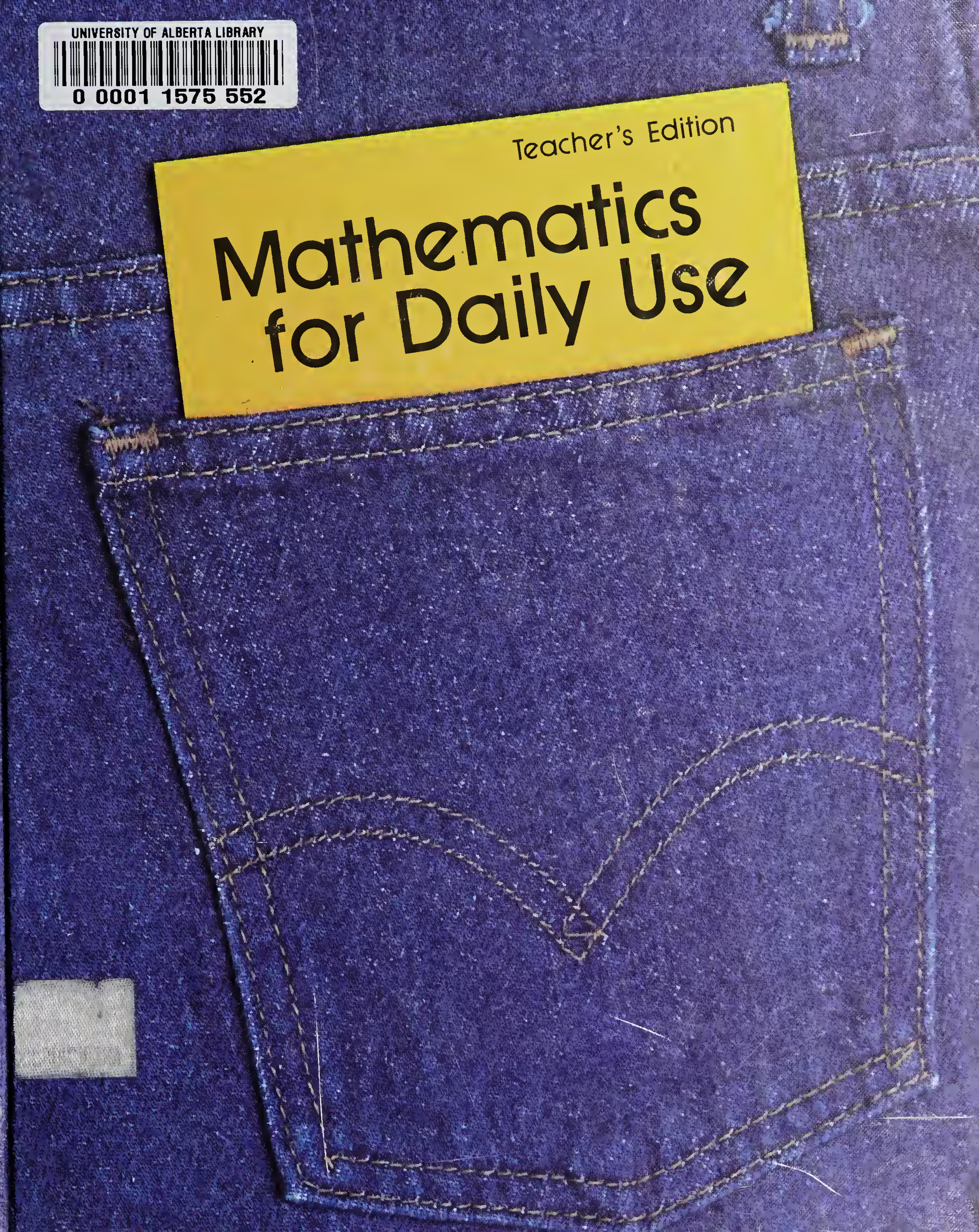

Teacher's Edition

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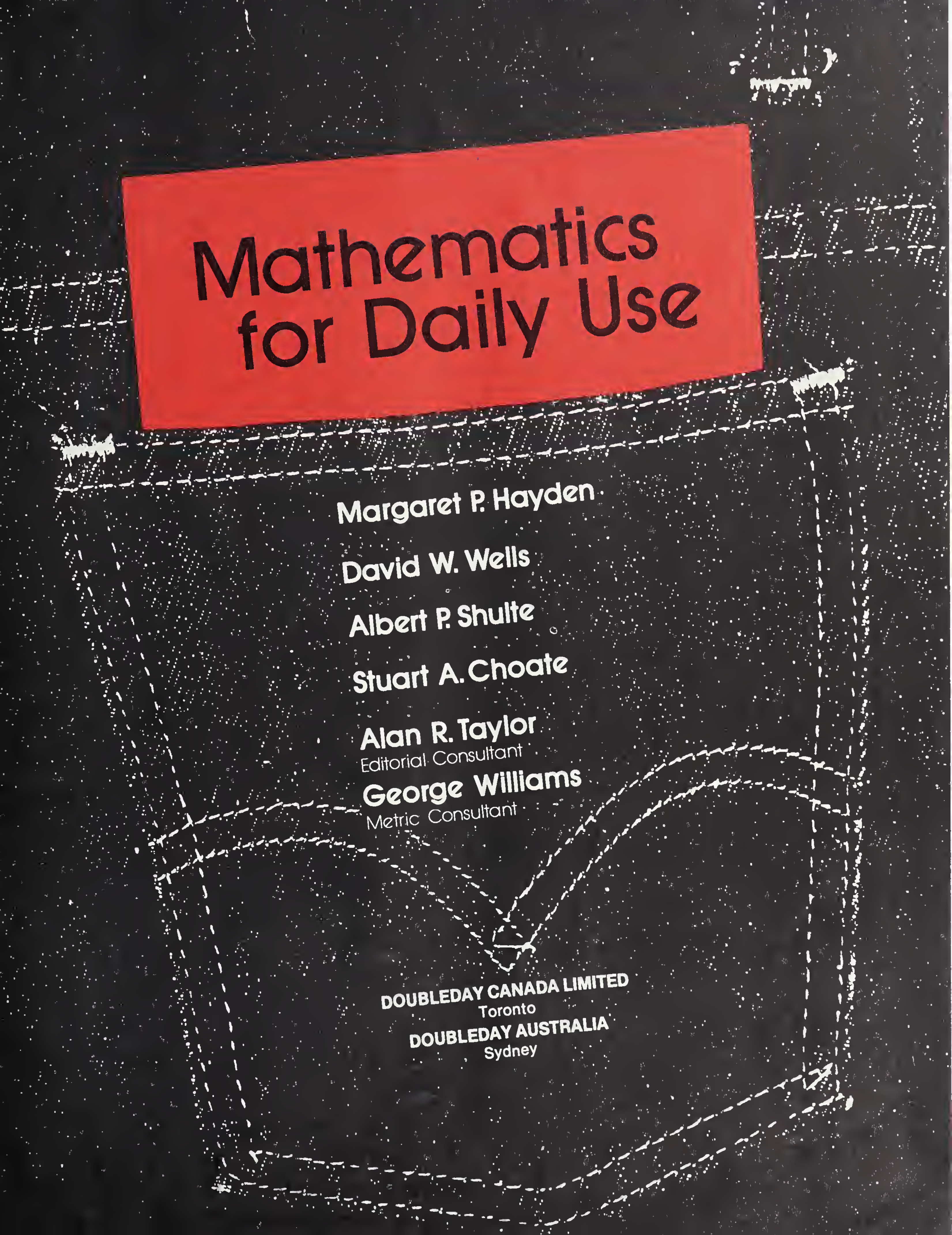

Mathematics for Daily Use

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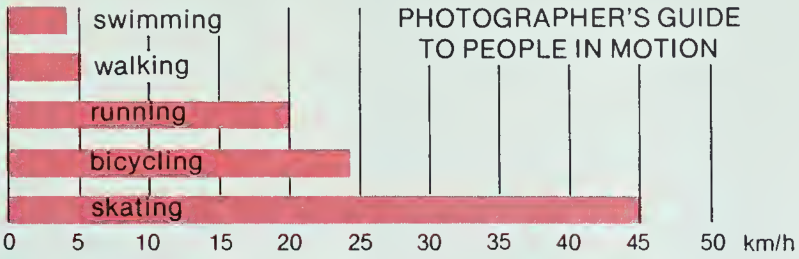
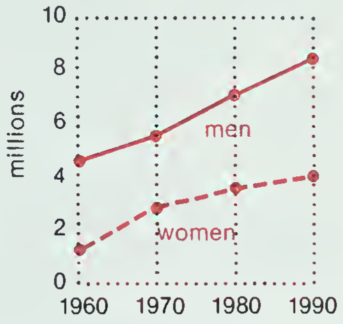
QUICK QUIZ: 15, 19, 36, 40, 44, 62, 72, 109, 113, 122, 164, 178, 207, 208, 242, 250, 261, 267, 295, 339, 365, 394, 411, 432, 435, 438

Preview Test for Chapter 1



Persons in the Labor Force 1960–1990



Source: Statistics Canada

Refer to the bar graph. (1.1)

1. Which is faster, swimming or walking? Walking
2. Skating is more than twice as fast as running.

From the line graph, about how many (1.2)

3. men were in the labor force in 1970? 5.6 million
4. women will be in the labor force in 1990? 4.0 million

MEN'S SHIRT PATTERN						
SIZE	34	36	38	40	42	44
chest size (cm)	87	92	97	102	107	112
Fabric required						
Width (cm)	metres needed					
90	2.90	2.90	2.90	3.00	3.10	3.10
115	2.20	2.20	2.30	2.30	2.40	2.50
140	1.80	1.80	1.80	1.90	1.90	2.00
150	1.70	1.70	1.70	1.80	1.80	1.80

From the table, how much fabric is needed (1.3)

5. for a size 42 shirt, using 90 cm wide fabric? 3.1 m
6. for a shirt for a man with a 92 cm chest, using 150 cm wide fabric? 1.7 m
7. The line graph has which of these? (1.4)
   a. a percent scale
   b. an informed guess at the future
8. If the bar graph were a pictograph and each picture stood for 5 km/h, how many pictures would there be for skating? 9

(1.5)

Use this set of numbers for Exercises 9–13.

3, 5, 5, 5, 6, 6, 9, 11, 14, 16

9. Find the mode. (1.6) 5
10. Find the median. (1.7) 6
11. Find the mean. (1.8) 8
12. Find the range. (1.9) 3–16 or 13
13. Make a frequency table. (1.10)
    See table at right.

Number	Frequency
3	1
5	3
6	2
9	1
11	1
14	1
16	1

14. Find the range, the mode, the median, and the mean.

3.4, 2.7, .3, 9.1, 8.2, 5.6, 6.2, 10.1, 4.2, 8.2 (1.11)

Range: .3–10.1 or 9.8; mode: 8.2; median: 5.9; mean: 5.8

1 Using Tables and Graphs

H. Armstrong Roberts

PICTURE PREVIEW: Graphs and tables are used to show facts in a simple and interesting way. The pictures on these pages give examples of tables and graphs and their uses. This chapter will take up some of the most common graphs and tables. It will also examine statistics which are used often.

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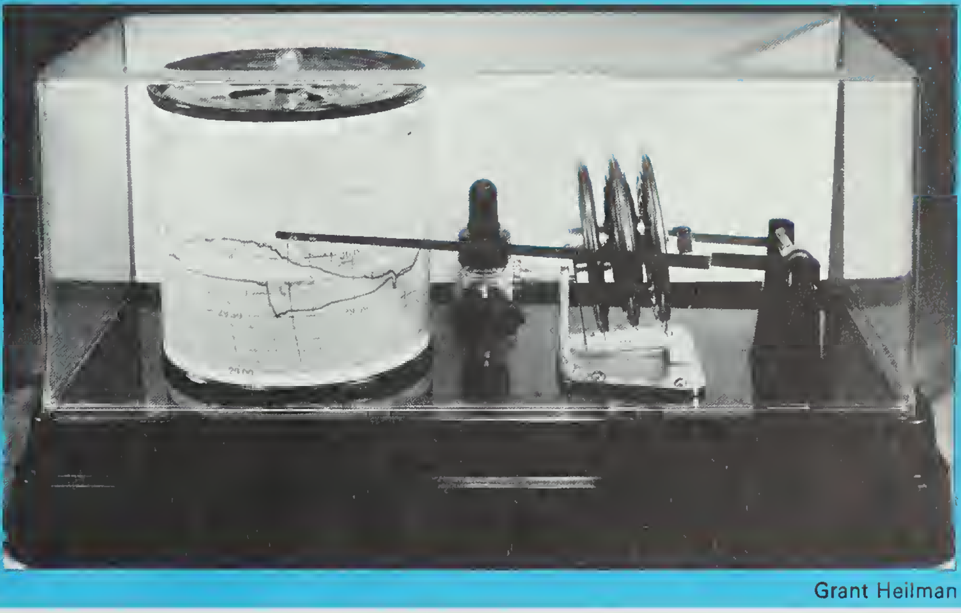


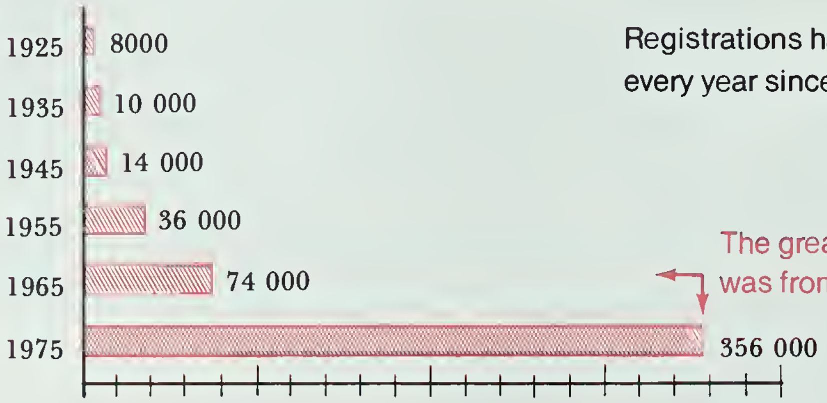
Grant Heilman

1.1 Reading Bar Graphs

Performance Objective: The student can read and interpret single, multiple, and divided bar graphs.

A **bar graph** is used to compare amounts. Here is a simple bar graph. It shows the number of motorcycles registered every tenth year since 1925.

MOTORCYCLE REGISTRATIONS (to the nearest thousand)



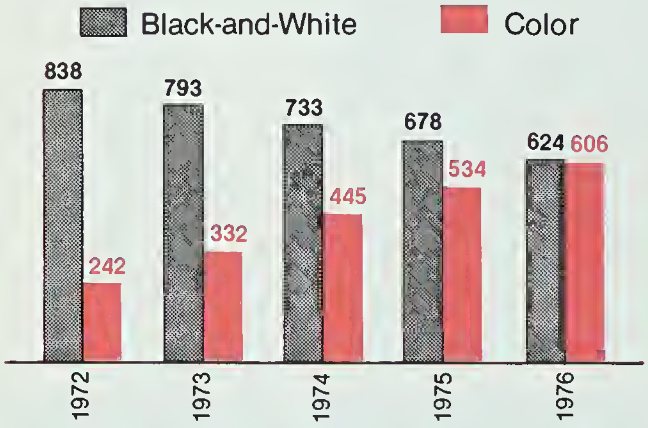
Registrations have increased every year since 1925

The greatest increase was from 1965 to 1975

Source: Statistics Canada

More facts can be shown on one graph by using double bars.

HOMES (out of every 1000) WITH TELEVISION SETS (1972–1976)



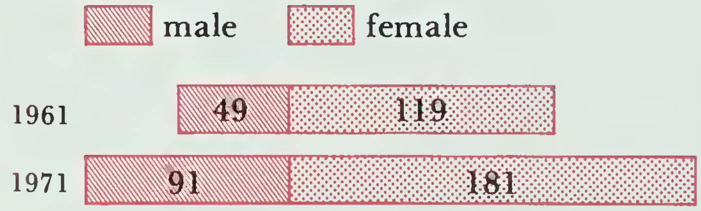
Source: Statistics Canada

The graph shows how many homes **out of every 1000** had black-and-white television sets and how many had color sets in each year from 1972 to 1976.

Graphs of this kind can also be made using three or more bars side by side.

Another way to show more facts on one graph is with divided bars.

TEACHERS (in thousands)



Source: Statistics Canada

The amounts given are in **thousands.**

There were about 119 000 female teachers in 1961.

Exercises

1—3. Use the top graph on page 12 to answer these questions.

1. How many motorcycles were registered in 1955? In 1975?
36 300; 355 500
2. What shows that the numbers beside the bars do not tell exactly how many motorcycles were registered?
Label (to the nearest thousand)
3. How could you compare the numbers of motorcycles registered in two different years if no numbers were listed beside the bars? Compare lengths of bars

4—8. Use the middle graph on page 12 to answer these questions.

4. How many homes out of every 1000 had black-and-white television sets in 1975? In 1972? 678;838

5. How many homes out of every 1000 had color television sets in 1976? In 1973? 606;332

6. Out of every 1000 homes, the number with black-and white TV decreased and the number with color TV increased from 1972 to 1976. Which was a greater change?
The number with color TV
7. Do you think either number will ever reach 1000? (Give reasons for your answer.) *Answers will vary.*

8. For any year, how could you find out how many homes out of every 1000 had no color TV? Subtract the number with color TV from 1000.

9—12. Use the bottom graph on page 12 to answer these questions.

9. In 1971, there were about how many male teachers? About how many female teachers? 91 thousand; 181 thousand

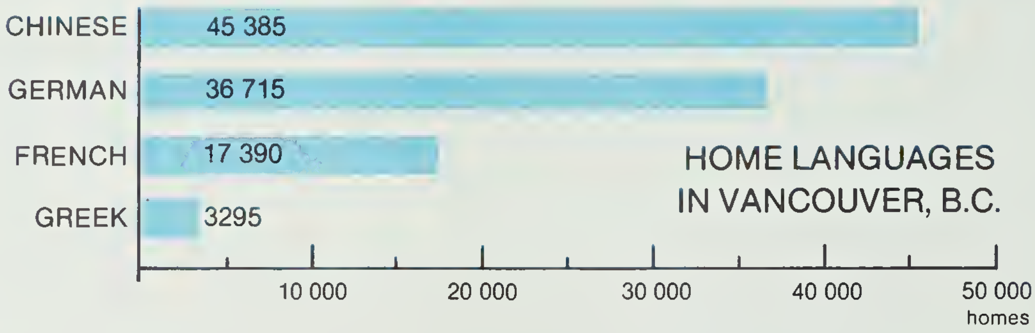
10. How could you find the *total* number of teachers in 1971?
Find (91 + 181) thousand or 91 000 + 181 000

From 1961 to 1971, there was a *net increase* in the number of teachers. Can you tell from the graph whether there was an increase during the period named?

11. from 1961 to 1965
No (numbers for 1965 not shown)

12. from 1969 to 1971
No (numbers for 1969 not shown)

13–20. A survey in urban Vancouver, B.C., shows that in about 21 out of every 100 homes English is not spoken at home. This graph compares four of the home languages.



Source: Statistics Canada

13. If the number labels were removed from the bars, how would you have to read this graph? Estimate using the scale.

14. Using the scale along the bottom of the graph, can you tell the number of homes as exactly as they are given by the labels? Why, or why not? No; you must estimate between divisions of the scale.

15. A bar two spaces long would represent how many homes? 10 000

16. A bar five spaces long would represent how many homes? 25 000

17. Italian is the home language in 2575 homes. About how long a bar would be needed to show this on the graph? 0.5 spaces

18. Ukrainian is the home language in 850 homes. About how long a bar would be needed to show this on the graph? 0.2 spaces

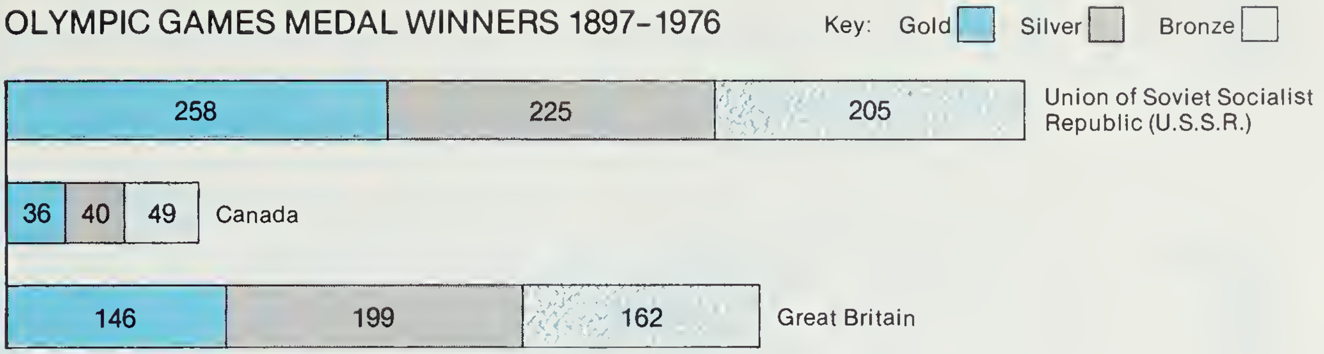
19. English is spoken in 924 000 homes. About how long a bar would be needed to show this on the graph? 184.8 spaces

20. If the graph were to represent all languages except English and French as "Other", about how long would that bar be? 17.6 spaces

What number please? If telephone books in Vancouver were to contain instructions in English and two other languages, which languages might be chosen in order to serve the largest number of customers?

Chinese, German

a. OLYMPIC GAMES MEDAL WINNERS 1897–1976



21–26. These two graphs show the same information.

21. Which graph is better for comparing the *total* number of medals won by each country? a

22. Which graph is better for comparing the number of silver medals won by each country? b

23. How could you find the total number of medals won by Great Britain? Add 146 + 199 + 162

24. How could you find the total number of medals won by the U.S.S.R.? Add 258 + 225 + 205

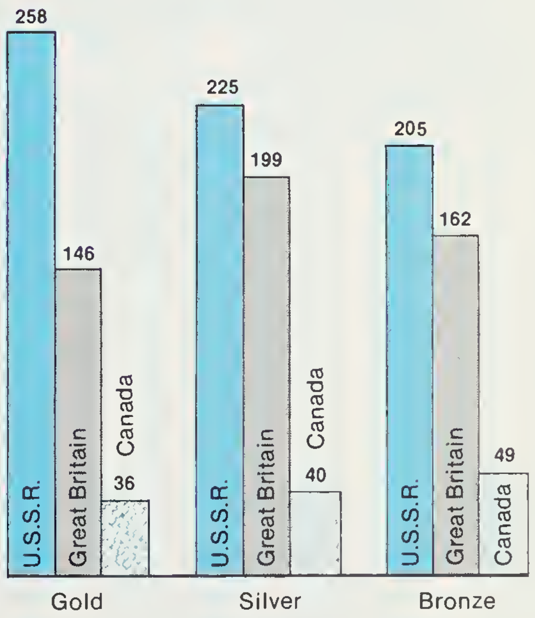
25. How could you find how many more silver medals were won by the U.S.S.R. than by Canada? Find 225 40.

26. How could you find how many more bronze medals were won by the U.S.S.R. than by Great Britain? Find 206 162.

b.

OLYMPIC GAMES MEDAL WINNERS 1897–1976

Union of Soviet Socialist Republic (U.S.S.R.)
Great Britain
Canada



1–4. Write out in words.

1. 5283 Five thousand two hundred eighty-three

2. 422 700 Four hundred twenty-two thousand seven hundred

3. 3 701 526 Three million seven hundred one thousand five hundred twenty-six

4. 20 030 010 Twenty million thirty thousand ten

5–8. Add.

5. $$\begin{array}{r} 347 \\ +251 \\ \hline 598 \end{array}$$

6. $$\begin{array}{r} 657 \\ +574 \\ \hline 1231 \end{array}$$

7. $$\begin{array}{r} 27 \\ 131 \\ 54 \\ +\ 20 \\ \hline 232 \end{array}$$

8. $$\begin{array}{r} 5723 \\ +\ 948 \\ \hline 6671 \end{array}$$

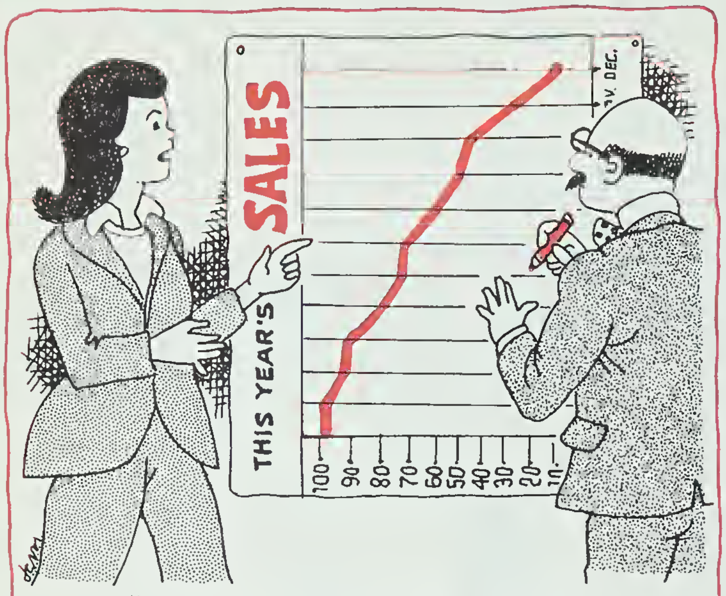
Quick Quiz

Check these skills:

- *writing words for whole numbers*
- *adding whole numbers*

1.2 Reading Line Graphs

Performance Objective: The student can read and interpret single and multiple line graphs.

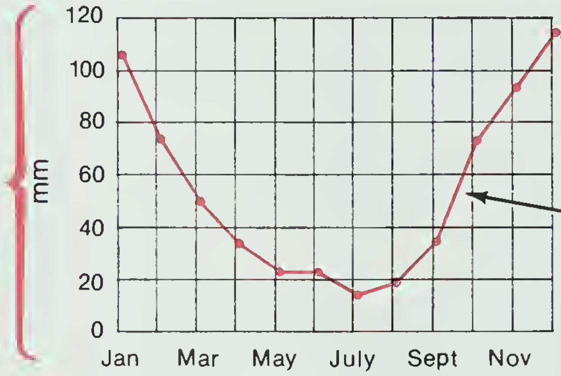


"Are you sure the boss hasn't noticed?"

Bar graphs are good for comparing amounts. But a **line graph** is better for showing change, like

temperature changes in a day,
changes in weight over 3 months,
or changes in car sales for 10 years.

NORMAL MONTHLY PRECIPITATION IN VICTORIA, B.C.



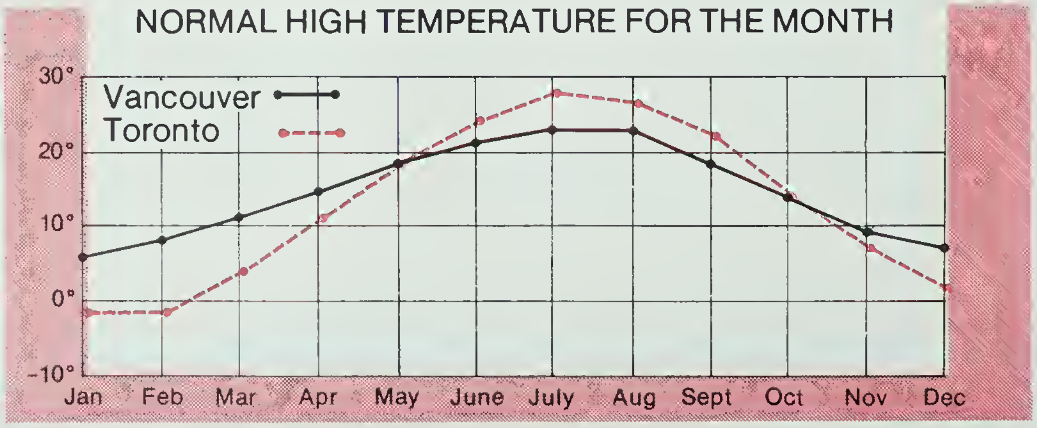
This scale cannot be read exactly—you must **estimate** the amount.

December has the most rain, about 115 mm.

Rainfall increases from July to December

Source: Canadian Almanac & Directory

Changes in several things can be compared by showing more than one line on the same graph.



NORMAL HIGH TEMPERATURE FOR THE MONTH

Source: Canadian Almanac and Directory

1—4. Refer to the graph of rainfall in Victoria (see page 16).

1. Which is the wettest month from November through May? December
2. Amounts are estimated when they (can, cannot) be read exactly.
3. About how much rain does Victoria have in July? In May? 13mm; 21mm
4. Rainfall increases from ______ to December. July

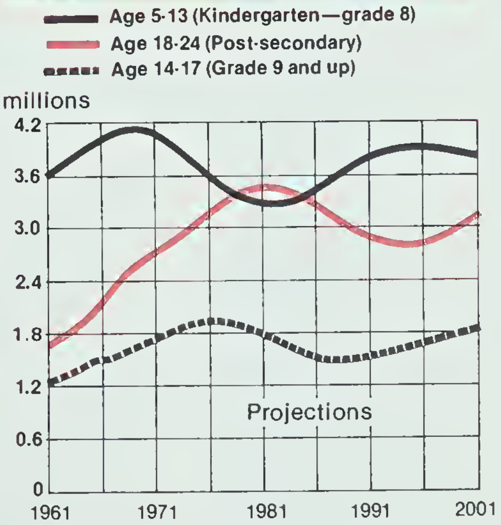
5—10. Refer to the graph at the bottom of page 16.

5. Which city has the lowest winter temperatures? Toronto
6. Which city has the highest summer temperatures? Toronto
7. Which city has the biggest change in temperature over a year's time? Toronto
8. For which scale must you estimate a reading? Temperature scale (Celsius scale)
9. About what is the temperature of Vancouver in April? 14°C
10. About what is the temperature of Toronto in November? 7°C

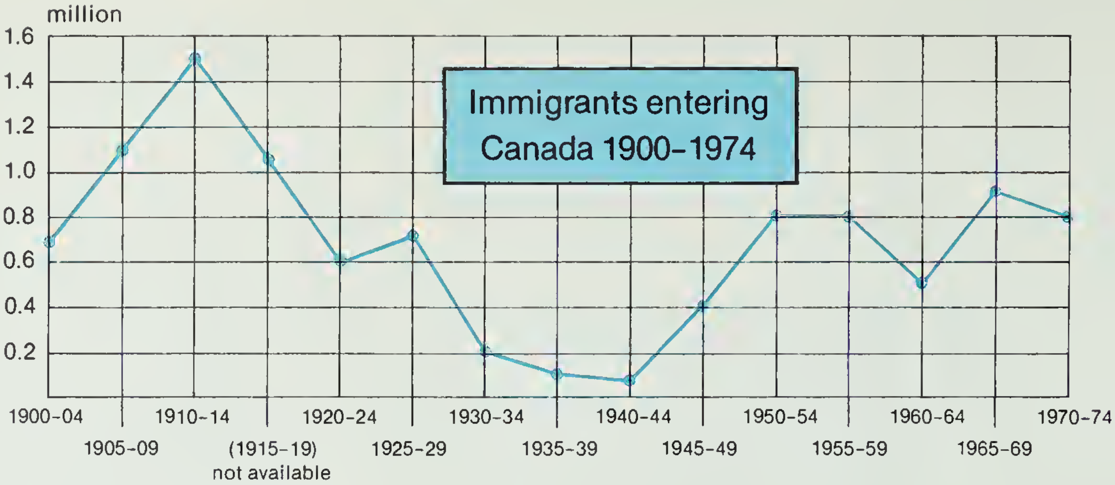
11—14. Refer to the graph at the right.

11. What was the number of students enrolled in kindergarten - grade 8 in 1966? In 1976? 4 million; 3.68 million
12. What was the school enrolment for grade 9 and up in 1961? In 1971? 1.25 million; 1.75 million
13. What is the projected enrolment for post-secondary in 1981? In 1990? Why the decrease? 3.45 million; 2.82 million. Decrease in number of students enrolled in schools from 1971 to 1981.
14. From 1961 to 1981, which enrolment's increase is the greatest? Which enrolment's decrease is the greatest? post secondary; kindergarten - grade 8

SELECTED AGE GROUP POPULATIONS RELEVANT TO SCHOOL ENROLMENT (1)



SOURCE: PERSPECTIVE CANADA



Source: Canadian Almanac and Directory

15. In which period did fewer immigrants come to Canada, 1900-1904 or 1945-1949? 1945-49

16. In which period did more immigrants come to Canada 1925-1929 or 1970-1974? 1970-74

17. In which five-year period did the greatest number of immigrants come to Canada? 1910-14

18. In which five-year period did the fewest immigrants come to Canada? 1940-44

Canada
Immigration
Immigration
du Canada

Robert Brown

19—20. Because there was a *net increase* in immigration from 1900 to 1914, we say the *trend* was up, or on the increase.

19. What was the trend from 1929 to 1944? Decrease

20. What was the trend from 1944 to 1954? Increase

21—26. About how many million immigrants came in each period?

21. 1940-44 0.07
22. 1965-69 0.9
23. 1925-29 0.7
24. 1935-39 0.1
25. 1905-09 1.1
26. 1900-04 0.7

27. How could you find about how many immigrants came from 1965 to 1974? Estimate numbers for 1965-69 and 1970-74 from the graph and add.

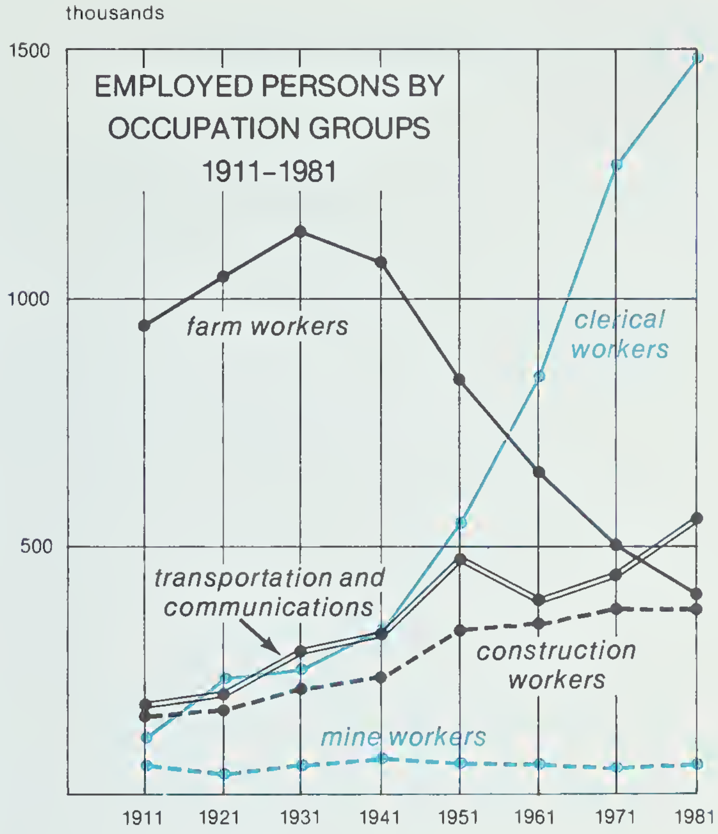
28. How could you find about how many more immigrants came from 1910 to 1914 than from 1960 to 1964? Subtract estimate (from graph) for 1910-14 from estimate (from graph) for 1960-64

29. About how many thousand farm workers were there in 1961? 650

30. How many thousand construction workers were there in 1951? 320

31. For which years may the graph be based on known facts? 1911 through 1971

32. For which year must the graph show what is expected to happen? 1981



Source: Statistics Canada

33–38. For which groups does the graph show the described change in the number of workers?

33. a decrease from 1951 to 1961 Mine workers, farm workers

34. little change from 1961 to 1971 Mine workers, construction workers

35. an expected decrease from 1971 to 1981 Farm workers

36. more rapid increase than any other group from 1951 to 1971 Clerical workers

37. a decrease every ten years from 1931 to 1971 Farm workers

38. both increases and decreases between 1951 and 1971 Transportation and communication workers

39–40. Which groups had about equal numbers of workers

39. in 1941? Transportation and communication and clerical workers

40. sometime between 1951 and 1961? Clerical workers and farm workers

Quick Quiz

Check these skills:

- *writing words for decimals*
- *rounding*

1–6. Write in words.

1. 12.4 Twelve and four-tenths

2. 3.72 Three and seventy-two hundredths

3. 5.06 Five and six-hundredths

4. 0.321 Three hundred twenty-one thousandths

5. 28.623 Twenty-eight and six hundred twenty-three thousandths

6. 1.2345 *Answer below.*

7–12. Round.

7. 6.34 (nearest tenth) 6.3

8. 9.688 (nearest hundredth) 9.69

9. 3.25 (nearest tenth) 3.3

10. 4.325 (nearest tenth) 4.3

11. 5746 (nearest hundred) 5700

12. 136 500 (nearest thousand) 137 000

6. One and two thousand three hundred forty-five ten-thousandths

1.3 Using Tables

Many times, information is presented in table form. Here is part of a table of sales taxes. Listed beside it are some of the facts the table reveals. (Of course, taxes are not the same in each province.)

COST OF PURCHASE	SALES TAX
$ .00– .15	.00
.16– .24	.01
.25– .49	.02
.50– .74	.03
.75– .99	.04
1.00–1.24	.05
1.25–1.49	.06
1.50–1.74	.07
1.75–1.99	.08

Purchases up to 15¢ are not taxed.

An item priced at 79¢ costs 79¢ + 4¢, or 83¢.

A purchase of $1.67 is taxed 7¢.

If the pattern continues, a purchase of $5.14 is taxed 25¢.

Other tables you may need to use include plane, train, and bus schedules.

Read DOWN for trains going FROM Montreal TOWARD Ottawa

rail diesel car

MONTRÉAL—OTTAWA

THE CANADIAN 3 Daily/Quot.	133 Daily/Quot.		Eastern Time Heure de l'Est		132 Daily/Quot.	134 Sun./Dim.	THE CANADIAN 4 Daily/Quot.
12 05	18 30	Dp	Montréal, Qué. ●	Ar	11 30	21 05	20 15
12 17	18 40		Montréal West		11 16	20 53	20 00
↓ Via Vankleek Hill	19 10		Rosemère		10 43	20 22	↑ Via Vankleek Hill
	19 30		Mirabel		10 22	20 04	
	19 50		Lachute		10 05	19 50	
	20 29		Montebello		09 26	19 10	
	21 15		Gatineau		08 43	18 28	
	21 24		Hull		08 35	18 20	
14 20	21 45	Ar	Ottawa, Ont.	Dp	08 15	18 00	18 05

train 133 stops at Gatineau at 21 15

train 132 leaves Ottawa at 08 15

Read UP for trains going FROM Ottawa TOWARD Montreal

Exercises

1—12. Use the tables on page 20 to answer these questions.

1. How much tax is there on a purchase of 32¢? 2¢
2. How much tax is there on a purchase of $1.19? 5¢
3. How much does a 60¢ item actually cost, including tax? 63¢
4. How much must you pay for a $1.30 book? $1.36
5. How much tax would you expect on a $2.05 purchase? 9¢
6. At what time does train 133 reach Ottawa? 21 45
7. At what time does train 132 reach Montreal? 11 30
8. At what time does train 134 reach Lachute? 19 50
9. What are the numbers of the trains you could take from Ottawa to Montebello? 132, 134
10. What trains could you take from Ottawa to Montreal? 132, 134, 4
11. When on Sunday is there a train from Hull to Mirabel? 08 35, 18 20
12. At what time on Thursday could you get a train from Rosemère to Gatineau? 19 10

13—18. Refer to the table of kilojoules needed by people of the given age and mass.

KILOJOULE REQUIREMENTS BY AGE AND SEX FOR MODERATE ACTIVITY

Persons	Age (yr)	Mass (kg)	Daily Kilojoules
Children	1-2	12	4 200
	2-3	14	5 250
	3-4	16	5 850
	4-6	19	6 700
	6-8	23	8 350
	8-10	28	9 200
Boys	10-12	35	10 450
	12-14	43	11 300
	14-18	58	12 555
Men	18-22	66	11 700
	22-35	69	11 700
	35-55	69	10 900
	55-75 +	69	10 050
Girls	10-12	35	9 400
	12-14	44	9 650
	14-16	51	10 050
	16-18	54	9 650
Women	18-22	58	8 350
	22-35	58	8 350
	35-55	58	7 750
	55-75 +	58	7 100

13. What mass is given for a 20-year-old man? 66 kg
14. What mass is given for a 17-year-old girl? 54 kg
15. How many kilojoules should a 58 kg, 22-year-old woman eat daily? 8350
16. How many kilojoules should a 69 kg, 34-year-old man eat daily? 11 700
17. Who needs more kiloujoules, a 10-year-old girl or a 60-year-old woman? 10-year-old girl
18. Who needs more kilojoules, a 75-year-old man or a 12-year-old boy? 12-year-old boy

19—30. Most highway maps show distances between small towns directly on the map, but distances between places are often shown in a table like one of these. (The distances are for the most direct routes. The colored rows and columns illustrate the examples below.)

	BANFF	BRANDON	CALGARY	CHARLOTTETOWN	CHICOUTIMI	DAWSON CREEK	EDMONTON	FLIN FLON
BANFF	●	1259	129	5079	4348	1019	428	1362
BRANDON	1259	●	1130	3864	3090	1741	1151	726
CALGARY	129	1130	●	4931	4220	890	299	1233
CHARLOTTETOWN	5079	3864	4931	●	935	5554	4963	4500
CHICOUTIMI	4348	3090	4220	935	●	4831	4241	3777
DAWSON CREEK	1019	1741	890	5554	4831	●	591	1642
EDMONTON	428	1151	299	4963	4241	591	●	1051
FLIN FLON	1362	726	1233	4500	3777	1642	1051	●
FREDERICTON	4706	3447	4588	373	562	5189	4598	4134
GASPÉ	4823	3565	4694	845	679	5306	4715	4252
HALIFAX	5121	3862	4973	280	977	5604	5013	4550
HAMILTON	3631	2372	3502	1854	1083	4113	3523	3059
JASPER	286	1519	415	5332	4609	752	369	1419
KENORA	1669	410	1540	3402	2680	2152	1561	1098

MORDEN	MORRIS	NEEPAWA	PINE FALLS	PORTAGE LA PRAIRIE	RIVERS	ROBLIN	RUSSELL	SELKIRK	SNOW LAKE	SOURIS	STEINBACH	STONEWAL
74												
212	233											
254	201	328										
114	135	98	216									
253	290	77	373	163								
449	472	237	552	335	243							
378	399	166	488	264	190	53						
158	105	232	114	119	283	449	388					
811	772	760	818	858	789	565	618	716				
217	243	119	370	175	87	241	188	274	806			
143	69	261	161	148	312	481	417	93	782	309		
156	108	224	140	111	180	396	380	35	679	272	103	
528	549	315	639	414	344	121	174	528	444	362	563	515
756	779	544	867	642	573	349	402	756	216	591	792	703
866	827	782	872	824	859	745	798	771	254	904	837	734
309	328	145	414	198	114	171	117	317	735	71	346	309
291	314	79	278	180	101	179	122	306	697	145	330	303
11	69	211	290	113	264	447	377	166	809	222	134	169
201	146	269	137	166	327	422	431	37	692	322	137	69
122	69	190	132	84	241	406	352	35	700	238	64	39

Examples: Calgary and Flin Flon are 1233 km apart.

Neepawa and Selkirk are 232 km apart.

19—26. Find the distance between the places.

19. Roblin, Steinbach
481 km

20. Jasper, Edmonton
369 km

21. Halifax, Banff
5121 km

22. Kenora, Chicoutimi
2680 km

23. Morris, Stonewall
108 km

24. Souris, Morden
217 km

25. Rivers, Souris
87 km

26. Snow Lake, Pine Falls
818 km

27. How far is it to Kenora from Calgary by way of Edmonton? How much shorter is the direct route?
299 + 1561 = 1860 km; 1860 − 1540 = 320 km

28. How far is it from Souris to Russell by way of Rivers? How much longer is this than the direct route?
87 + 190 = 277 km; 277 − 188 = 89 km

29. Is it farther from Banff to Brandon through Calgary or through Edmonton? How much farther?
Edmonton; 320 km (428 + 151 = 1579, 129 + 1130 = 1259)

30. Is it farther from Neepawa to Steinbach through Selkirk or through Morris? How much farther?
Selkirk; 23 km (232 + 93 = 325, 233 + 69 = 302)

31—38. The table below shows results of road tests on four 1975 car models.

km/h means kilometres per hour

L/100 km means litres per 100 km

See the footnote with this label

	Plymouth Valiant	Chevrolet Nova	Mercury Monarch	Ford Maverick
ACCELERATION				
ON LEVEL ROAD				
0-95 km/h (seconds)	16	16	21	19.5
0.4 km from rest (seconds)	20.5	20.5	22	21.5
speed at end of 0.4 km (km/h)	110	110	99	101
Passing: 70 to 105 km/h (seconds)	10	10	15	14
FUEL ECONOMY				
RANGE OF GAS CONSUMPTION TO BE EXPECTED IN NORMAL USE (L/100 km)	26-13	23-15	28-18	31-18
GAS CONSUMPTION OBSERVED ON 312 km TEST TRIP (L/100 km)	15	16	19	19
CONSTANT-SPEED GAS CONSUMPTION (L/100 km)				
at 65 km/h	12	13	17	17.5
at 80 km/h	12.5	13.5	17	18
at 95 km/h	14.5	15.5	18	18
BRAKING				
LEVEL BRAKING FROM 95 km/h Minimum-distance stop with no wheels locked (metres)	50	63	67	61 [D]

[D] *With all-drum brakes.*

31. How long did the Nova take to reach 95 km/h from 0 km/h? 16 s

32. How long did the Maverick take to travel 0.4 km from rest? 21.5 s

33. How fast was the Valiant going after accelerating for 0.4 km? 110 km/h

34. How long did it take the Monarch to accelerate from 70 km/h to 105 km/h? 15 s

35. Which car had the lowest gas consumption on the 312 km test trip? Plymouth Valiant

36. Which car had the highest gas consumption at a constant 65 km/h? Ford Maverick

37. Which car stopped in the shortest distance when the brakes were applied at 95 km/h? Plymouth Valiant

38. Which car took the greatest distance to stop when the brakes were applied at 95 km/h? Mercury Monarch

Train Time

Use the train schedule on page 20. Make a table of times needed for the trip between Montreal and Ottawa (in either direction). Find out how much faster the fastest train is than each of the others.

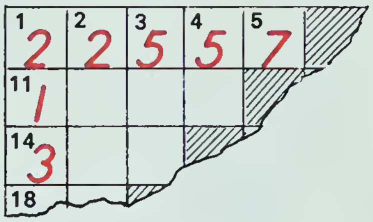
Train	3	133	132
Time	2 15		

Train	3	133	132	134	4
Time	2 15	3 15	3 15	3 05	2 10
Minutes slower than fastest train	5	75	75	55	—

CROSS-NUMBER PUZZLE

1	2	3	4	5	▨	6	7	8	9	10
11				▨	12	▨	13			
14			▨	15		16	▨	17		
18		▨	19				20	▨	21	
22	▨	23			▨	24		25	▨	26
▨	27			▨	▨	▨	28		29	▨
30	▨	31		32	▨	33			▨	34
35	36	▨	37		38			▨	39	
40		41	▨	42			▨	43		
44			45	▨	46	▨	47			
48				49	▨	50				

Copy this pattern on grid paper, but put each number in one corner of its square. Fill in squares by using either the clues for numbers to be written ACROSS the page or the clues for numbers to be written DOWN the page. (In this puzzle, the ACROSS clues are harder than the DOWN clues.) Here is what the corner of the puzzle should look like after you fill in **1** ACROSS and **1** DOWN.



2	2	5	5	7	▨	7	1	5	3	5
1	4	0	4	▨	9	▨	2	3	0	4
3	2	0	▨	5	0	4	▨	1	1	0
6	4	▨	1	0	0	0	1	▨	1	9
6	▨	3	4	5	▨	4	0	7	▨	5
▨	9	2	0	▨	▨	▨	6	4	5	▨
1	▨	3	0	4	▨	1	3	2	▨	8
1	6	▨	1	5	2	6	8	▨	2	2
4	4	4	▨	6	2	0	▨	6	8	0
2	5	9	2	▨	1	▨	1	3	7	7
1	1	1	1	1	▨	1	2	3	4	5

ACROSS

1. 32658 − 10101
6. 71533.7 + 1.3
11. 27 × 52
12. $8\frac{2}{3} + \frac{1}{3}$
13. 7200 × 0.32
14. 10 × 32
15. 18 × 28
17. 3.41 ÷ 0.031
18. 8^2
19. 9999.98 + 1.02
21. $16\frac{5}{8} + 2\frac{3}{8}$
22. $\frac{1}{2} \times 12$
23. 352.4 − 7.4
24. $4 \times 101\frac{3}{4}$
26. $\sqrt{25}$
27. $922\frac{1}{2} - 2\frac{1}{2}$
28. 322.5 ÷ 0.5
30. $\frac{5}{12} + \frac{1}{3} + \frac{1}{4}$
31. 9 + 4 + 291
33. $\frac{1}{3} \times 396$
34. (−2) × (−4)
35. 4^2
37. 3817 ÷ 0.25
39. 11, ____, 44, 88, . . .
40. 123 + 321
42. (−1860) ÷ (−3)
43. 680 × 2 × 0.5
44. 36 × 6 × 12
46. 2.5 × 0.4
47. 9(72 + 81)
48. 11111.1 − 0.1
50. 74070 ÷ 6

DOWN

1. 10683 × 2
2. 2400 + 24
3. 501 − 1
4. 3 × 18
5. 3.5 × 2
6. 3661 ÷ 523
7. 2 + 7 + 3
8. 528.2 + 2.8
9. 9033 ÷ 3
10. 5 × 10819
12. $200 \times 4\frac{1}{2}$
15. 512.2 − 7.2
16. 4 × 101
19. $\frac{1}{2}(28002)$
20. 32638 − 22000
23. 299 + 15 + 9
25. 739.4 + 2.6
27. 4707 ÷ 523
29. $\frac{125}{25}$
30. 10983 + 438
32. 4241 − 3785
33. 1760 ÷ 11
34. 3283 × 25
36. 7000 − 549
38. (11 × 11) + 100
39. 2 × 3 × 479
41. 2946 ÷ 6
43. 3 × 200 + 3 × 11
45. $\frac{1}{3} \times 63$
47. 9 + 7 − 4
49. 10 × 0.1
50. $\frac{1}{13} \times 13$

Performance Objective: The student can read and interpret line and bar graphs that use such common techniques as a broken scale or opposite double bars.

A Further Look at Graphs

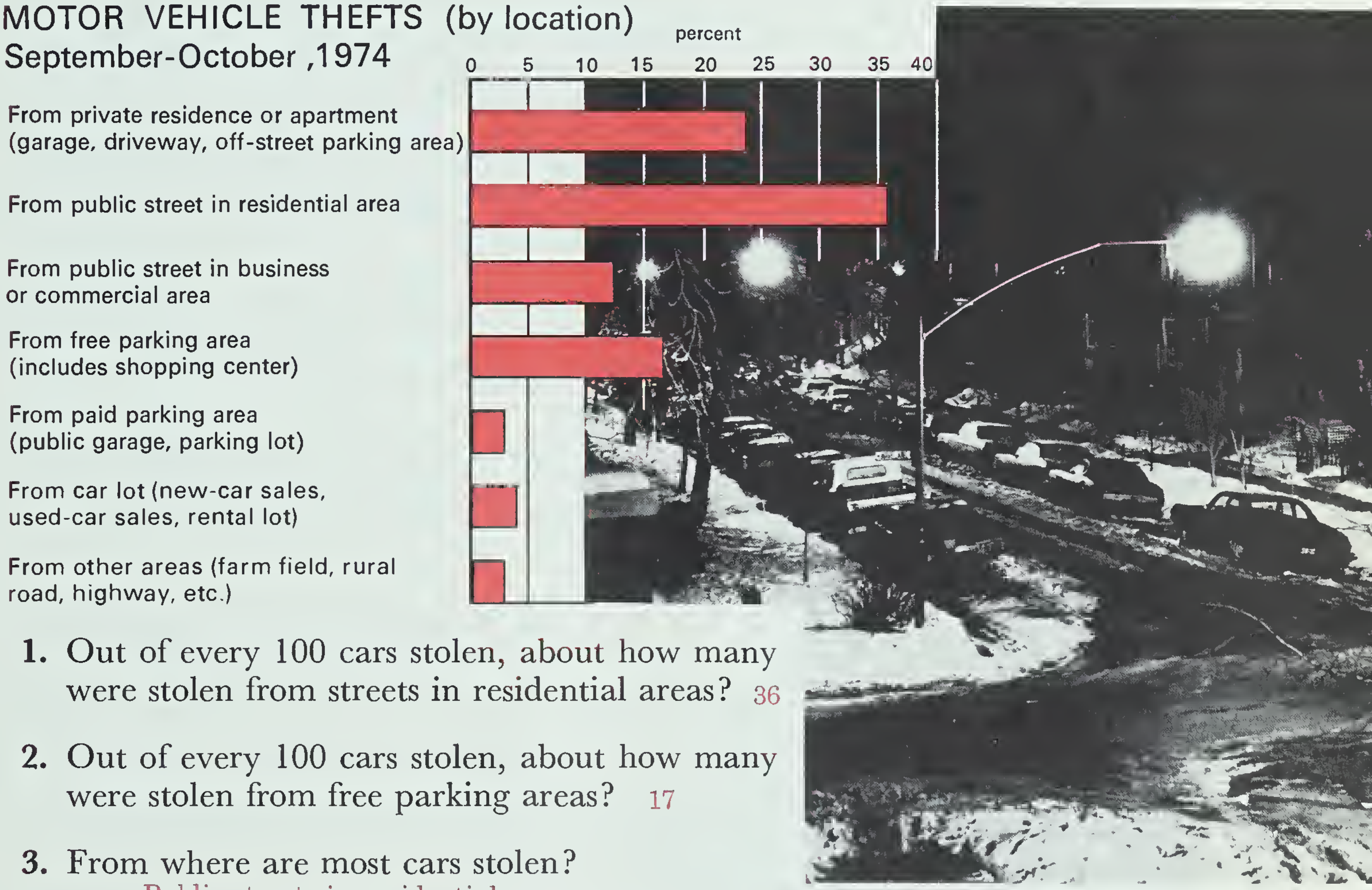
1.4

Now let's look at some bar graphs and line graphs that are a little different from those you worked with earlier.

Exercises

One scale of the graph gives **percents.** On a graph like this, 30 percent means *30 out of every 100.*

MOTOR VEHICLE THEFTS (by location)
September-October ,1974



Frank Kuechmann

1. Out of every 100 cars stolen, about how many were stolen from streets in residential areas? 36

2. Out of every 100 cars stolen, about how many were stolen from free parking areas? 17

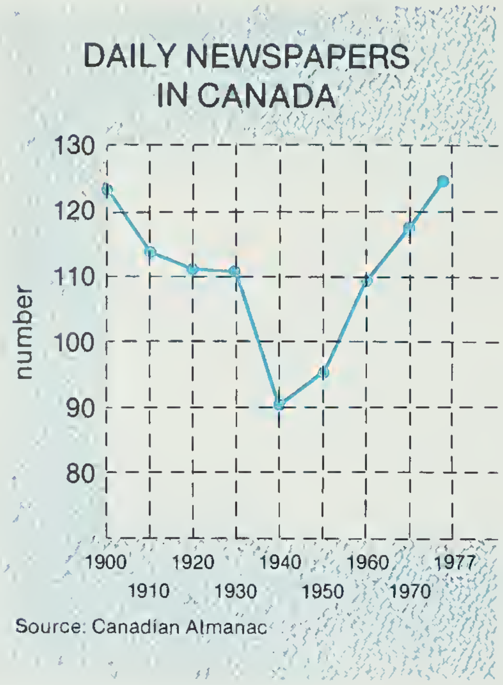
3. From where are most cars stolen?
Public streets in residential areas

4. From where is the second largest number of cars stolen?
Private residences or apartments

5–6. From which place listed, **a** or **b**, is your car least likely to be stolen?

5. **a.** a paid parking area **b.** a free parking area

6. **a.** the street in front of a house **b.** the driveway beside a house


DAILY NEWSPAPERS IN CANADA

Source: Canadian Almanac

The lower part of the scale is not needed. A break or ≀ shows that it is left off.

7. How many points are used to make the graph? 9

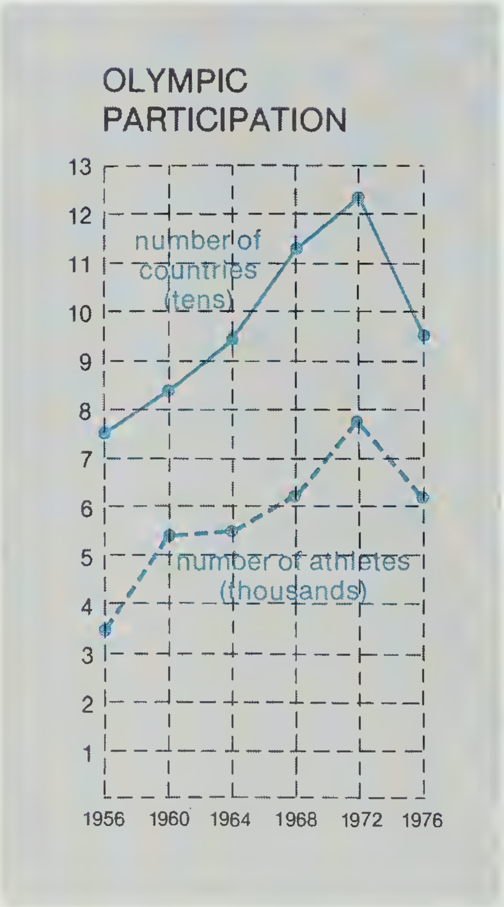
8. For what years are the numbers of daily newspapers graphed? 1900 to 1977

9. For which year does the graph show the most daily papers? 1977

10. For which year does the graph show the fewest daily papers? 1940

11. The graph shows an upward trend in the number of daily papers during the period from 1940 to 1977.

12. The graph shows a downward trend in the number of daily papers during the period from 1900 to 1940 .


OLYMPIC PARTICIPATION

The lines are graphed using different values for the vertical scale.

13. What has happened to the number of countries and athletes from 1956 to 1972? Increased

14. What happened to the number of countries and athletes in 1976? Decreased

15. About how many countries entered in 1960? 84

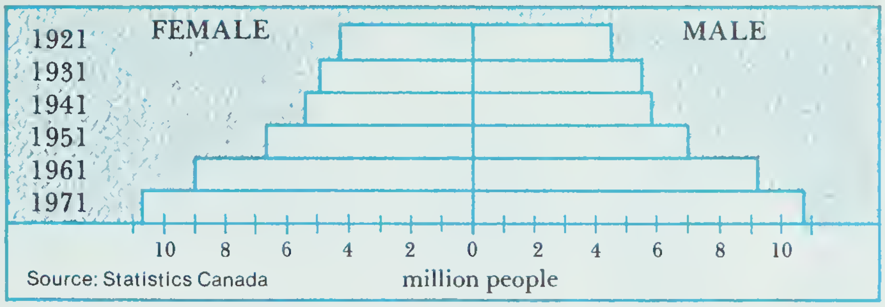
16. About how many countries entered in 1972? 122

17. About how many athletes took part in 1956? 3.4 thousand

18. About how many athletes took part in 1976? 6.1 thousand

The bars going to the right show data for men; the bars going to the left show data for women. (This idea is also good for income-expense graphs.)

POPULATION OF CANADA



Source: Statistics Canada

19—20. Which comparison is easier to make?

19. a. number of females in 1921 and in 1931
    b. number of females in 1921 and number of males in 1921

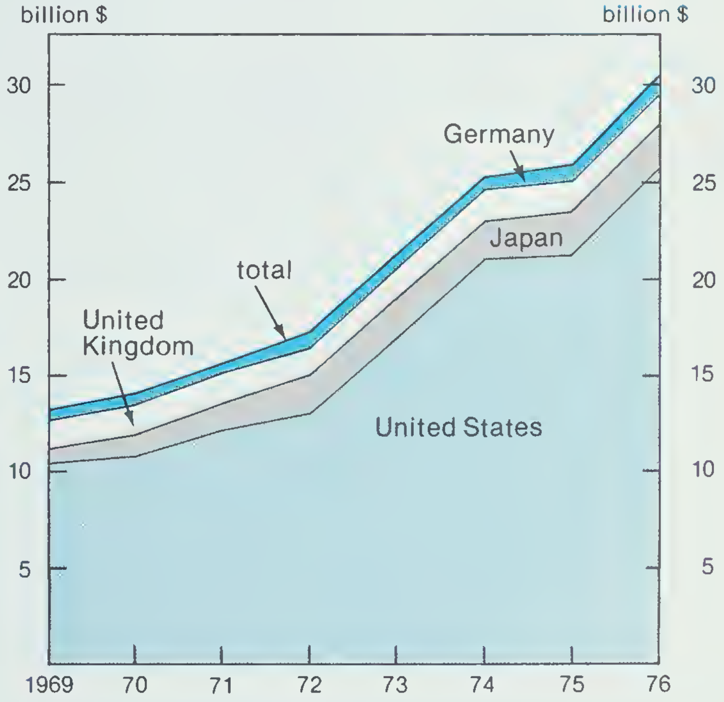
20. a. number of people in 1941 and in 1951
    b. number of females in 1941 and in 1951

21. In what ten-year period was there the smallest increase in population? 1931-1941

22. In what ten-year period was there the largest increase in population? 1951-1961

Each line shows the sum of the exports to all the countries below it (rather than the exports to one country). The top line then shows the total exports.

CANADIAN EXPORTS



23—26. About how much was exported to the countries below in the year listed?

23. United States, 1969 $11 billion

24. United States and Japan, 1976 $28 billion

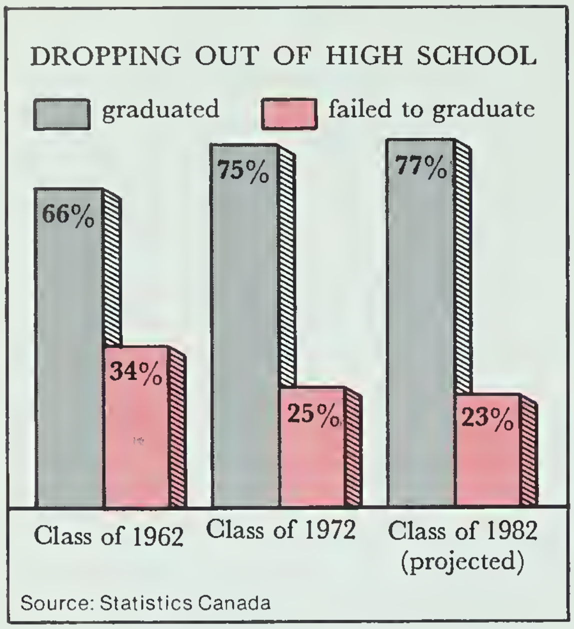
25. All exports, 1973 $21 billion

26. All exports except Germany, 1974 $24.5 billion

27—28. How could you find the exports to each country in the year listed?

27. Japan, 1970 From the graph, estimate 1970 amounts for the top two lines; subtract.

28. United Kingdom, 1972 From the graph, estimate 1972 amounts for the two middle lines; subtract.



Source: Statistics Canada

An informed guess (projection) is made for 1982 and included on the graph. Also, the bars are drawn to look three dimensional. That is, they look like boxes, not flat bars. (Remember, 66% means 66 percent, or 66 out of every 100.)

29. Which of these is meant by *the class of 1962?*

a. the people who should have graduated from a certain high school in 1962

b. the people who should have graduated from all high schools in the nation in 1962

30. Which of these is meant by *34% of the class of 1962 failed to graduate?*

a. In each school, 34 out of every 100 people who should have graduated in 1962 failed to graduate.

b. Of all the people in the nation who should have graduated in 1962, 34 out of every 100 failed to graduate.

31. What do the gray bars stand for? What do the red bars stand for? Those who graduated; those who failed to graduate

32. Why must the numbers on each pair of bars add up to 100? Out of every 100 people, all 100 either graduate or don't.

33. Is the percent of people graduating expected to increase by 1982? Yes

34. Is the percent of people failing to graduate expected to increase by 1982? No

35. Can you tell from the graph whether the percent of graduates was more in 1968 than in 1962? Explain. No; no information for 1968

36. Can you tell from the graph whether the percent of dropouts was less in 1974 than in 1972? Explain. No; no information for 1974.

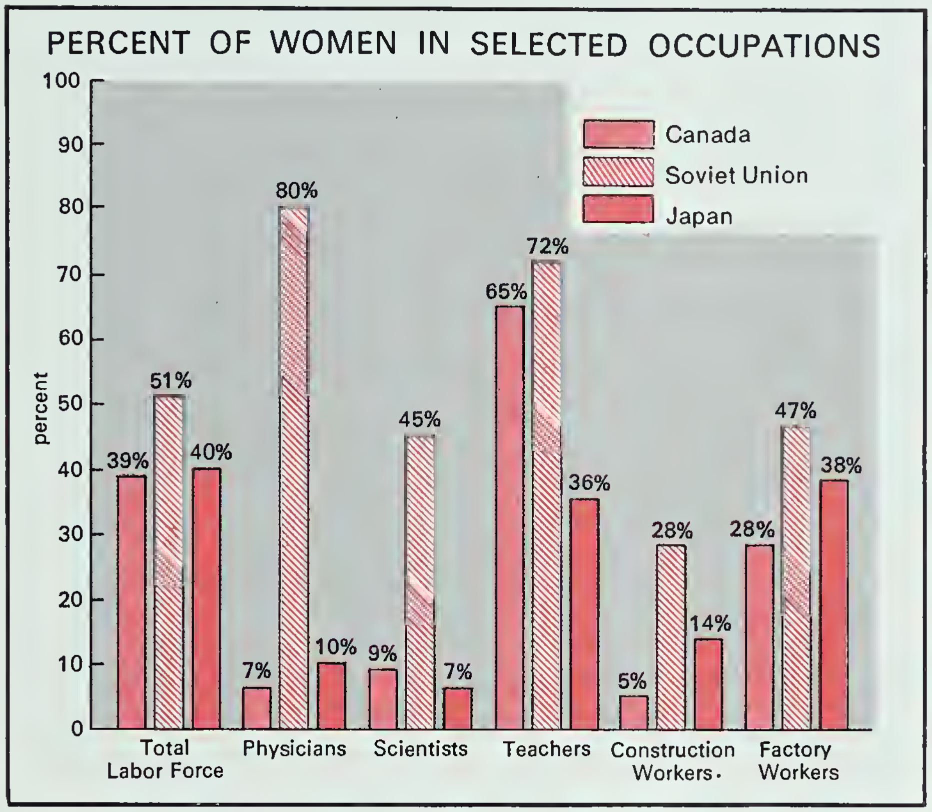
Look Around You

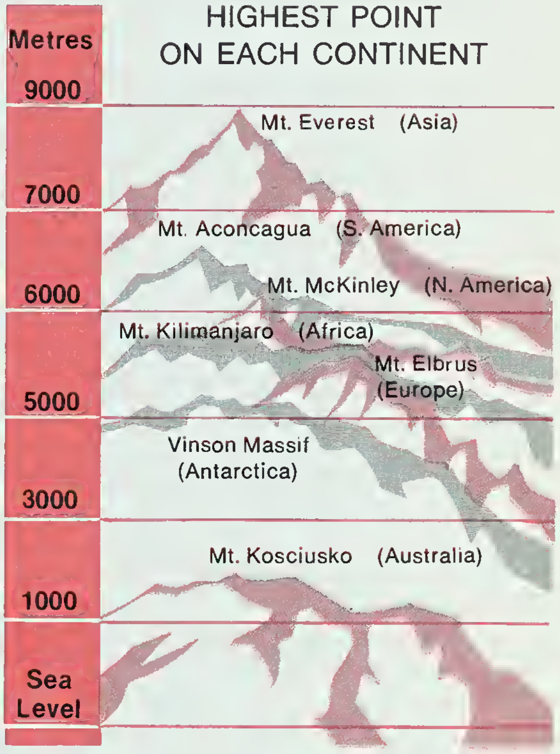
Find bar graphs, line graphs, and tables in hobby magazines or books, in newspapers, in newsmagazines, or in any other source. If possible, bring copies of some of them to class.

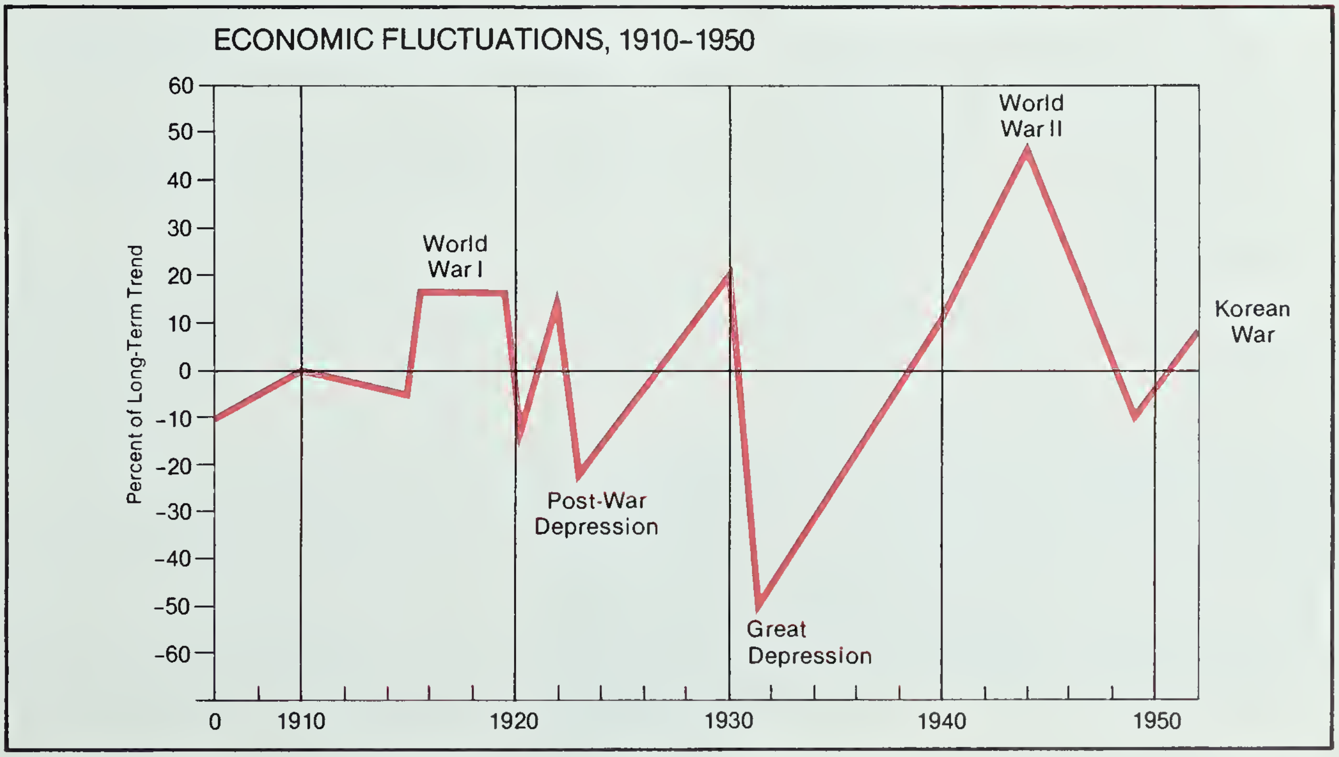
CAREER CORNER

MATH IN THE SOCIAL SCIENCES

Graphs and tables are used often by people who work with history, civics, economics, or geography. You will probably take a course in at least one of these subjects while you are in high school. Here are some graphs like those you may find in your textbooks. See how well you can read them.


PERCENT OF WOMEN IN SELECTED OCCUPATIONS


HIGHEST POINT ON EACH CONTINENT

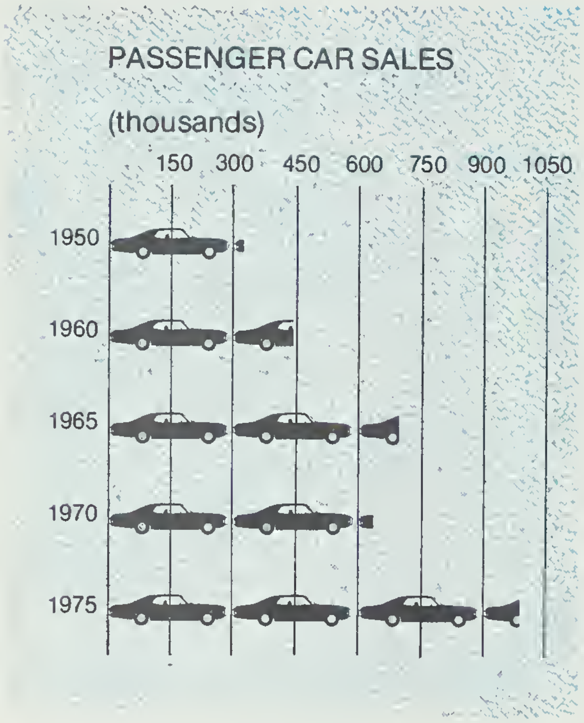

ECONOMIC FLUCTUATIONS, 1910–1950

1.5 Pictographs

Performance Objective: The student can read and interpret pictographs.

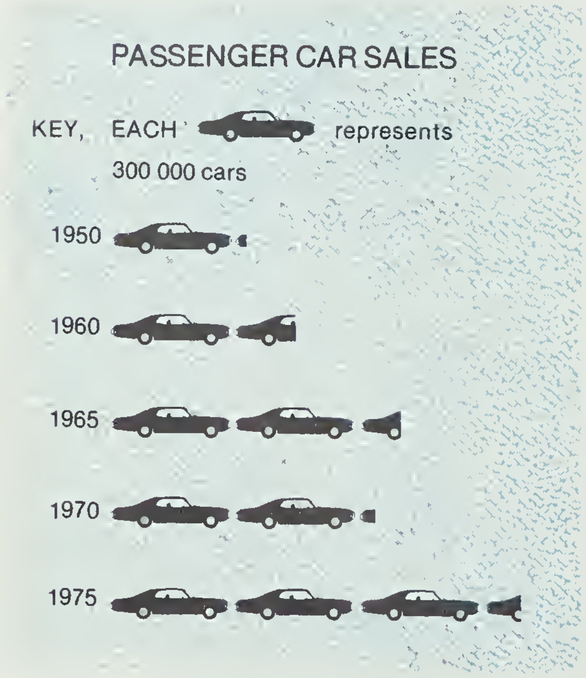
There are many ways of using pictures to make graphs more fun to read. From the pictures alone, you can often tell what the subject of the graph is.

Exercises



1—6. Refer to the graph at the left.

1. This is a graph of sales of Passenger cars
2. Explain why this is really a bar graph.
3. What is the *scale value?* (How much does each small part of the scale stand for?) 300 thousand (cars)
4. About how many cars were sold in 1970? 640 thousand
5. Does the graph show an upward trend in car sales from 1950 to 1960? Yes
6. Can you tell from the graph whether sales went up every year from 1950 to 1960? No



Source: Statistics Canada

7—10. The scale on the top graph can be left off if a *key* is included with the graph.

7. Did each whole car in the top graph stand for about 300 thousand cars? Yes
8. Is it as easy to tell from the bottom graph as it is from the top graph about how many cars were sold in 1970? Why, or why not? No; lack of scale makes estimating more difficult.
9. Is it as easy to tell from the bottom graph as it is from the top graph whether sales were greater in 1960 than in 1970? Yes
10. About how many more cars were sold in 1960 than in 1965? 250 thousand

11–20. Refer to the graph at the right.

11. For each year shown, the life expectancies of Men and of Women are compared.

12. The graph is really a (*line*, [✓ *bar*]) graph.

13. Was men's life expectancy as great as women's in any year shown? No

14. Did men's life expectancy increase as much as women's from 1920 to 1970? No

15. Men's life expectancy increased very little after 1950 .

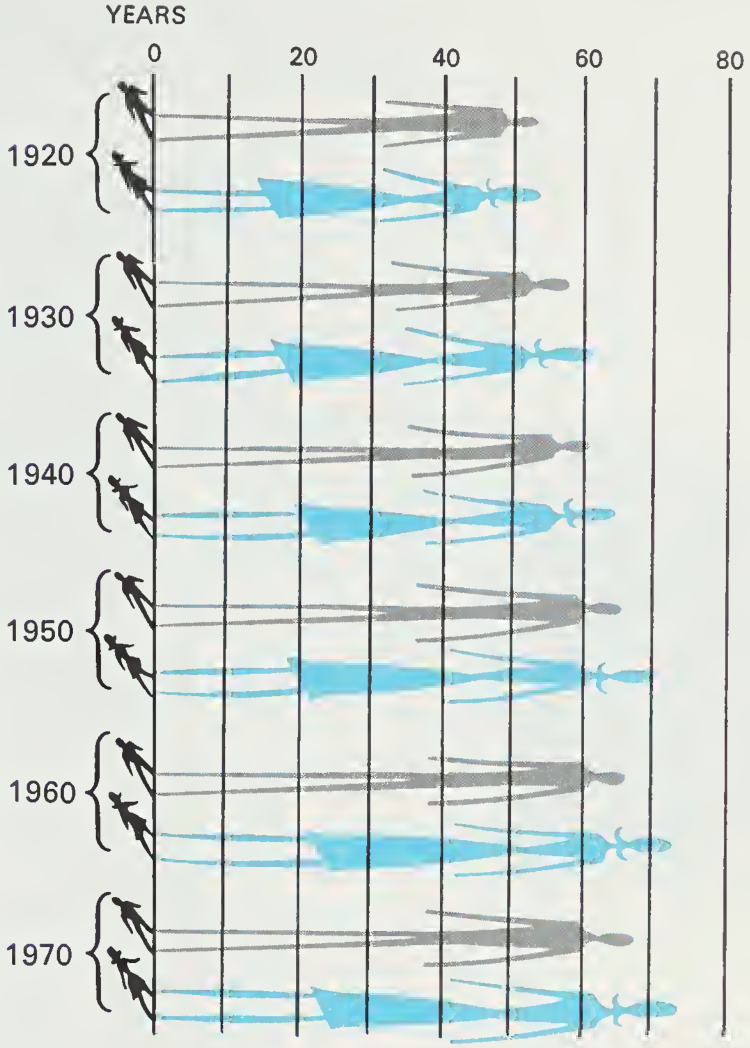
16. Women's life expectancy increased very little after 1960 .

17. When did women's life expectancy first go over 60 years? 1930

18. When did men's life expectancy first go over 60 years? 1940

19. Was women's life expectancy more than 70 years in 1970? Yes

Life Expectancy at Birth in Canada



Source: Statistics Canada

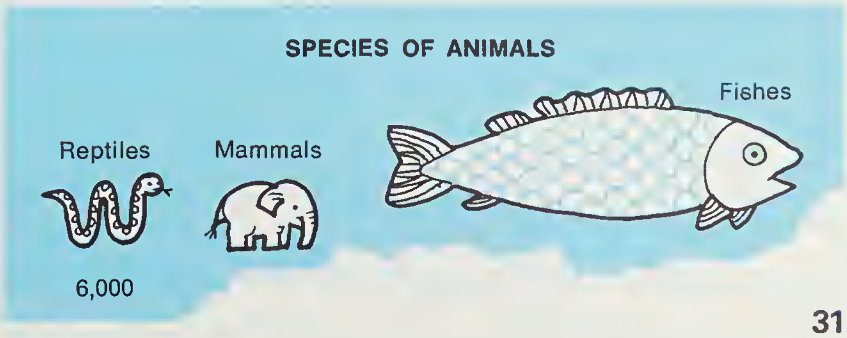
20. If equal numbers of men and women are born, would you expect to find more men over 65 or more women over 65? Women

Poor Pictographs

Sometimes drawings of three-dimensional objects of different sizes are used in pictographs and charts. Comparisons are very difficult to make from such drawings.

For example, could you guess that the elephant stands for 5000 species of mammals? How many species of fishes do you think the fish stands for?

There are about 20 000 species of fishes.



1.6 The Mode

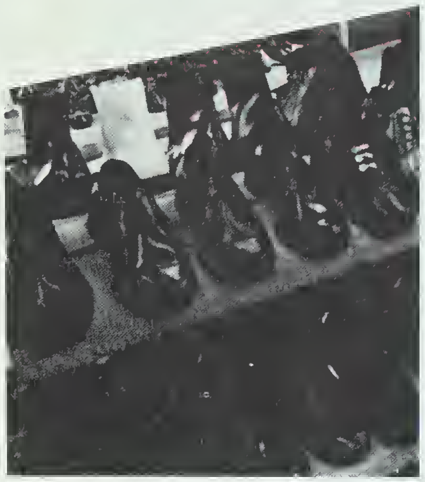
If you owned a shoe store, which of these shoe sizes would you find it most useful to know?

Performance Objective: The student can find the mode of a set of numbers.

the largest

the smallest

the most common



You would probably find it useful to know the most common shoe size—the one that occurs most often.

Editorial Phot

The item that occurs most often in any list is the **mode.**

3
4
4
5
6
6
6
7
7

The mode is 6.

2
1
7
7
1
1
3
5
1

The mode is 1.

There may be more than one mode.

$$7, 7, 8, 9, \underbrace{12, 12, 12}, 15, 21, 23, 23, \underbrace{29, 29, 29}, 35, 37$$

modes

Exercises

1. Which of these mean the same thing?

a. The most common test score was 82.

b. The highest test score was 82.

c. The mode on the test was 82.

d. The test score that occurred most frequently was 82.

2. The mode is found by (multiplying, counting, discarding) items in the list.

3. Three numbers in a list of fourteen occur the same number of times, and no other number occurs more often. How many modes are there? 3

4. Two numbers in a list of twenty numbers occur the same number of times, but one other number occurs more often. How many modes are there? 1

5–14. Find the mode or modes.

5. 3, 4, 4, 5, 5, 5, 6, 6, 7 5

6. 9, 9, 8, 7, 7, 7, 6, 5, 5 7

7. 20, 13, 17, 13, 20, 17, 17, 15 17

8. 12, 18, 15, 12, 18, 15, 15, 20 15

9. 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 5, 5, 6, 6 2, 5

10. 9, 9, 9, 8, 8, 7, 7, 7, 7, 7, 6, 6, 5, 5, 4, 4, 4, 4, 4 7, 4

11. $2, 2\frac{1}{2}, 3, 2, 4\frac{1}{2}, 3, 2\frac{1}{2}, 4, 2\frac{1}{2}, 2, 4, 3, 2\frac{1}{2}, 4, 4\frac{1}{2}, 3\frac{1}{2}, 2\frac{1}{2}$ $2\frac{1}{2}$

12. $\frac{2}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1}{3}, \frac{1}{5}, \frac{2}{3}, \frac{4}{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{2}{3}, \frac{4}{3}, \frac{1}{5}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}$ $\frac{1}{3}$

13. 1.7, 1.4, 1.4, 1.6, 1.5, 1.7, 1.4, 1.6, 1.3 1.4

14. 2.5, 3.6, 4.2, 2.5, 4.3, 8.6, 2.5, 3.1, 4.2 2.5

Look Around You

At a local clothing store, find out the most common sizes for men's shirts, for women's dresses, and for men's and women's belts, gloves, and shoes. Why is this information important to a clothing-store manager?

A store should stock more clothes in the most common sizes than in other sizes.

1.7 Finding the Middle

Performance Objective: The student can find the median of a set of whole numbers.

On some highways, the *median* is a strip down the *middle* that separates the lanes of traffic.

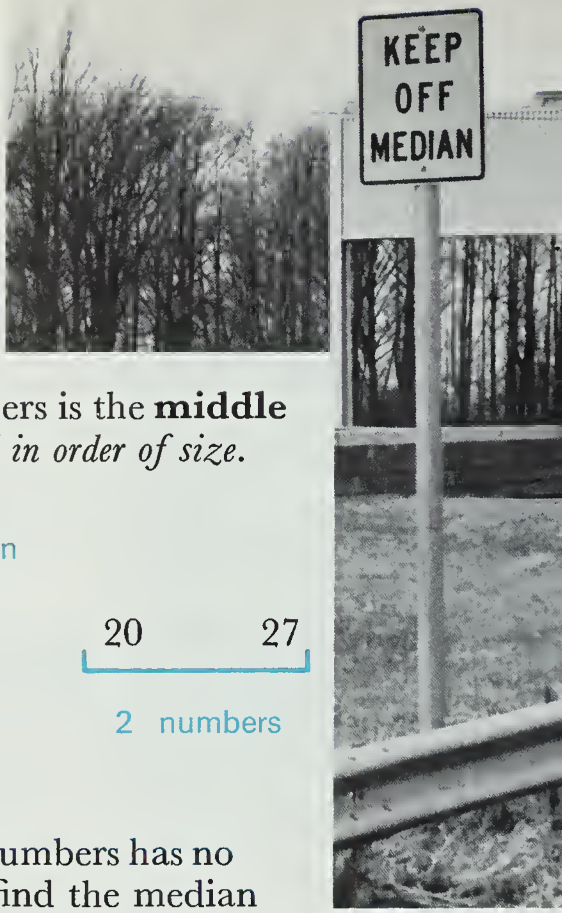


Photo Research Intern

The **median** of a set of numbers is the **middle** number *when the numbers are listed in order of size.*

Example 1:

$$\underbrace{3 \qquad 14}_{\text{2 numbers}} \qquad \overset{\substack{\text{median}\\ \blacktriangledown}}{14} \qquad \underbrace{20 \qquad 27}_{\text{2 numbers}}$$

An even number of scores or numbers has no middle number. In this case, to find the median

- Add the two middle scores
- Divide by 2

Example 2:

$$\frac{6+12}{2} = \frac{18}{2} = 9 \quad \blacktriangleleft \text{ median}$$

$$\underbrace{1 \qquad 3 \qquad 6}_{\text{3 numbers}} \qquad \underbrace{12 \qquad 20 \qquad 28}_{\text{3 numbers}}$$

The numbers above are listed from smallest to greatest. It would be just as easy to find the median if they were listed in the opposite order. (But if they were not in either order, it would be difficult to find the median.)

Example 3:

Given these numbers, ▶	15	12	7	16	2	1	7	3	9
list them in this order ▶	1	2	3	7	$\boxed{7}$	9	12	15	16
or this order ▶	16	15	12	9	$\boxed{7}$	7	3	2	1
to find the median. ▶					median				

1–6. True or False?

1. The median of a set of numbers may be one of the numbers. T
2. The median of a set of numbers must be one of the numbers. F
3. The median of 1, 2, 3, 4, 5, 6, 7 is 4. T
4. The median of 1, 2, 3, 5, 6, 7 is 8. F
5. The median of 1, 4, 4, 4, 5, 6, 7 is 4. T
6. The median of 3, 3, 3, 5, 5, 5, is 4. T

7–20. Find the median.

7. 52, 66, 74, 82, 97 74
8. 46, 58, 62, 76, 85 62
9. 30, 121, 7, 12, 59, 3, 20 20
10. 91, 32, 49, 6, 11, 73, 18 32
11. 7, 11, 18, 26, 30, 71 22
12. 3, 19, 20, 26, 32, 47 23
13. 19, 8, 36, 25, 50, 14 22
14. 23, 12, 1, 27, 23, 30 23
15. 1.1, 4.9, 6.2, 7.4, 9.6, 11.5, 16.2, 23.1, 27.7, 27.9, 30.2 11.5
16. 3.2, 5.7, 7.1, 9.5, 9.7, 10.6, 13.9, 20.0, 22.3, 30.7, 43.2 10.6
17. $75.12, $91.80, $316.05, $224.06, $95.73 $95.73
18. $102.50, $107.30, $210.12, $97.80, $43.10 $102.50
19. 7852, 5324, 658, 1840, 3526, 2044, 1746 2044
20. 1324, 1752, 1206, 944, 1258, 652, 1834 1258

21–22. Thirteen teams of the Lucky League have these records.

Team	A	B	C	D	E	F	G	H	I	J	K	L	M
Games won	0	2	3	1	4	1	2	5	3	1	3	4	3
Games lost	5	3	2	4	1	4	3	0	2	4	2	1	2

21. What is the median number of games won? 3
22. What is the median number of games lost? 2

MEDIAN INCOMES OF YEAR—ROUND, FULL—TIME WORKERS 25—29 years old 1971 figures

Education completed	Male	Female
Less than 9 years	\$5247	\$2368
1—3 years of high school	6556	3279
4 years of high school	7335	4264
1—3 years of college	6884	5064
4 years of college or more	8052	6301

23—28. What was the median income in the group described?

23. Women, less than 9 years education 2368

24. Men high-school graduates 7335

25. Women high-school graduates 4264

26. Women college graduates 6301

27. Men college graduates 8052

28. Men less than 9 years education 5247

29. Median income for men with one to three years of high school was \$779 less than for male high-school graduates.

30. Median income for women high-school graduates was \$985 more than for women with one to three years of high school.

31. Median income for male high-school graduates was \$1034 more than for female college graduates.

32. Median income for female high-school graduates was \$3071 less than for male high-school graduates.

Quick Quiz

Check this skill:

- *dividing whole numbers by whole numbers*

1–8. Divide.

1. $\overset{13}{12\overline{)156}}$ **2.** $\overset{13}{17\overline{)221}}$ **3.** $1012 \div 11$ 92 **4.** $\frac{189}{9}$ 21

5. $14\overline{)179}$ **6.** $21\overline{)326}$ **7.** $452 \div 8$ **8.** $\frac{356}{9}$

12R11 or $12\frac{11}{14}$ 15R11 or $15\frac{11}{21}$ 56R4 or $56\frac{1}{2}$ or 56.5

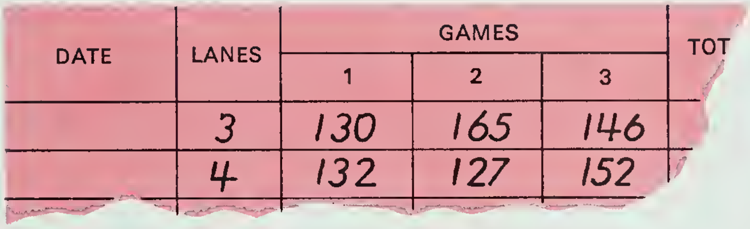
8. 39R5 or $39\frac{5}{9}$ or 3

Performance Objective:
The student can find the mean
of a set of whole numbers.

What Does MEAN Mean?

1.8

Averages are important to bowlers. The higher a bowler's average, the better the bowler. (A "perfect" bowler would average 300!) One bowler's six-game record looks like this.



DATE	LANES	GAMES			TOT
		1	2	3	
	3	130	165	146	
	4	132	127	152	

A. Devaney, Inc.

130
165
146
132
127
152
852

What is this bowler's average?

852 ÷ 6 = 142

Add the numbers.

Divide by how many numbers there are.

Bowler's average

Because the *mode* and the *median* are also sometimes called averages, the average found above is called the **mean**. (Most "averages" in athletics are means.)

Examples:

1. What is the mean number of letters in the words of this question?

4 2 3 4 6 2 7 2 3 5 2 4 8 ◀ letters per word

Add to find the total number of letters. → 52

Count the number of words. → 13

52 ÷ 13 = 4 ◀ mean number of letters per word

2. Find the mean. 3, 4, 7, 16, 23

Add. 3 + 4 + 7 + 16 + 23 = 53

Count. 1 2 3 4 5

$$\begin{array}{r} 10\frac{3}{5} \\ 5\overline{)53} \end{array}$$

◀ mean

Exercises

1. To find the mean of 2, 4, 6, and 8, add the numbers and divide by __4__ .

2. The mean of 2, 4, 6, and 8 is __5__ .

3–16. Find the mean.

3. 7, 3, 2, 4 4

4. 5, 3, 8, 8 6

5. 12, 17, 8, 5, 23 13

6. 16, 13, 9, 8, 29 15

7. 5231, 7000, 1293, 304 3457

8. 6000, 4377, 526, 2597 3375

9. 62, 7, 23, 12, 21, 1 21

10. 74, 18, 3, 31, 7, 41 29

11. 28, 39 $33\frac{1}{2}$

12. 16, 41 $28\frac{1}{2}$

13. 17, 32, 4, 12, 19, 75, 42 $28\frac{5}{7}$

14. 15, 6, 52, 12, 19, 31, 4 $19\frac{6}{7}$

15. 320, 401, 624, 218 $390\frac{3}{4}$

16. 603, 128, 434, 300 $366\frac{1}{4}$

17. The mean of 15 equal scores is 75. What is each score? 75

18. The mean of 10 equal scores is 37. What is each score? 37

19. Jim Parito is a construction worker. His income depends on the weather. The first six months of last year he earned $720, $842, $1212, $1675, $2900, and $3757. What was his mean monthly income? $1851

20. Joyce Sanford is a commissioned salesperson. Her income varies from week to week. In six weeks she earned $410, $580, $640, $620, $530, and $490. What was her mean weekly income? $545

21. Marie Jacobs bowled 34 games, getting a total of 4216 pins. Sally Jacobs bowled 12 games, getting a total of 1608 pins.

a. Whose average was higher? Sally Jacobs

b. What is the difference between the averages? 10

22. Nate Archibald scored 2720 points in 80 games. Kareem Abdul-Jabbar scored 2280 points in 76 games that same year.

a. Whose average was higher? Nate Archibald

b. What is the difference between the averages? 4

23–26. A "traffic counter" counts the number of cars and trucks passing a certain point over a period of time. The count is used to plan such things as road improvements, and traffic-light and warning-sign placement. Find each mean.

	TIME PERIOD	NUMBER OF VEHICLES COUNTED	NEED THIS MEAN	
23.	8 hours	21-52-120-64-27-19-42-23	vehicles per hour	46
24.	7 hours	31-35-58-62-36-24-41	vehicles per hour	41
25.	12 days	122-130-116-127-156-38-21-128-132-110-125-163	vehicles per day	114
26.	14 days	134-159-201-52-36-162-174-128-161-195-68-42-170-166	vehicles per day	132

27–32. Many teachers give students grades or scores by finding the mean (or "average") of each student's test scores or assignment scores. Find the mean of each student's scores.

	STUDENT	SCORES	
27.	Sue	82, 94, 86, 79, 83, 92	86
28.	Joe	93, 87, 83, 90, 84, 85	87
29.	Shawn	83, 76, 72, 80, 74, 68, 75, 80	76
30.	Juanita	73, 83, 77, 79, 73, 69, 78, 84	77
31.	Igor	91, 83, 87, 79, 81, 85, 82, 80	$83\frac{1}{2}$
32.	Chung	84, 92, 87, 83, 79, 88, 93, 86	$86\frac{1}{2}$

33–38. Some typical sports statistics are given below. In each case, find the average (mean) called for.

33. Baseball: pitcher's earned-run average 3

game	1	2	3	4	5	6
runs given up	4	3	0	4	3	4

34. Hockey: goalie's goals-against average 4

game	1	2	3	4	5	6
goals against	4	0	5	5	4	6

35. Soccer: goalie's goals-against average 3

game	1	2	3	4	5	6	7
goals against	3	5	2	5	0	2	4

36. Football: player's metres-per-carry average 6

carry	1	2	3	4	5	6	7
metres	3	1	21	6	0	9	2

37. Basketball: player's points-per-game average 20

game	1	2	3	4	5	6
points scored	12	20	32	24	18	14

38. Football: team's punt-return average 28

punt return	1	2	3	4	5	6	7
metres returned	33	24	41	25	36	12	25

Quick Quiz

Check these skills:

- *subtracting whole numbers and decimals*
- *multiplying whole numbers*

1–6. Subtract.

1.
$$\begin{array}{r} 658 \\ -236 \\ \hline 422 \end{array}$$

2.
$$\begin{array}{r} 371 \\ -158 \\ \hline 213 \end{array}$$

3.
$$\begin{array}{r} 423 \\ -379 \\ \hline 44 \end{array}$$

4.
$$\begin{array}{r} 79.5 \\ -31.2 \\ \hline 48.3 \end{array}$$

5.
$$\begin{array}{r} 84.1 \\ -53.6 \\ \hline 30.5 \end{array}$$

6.
$$\begin{array}{r} 45.42 \\ -\ \ 3.59 \\ \hline 41.83 \end{array}$$

7–12. Multiply.

7. 21 × 7 147

8. 9 × 36 324

9. 43 × 12 516

10. 23 × 37 851

11. 271 × 6 1626

12. 534 × 16 8544

Performance Objective: The student can find the range of a set of numbers and express it in either of two ways.

The Range

1.9

The numbers on the cards are the scores awarded in one of the Olympic competitions. These scores have a **range** of

5.3 – 5.5 or **.2**

Name the high and the low scores. or Give the difference between the high and the low scores.

David Leonardi

The *range* of a set of scores tells how much the scores are spread out. The *mean*, *median*, and *mode* give an idea of a typical score. The four numbers together give a fairly good picture of a set of numbers.

Exercises

1. To find the range of a set of numbers, you need to know
a. any two of the numbers. **b.** the highest number.
c. the highest and lowest numbers. **d.** the mean.

2. The range tells
a. the most frequent score. **b.** the spread of the scores.
c. a typical score. **d.** the number of scores.

3–6. Give the range of each set of numbers in two ways.

3. 1, 5, 5, 7, 8, 9, 9, 13, 15
1–15; 14

4. 2, 5, 7, 8, 15, 15, 18
2–18; 16

5. 37, 27, 26, 25, 23, 19, 18
18–37; 19

6. 23, 22, 22, 21, 16, 14, 8
8–23; 15

7–20. Find the range of each set of numbers in Exercises **7–20** on page 35. Give each range in two ways.
Answers at right.

7. 52–97; 45
8. 46–85; 39
9. 3–121; 118
10. 6–91; 85
11. 7–71; 64
12. 3–47; 44
13. 8–50; 42
14. 1–30; 29
15. 1.1–30.2; 29.1
16. 3.2–43.2; 40
17. \$75.12–\$316.05; \$240.93
18. \$43.10–\$210.12; \$167.02
19. 658–7852; 7194
20. 652–1834; 1182

Trimming the Range

In competition, one very high or very low score may unfairly help or spoil a person's chances of winning. So the highest and lowest scores may be "trimmed off," leaving a *trimmed range*. Find the mean in Exercises **3–6**, above. Then trim the range, and find the mean again. Does trimming ever change the mean? Does it always change the mean? Yes; no

3. 8; 8
4. 10; 10
5. 25; 24
6. 18; 19

1.10 Frequency Tables

Performance Objective: The student can
- make a frequency table for a set of numbers.
- use such a table to find mode, median, mean, and rang

EXCLUSIVE LANE
3 PERSONS
MINIMUM
PER VEHICLE
3-7 PM
MON-FRI

Many cities urge workers to save gasoline and parking space by using car pools to get to work. As a check, one company recorded how many people arrived in each car on one working day. Here is the record.

2 4 2 3 5 1 3 1 1 6 2 4 1
2 1 7 4 1 5 2 3 2 1 4 2 5
2 3 6 1 5 2 4 3 6 4 1 4 2

One way to find the range, mode, median, and mean of such a list of numbers is to make a *frequency table* showing how many times each number occurs.

Record each number in order of size.

Count how many times each number occurs.

Number of people	Number of cars
1	9
2	10
3	5
4	7
5	4
6	3
7	1

greatest frequency

mode ▶

Range is 1–7 or 6.

19 cars

1

4

So the **median** number of people per car is 3.

19 cars

To find the mean, you can add all the numbers in the list and divide by 39. Or you can find the mean by using the table.

Number of people	Number of cars	Number of people in these cars
1	9	1 × 9 = 9
2	10	2 × 10 = 20
3	5	3 × 5 = 15
4	7	4 × 7 = 28
5	4	5 × 4 = 20
6	3	6 × 3 = 18
7	1	7 × 1 = 7
Totals:	39	117

$\frac{117}{39} = 3$, so the **mean** number of people per car is 3.

Exercises

1. A ______ shows how many times each number is listed. Frequency table

2. The mode is the number that occurs with the (greatest, least) frequency.

3. The middle number in the list on page 42 is 3 when the numbers are arranged according to __Size__ .

4. The sum of the numbers in the list on page 42 is __117__ .

5. If 35 numbers are listed in order of size, which number is the median, or middle number? 18th

6. If 40 numbers are listed in order of size, the median is the mean of the __20th__ and __21st__ numbers.

7–10. Find the range, the mode, the median, and the mean.
Answers below.

7. Gasoline Sales, September

litres	days
2500	3
2700	4
2900	3
3600	5
4100	8
4500	4
4530	1

8. Fish Catch, Angler's Club

mass of catch (kg)	number of members
12	22
16	9
20	4
24	1
28	2
32	1
36	0
40	1

9. Household Water Use

litres	days
1000	4
1050	3
1200	6
1250	5
1350	4
1400	4
2700	5

10. Sampling of 450 g Cans of Pear Halves

number halves	number cans
6	9
7	22
8	37
9	24
10	8

	range	mode	median	mean
7.	1800-4530 or 2750	4100	3600	3531
9.	1000-2700 or 1700	1200	1250	1470

	range	mode	median	mean
8.	12-40 or 28	12	12	16
10.	6-10 or 4	8	8	8

11. size	no.
4	2
5	5
6	12
7	14
8	3
9	3
10	2
11	2
12	1

12. brothers, sisters	no.
0	8
1	11
2	12
3	6
4	1
5	1
7	2
8	1

13. score	no.
50	1
55	1
65	2
70	4
75	7
80	5
85	7
90	4
95	3
100	2

14. price	no.
58	3
60	3
62	1
64	1
66	4
68	8
70	3
72	8
74	3
76	2

15. depth	no.
46	2
48	5
52	2
54	1
56	4
57	1
58	2
60	3
62	6
64	2
66	2
68	1

16. temp.	no.
18	1
20	4
21	2
22	5
23	4
24	2
25	2
26	3
27	1
28	1
29	3
30	2

	mode(s)	median	mean
17.	7	7	7
18.	2	2	2
19.	75, 85	80	80
20.	68, 72	68	68
21.	62	58	57
22.	22	23	24

11–16. Make a frequency table for each set of numbers.

11. Shoe sizes of 44 class members: 7, 6, 5, 7, 12, 9, 7, 6, 8, 6, 4, 5, 6, 7, 7, 6, 7, 5, 9, 7, 6, 8, 6, 8, 7, 6, 7, 5, 7, 9, 6, 7, 5, 7, 6, 4, 11, 7, 6, 10, 11, 7, 6, 10

12. Number of brothers and sisters, 42 class members: 3, 1, 0, 8, 1, 2, 0, 4, 1, 1, 2, 7, 0, 2, 1, 2, 3, 0, 1, 2, 1, 2, 2, 3, 0, 2, 3, 1, 2, 0, 7, 2, 0, 1, 3, 2, 1, 5, 2, 1, 3, 0

13. Driver's Education test scores: 75, 85, 90, 70, 80, 65, 85, 75, 95, 55, 80, 85, 70, 90, 75, 85, 80, 70, 95, 75, 100, 70, 85, 90, 50, 95, 75, 85, 100, 75, 80, 65, 80, 90, 85, 75

14. Weekly price per 100 g of frying chicken (in cents): 72, 72, 68, 72, 72, 68, 68, 66, 68, 66, 60, 58, 58, 58, 60, 66, 62, 60, 64, 66, 68, 68, 70, 72, 74, 76, 74, 76, 74, 72, 72, 70, 68, 68, 70, 72

15. Stream depth (in centimetres), Shallow River, September 1978: 64, 66, 62, 68, 66, 62, 60, 62, 64, 60, 58, 52, 48, 46, 48, 46, 48, 52, 56, 60, 62, 62, 62, 57, 56, 56, 58, 56, 54, 48, 48

16. Daily high temperatures (in degrees Celsius), Inland City, June, 1976: 18, 20, 22, 20, 22, 24, 25, 23, 24, 23, 21, 22, 23, 21, 22, 20, 20, 22, 23, 26, 26, 27, 29, 30, 30, 29, 28, 29, 25, 26

17–22. Using the frequency table for the exercise named, find the range, the mode, the median, and the mean. *Ranges below*

17. Exercise 11 4–12 or 8 **18.** Exercise 12 0–8 or 8 **19.** Exercise 13 50–100 or 50

20. Exercise 14 58–76 or 18 **21.** Exercise 15 46–68 or 22 **22.** Exercise 16 18–30 or 12

Quick Quiz

Check these skills:

- *adding decimals*
- *dividing decimals by whole numbers*

1–3. Add.

1. 3.45 + 7.21 + 4.5 + 2.36 + 5.72 23.24

2. $\begin{array}{r}3.61\\+4.27\\\hline 7.88\end{array}$

3. 6.8 + 9.04 + 3.26 + 50.07 69.17

4–7. Divide.

4. $8\overline{)\overset{9.6}{76.8}}$ **5.** $7\overline{)\overset{7.55}{52.85}}$ **6.** $11\overline{)\overset{13.52}{148.72}}$ **7.** $12\overline{)\overset{79.38}{952.56}}$

TAKING THE CENSUS

Do you know what these words mean?

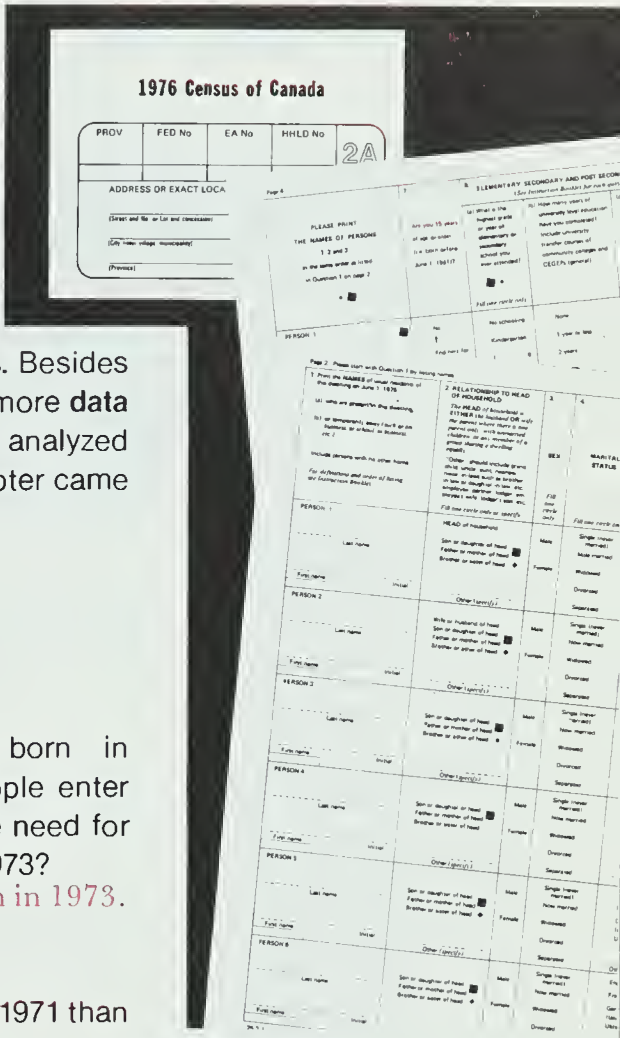
poll *census* *data* *statistics*

poll: a survey to get facts (or opinions)

census: a count of the people

data: facts

statistics: facts involving numbers



1976 Census of Canada

PROV | FED No | EA No | HHLD No | 2A

ADDRESS OR EXACT LOCA

Robert Brown

Every ten years in this country, the government takes a **census**. Besides just counting the people, the government also takes a **poll** to get more **data** on housing, education, and many other things. These **statistics** are analyzed and put into tables and graphs. Many of the facts used in this chapter came from Statistics Canada.

Here are some questions the census can help to answer.

EDUCATION: In 1961, there were about 475 700 babies born in Canada. In 1971, about 362 200 babies were born. Most people enter junior high school about twelve years after birth. How will the need for junior-high-school teachers in 1983 compare with the need in 1973? Need will be less in 1983 than in 1973.

• • •

HOUSING: There were 556 000 more people in British Columbia in 1971 than in 1961. How would this change housing needs? Needs would increase.

• • •

SERVICES: In 1971, Saskatchewan had about twice as many people living in the country as in its cities. However, Manitoba had about two times as many people in its cities as in the country. Which province must be ready to provide more help and information to farmers and small-town businesses? Saskatchewan

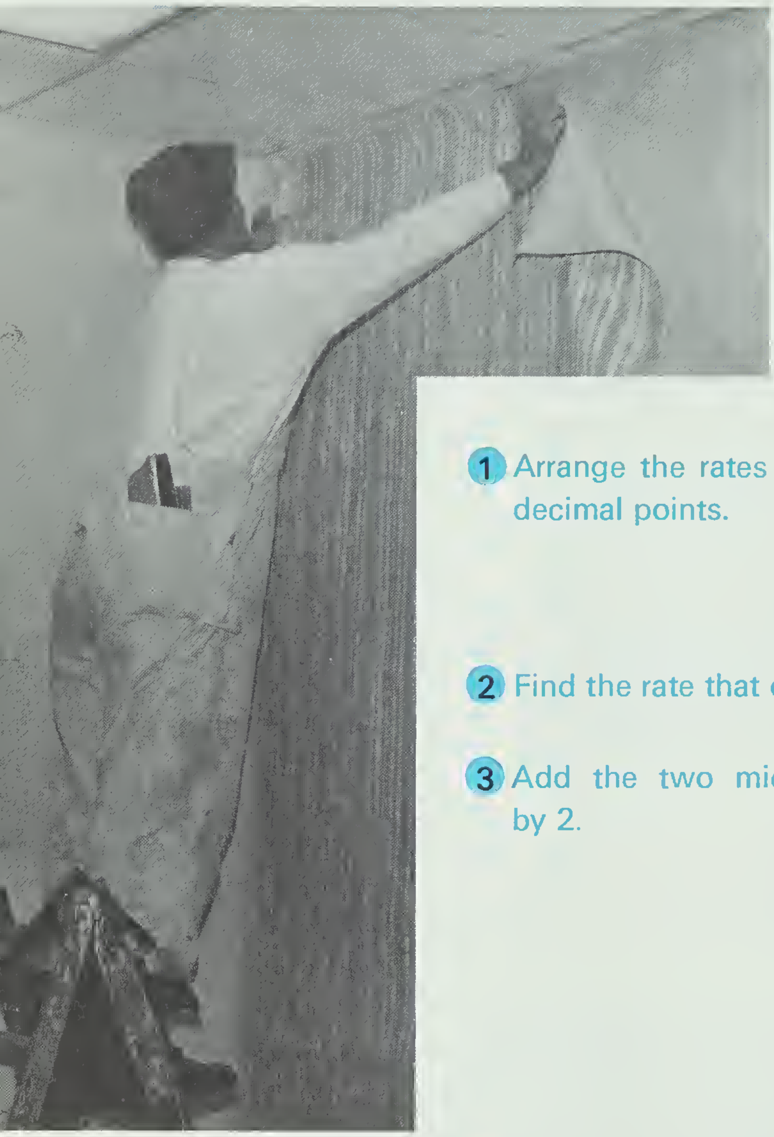
• • •

HEALTH CARE: In 1974, Ontario had about 8 000 000 people. There were about 14 000 doctors in Ontario. Prince Edward Island, which had 117 000 people at that time, had 113 doctors. Which province had better health care in terms of having fewer people per doctor? Ontario

1.11 More About Averages

Performance Objective: The student can find the mode, median, and mean of a set of decimals.

If the table shows the minimum hourly wage rate for paperhangers in twelve cities, how can you find the mode, median, and mean of the hourly rates?



Means/Van Cleve

Ottawa	$7.30	St. John's	$5.38
Vancouver	8.28	Toronto	7.14
Sudbury	7.93	Charlottetown	4.65
Victoria	8.70	Halifax	6.02
Calgary	7.95	Edmonton	7.93
Saint John	6.36	Montreal	6.78

① Arrange the rates in order of size. Align decimal points.

② Find the rate that occurs most often.

$$\begin{array}{r} 8.70\\ 8.28\\ 7.95\\ \left.\begin{array}{r}7.93\\ 7.93\end{array}\right] \Leftarrow \text{mode}\\ \left.\begin{array}{r}7.30\\ 7.14\end{array}\right]\phantom{\Leftarrow \text{mode}}\\ 6.78\\ 6.36\\ 6.02\\ 5.38\\ \underline{4.65}\\ 84.42 \end{array}$$

③ Add the two middle rates and divide by 2.

$$\begin{array}{r} 7.30\\ \underline{7.14}\\ 14.44 \end{array} \qquad \begin{array}{r} \text{median}\\ \Downarrow\\ 7.22\\ 2\overline{)14.44} \end{array}$$

④ Add all the rates and divide by 12. [Carry the division out only far enough to give a sensible answer to the problem. In this case, either 7.035 or 7.04 will do.]

$$\begin{array}{r} \text{mean}\\ \Downarrow\\ 7.035\\ 12\overline{)84.420}\\ \underline{84}\\ 42\\ \underline{36}\\ 60\\ \underline{60}\\ 0 \end{array}$$

Exercises

1–6. Refer to the example on page 46.

1. The most common rate is $7.93 .

2. In half the cities, the minimum rate is $7.22 or more.

3. In half the cities, the minimum rate is $7.22 or less.

4. If there were only eleven cities listed, how would you find the median? Find the 6th rate.

5. If there were only eleven cities listed, how would you find the mean? Add rates; divide by 11.

6. In finding the mean of any list of numbers, how far should the division be carried out? Far enough to give a sensible answer

7–14. Find the mode, median, and mean.
Ex. 7–14: See table at right.

7. 3.2, 7.1, 5.9, 12.2, 6.5, 4.7, 9.5, 5.9, 7.1

8. 6.7, 5.1, 10.9, 3.5, 8.8, 5.1, 12.6, 8.8, 4.2

9. 2.41, 3.45, 6.42, 4.91, 9.28, 1.07, 3.45, 5.89

10. 4.32, 8.95, 2.36, 5.80, 3.55, 6.56, 8.95, 7.91

11. 12.65, 9.41, 6.57, 12.65, 13.81, 15.23

12. 18.47, 20.16, 15.61, 9.85, 13.90, 15.61

13. 132.81, 164.26, 153.79, 121.21, 146.13, 121.21, 170.2

14. 181.43, 179.2, 160.02, 177.2, 186.3, 160.02, 190

	mode(s)	median	mean
7.	5.9, 7.1	6.5	6.9
8.	5.1, 8.8	6.7	7.3
9.	3.45	4.18	4.61
10.	8.95	6.18	6.05
11.	12.65	12.65	11.72
12.	15.61	15.61	15.60
13.	121.21	146.13	144.23
14.	160.02	179.2	176.31

15. Bulldozer operators in ten cities work at the following hourly rates (in dollars). Find the median rate and the mean rate.
6.97, 8.59, 8.48, 5.65, 9.08, 6.65, 7.81, 8.02, 4.70, 6.55
Median: 7.39, mean: 7.25

16. In the last ten months, one supermarket has sold lean hamburger at the following prices per 150 g. Find the median price and the mean price.
1.19, 1.29, 1.21, 1.13, 1.09, 0.98, 0.98, 1.05, 1.09, 1.19
Median: 1.11, mean: 1.12

17–20. The normal precipitation (in millimetres) is given for each month. Find the mean for the year.

	17. 139.9	**18.** 124.4	**19.** 90.3	**20.** 109.8
	Saint John	Halifax	Montreal	Moncton
Jan.	205.7	164.1	100.1	141.5
Feb.	145.1	120.4	108.5	117.6
Mar.	96.5	90.7	89.2	103.4
Apr.	99.5	66.8	81.5	69.3
May	192.4	142.2	119.4	89.2
June	75.4	59.2	84.6	97.8
July	170.8	131.8	104.4	149.9
Aug.	122.8	76.7	80.0	85.6
Sept.	100.6	122.7	109.0	78.7
Oct.	213.5	178.6	115.1	138.4
Nov.	69.8	99.8	43.7	78.0
Dec.	187.0	239.8	47.7	167.6

21–24. The mean daily temperature (in degrees Celsius) is given for each month. Find the mean for the year.

	21. 4.7	**22.** 6.3	**23.** 5.0	**24.** 4.8
	Saint John	Halifax	Montreal	Moncton
Jan.	−9.9	−6.2	−14.7	−10.8
Feb.	−5.4	−4.7	−7.1	−7.0
Mar.	−2.6	−1.1	−3.1	−2.9
Apr.	4.3	5.4	7.6	4.2
May	9.0	10.4	12.2	11.5
June	14.8	17.6	19.1	16.4
July	16.6	17.9	18.8	17.6
Aug.	17.5	18.2	18.4	17.5
Sept.	12.6	14.8	14.7	13.9
Oct.	6.8	8.0	6.5	7.3
Nov.	0.4	1.3	−0.9	−0.8
Dec.	−7.3	−5.8	−11.3	−9.6

WHAT'S AVERAGE?

Have you seen ads like this one? What does it mean by average income? Look at the examples below. In each of them, at least one of the averages is $10!

EARN TOP PAY!

Average income of our graduates is $10 an hour. Employment after graduation guaranteed. Write for our catalog today.

A

	HOURLY INCOME	NUMBER OF PEOPLE
	$13	6
	12	7
mode median ▶	10	22
	8	6
	5	4

mean: $\frac{450}{45} = \$10$

B

	HOURLY INCOME	NUMBER OF PEOPLE
	$50	2
	26	3
	13	8
	11	8
median ▶	6	8
mode ▶	2	16

mean: $\frac{450}{45} = \$10$

C

	HOURLY INCOME	NUMBER OF PEOPLE
	$15	3
	14	6
	13	7
median ▶	12	7
mode ▶	10	8
	2	7
	1	7

mean: $\frac{405}{45} = \$9$

Which of the the three examples illustrates each of these statements?

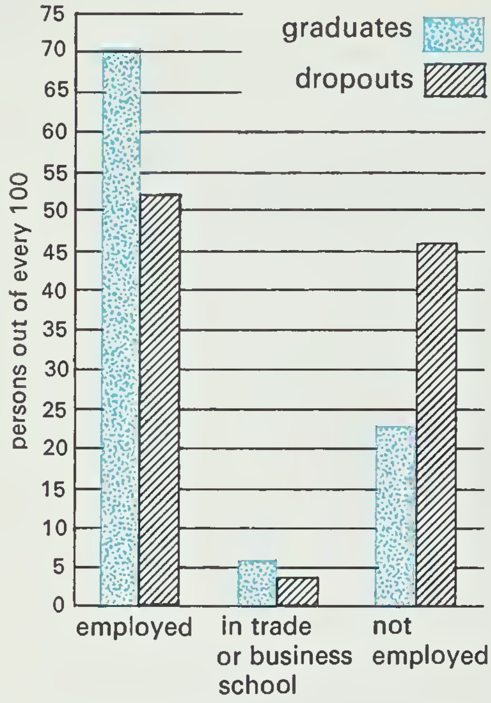
The mean, median, and mode may be equal. A

The mode need not occur much more often than other amounts. C

When the *median* is *greater than* the *mean*, more than half the people earn above the mean. But there must be some low earnings to balance the great number of high ones. C

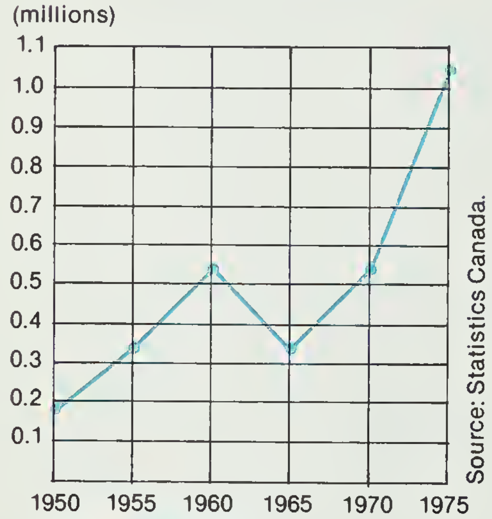
When the *median* is *less than* the *mean*, more than half the people earn below the mean. But there must be some high earnings to balance the great number of low ones. B

October, 1973, employment of 16–24 year olds leaving high school in 1972–73 (not enrolled in college)



Source: Statistics Canada

UNEMPLOYMENT INSURANCE COMMISSION CLAIMANTS



Source: Statistics Canada.

MOTORCYCLE SPECIFICATIONS					
model	gears	wheel-base (cm)	mass (kg)	approximate km/h	price
1	1	80	40	56	$254
2	4	97	45	86	349
3	4	97	43	86	329
4	1	94	41	61	299
5	4	112	44	74	369

Refer to the bar graph. (1.1)

1. About half the ______ are employed. Dropouts

2. About twice as many ______ as ______ are unemployed. Dropouts; graduates

From the line graph, in which five-year period did the number of claimants (1.2)

3. decrease? 1960-1965

4. increase most rapidly? 1970-1975

From the table, which motorcycle model (1.3)

5. has a 97 cm wheelbase and the least mass? 3

6. has 4 gears and the lowest speed? 5

7. Which graph has a percent scale? (1.4) Bar graph

8. If the bar graph were a pictograph and each picture stood for 10 people, how many whole pictures would there be for employed graduates? 7 (1.5)

Use this set of numbers for Exercises **9–13**.

3, 4, 4, 5, 6, 8, 12, 12, 12, 14

9. Find the mode. (1.6) 12

10. Find the median. (1.7) 7

11. Find the mean. (1.8) 8

12. Find the range. (1.9) 3–14 or 11

13. Make a frequency table. *See table at right.* (1.10)

no.	freq
3	1
4	2
5	1
6	1
8	1
12	3
14	1

14. Find the range, the mode, the median, and the mean.
3.27, 5.43, 6.95, 2.73, 3.27, 6.95, 4.21, 3.27 (1.11)
Range: 2.73–6.95 or 4.22; mode: 3.27; median: 3.74; mean: 4.51

RATE YOURSELF

Number Correct	
14	Superior
12–13	Excellent
10–11	Good
8–9	Fair
0–7	Poor

Chapter 1 Review

1–4. Refer to the bar graph. (1.1)

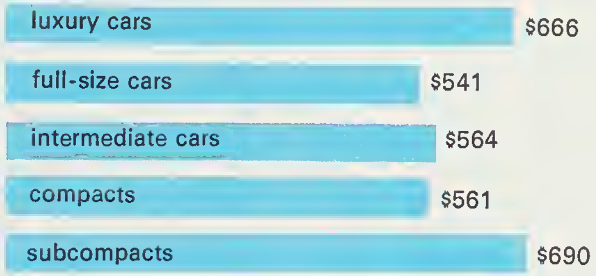
1. For which type of car were average insurance claims the highest? Subcompacts

2. For which type of car were average insurance claims the lowest? Full-size

3. How much higher were average claims for intermediate cars than for compacts? $3

4. How much lower were average claims for luxury cars than for subcompacts? $24

AVERAGE INSURANCE CLAIMS
1975 model cars



Source: Insurance Institute for Highway Safety

5–8. Refer to the line graph. (1.2)

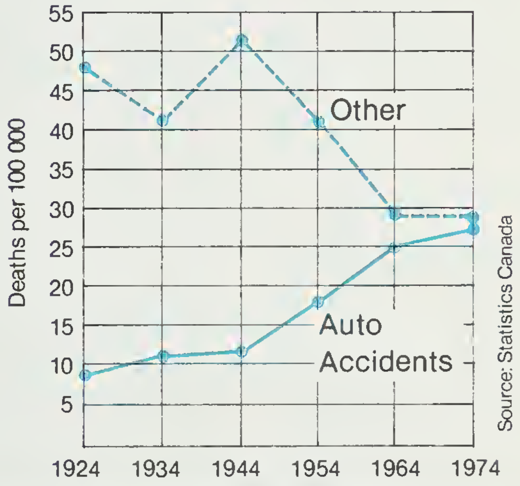
5. From 1924 to 1974, the trend in death rates for auto accidents was Up

6. The biggest overall increase in auto accidents deaths occurred in what ten-year period? 1954-1964

7. Out of every 100 000 people, about how many died in auto accidents in 1974? 28

8. In what year did about 41 people out of every 100 000 die in accidents other than auto accidents? 1954

ACCIDENTAL DEATH RATES
1924–1974



9–12. The table shows the chilling effect of wind. When there is a temperature of 10° C and a 32 km/h wind, your body loses heat as fast as on a calm day at 0° C. (1.3)

9. What is the effect of a 16 km/h wind when the thermometer reads -5°C? A body loses heat as fast as on a calm day at -14°C.

10. How strong a wind gives the effect of -10°C when the thermometer reads 5°C? 48 km/h

11. If the temperature is 0°C and the wind is 24 km/h, for what temperature must you dress? -12°C

12. At 10°C, a wind over 32 km/h gives the effect of below-zero temperatures.

WINDCHILL CHART

WIND SPEED in km/h	THERMOMETER READING °C					
	10	5	0	-5	-10	-15
	Equivalent temperature °C					
calm	10	5	0	-5	-10	-15
8	9	4	-2	-8	-13	-20
16	4	-1	-8	-14	-22	-30
24	2	-5	-12	-19	-27	-35
32	0	-7	-14	-20	-30	-37
40	-1	-8	-16	-24	-32	-40
48	-2	-10	-17	-26	-34	-42
56	-3	-11	-19	-27	-35	-44
64	-4	-12	-20	-28	-36	-45

13–15. Refer to the graphs on page 51. (1.4)

13. Do any of the graphs have a percent scale? No

14. If a line were added to the line graph to show death rates for all accidents, it would be (above, below) the line for "other" accidents.

15. Are auto and other accidents graphed using the same scale values? Yes

16–17. In a pictograph, the symbol [$500] stands for $500. How many symbols would be needed to stand for each amount? (1.5)

16. $3500 7

17. $250 $\frac{1}{2}$

18–19. If the bar graph were a pictograph in which one picture stood for $10, how many whole pictures would there be for

18. subcompacts? 69

19. full-size cars? 54 (1.5)

20–27. Use these sets of numbers to find each mode, median, mean, and range.

A: 4, 4, 7, 11, 15, 15, 17, 21, 21, 21, 26, 30

B: 6, 8, 8, 8, 10, 12, 13, 13, 13, 15, 15

20. the mode for set A 21

21. the mode for set B 8, 13 (1.6)

22. the median for set A 16

23. the median for set B 12 (1.7)

24. the mean for set A 16

25. the range for set B 11 (1.8)

26. the range for set A 4–30 or 26

27. the range for set B 6–15 or 9 (1.9)

28. Make a frequency table for set A above.

29. Make a frequency table for set B above. (1.10)

28–29: *See tables at left.*

28.

no.	freq.
4	2
7	1
11	1
15	2
17	1
21	3
26	1
30	1

29.

no.	freq.
6	1
8	3
10	1
12	1
13	3
15	2

30. Find the range, mode, median, and mean ages of this class. (1.10)

Age	12	13	14	15	16	17
Number of Students	4	7	12	8	2	1

	range	mode(s)	median	mean
30.	12–17 or 5	14	14	14

31–32. Find the range, mode, median, and mean. (1.11)

31. 15.31, 12.72, 10.16, 15.31, 9.72, 13.21, 10.16

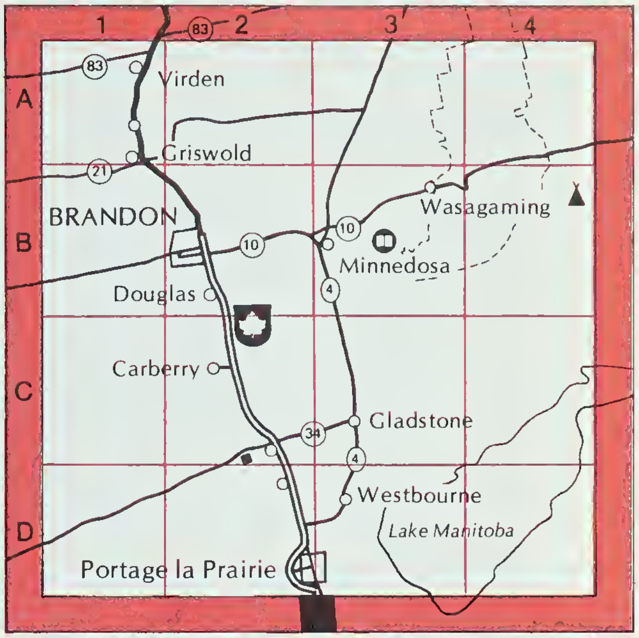
31. 9.72–15.31 or 5.59 10.16, 15.31 12.72 12.37

32. 7.6, 8.9, 3.2, 6.2, 4.7, 8.2, 7.5, 10.5, 6.2

32. 3.2–10.5 or 7.3 6.2 7.5 7.0

Preview Test for Chapter 2

Refer to the map. (2.1)



1
2
3
4
A
B
C
D
83
83
21
10
10
4
34
4
Virden
Griswold
BRANDON
Wasagaming
Minnedosa
Douglas
Carberry
Gladstone
Westbourne
Lake Manitoba
Portage la Prairie

1. In which region is Griswold located?
D-4

2. What town is located in region C-3?
Lomax

Graph and label each point.

3–5. *Graphs*

3. (0, 2), (6, 4), (2, 5), (4, 1) (2.2)

4. (2.3)

x	5	3	6	$4\frac{1}{2}$	7	2
y	0	7	6	4	5	$3\frac{1}{2}$

5. Make a line graph from this table. (2.4)

Hour	1	2	3	4	5	6	7	8
Degrees C	5	6	8	9	8	7	5	3

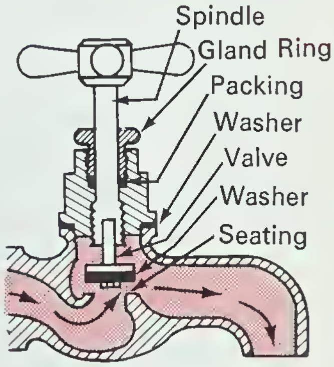
6. Graph the points; join them in order. (2.5) *Flower in pot*

x	$3\frac{1}{2}$	3	$2\frac{1}{2}$	2	$3\frac{1}{2}$	$2\frac{1}{2}$	2	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	5	4	$3\frac{1}{2}$	$3\frac{1}{2}$	2	. . .
y	8	7	7	$7\frac{1}{2}$	8	8	$8\frac{1}{2}$	9	8	9	$9\frac{1}{2}$	$8\frac{1}{2}$	8	8	$7\frac{1}{2}$	7	8	5	$6\frac{1}{2}$	

. . .	2	$3\frac{1}{2}$	$3\frac{1}{2}$	5	5	$3\frac{1}{2}$	$3\frac{1}{2}$	6	5	6	5	2	1	2	1	$3\frac{1}{2}$
	6	5	4	6	$5\frac{1}{2}$	4	3	3	1	1	0	0	1	1	3	3

7. The list gives 1973-74 part-time credit and non-credit continuing education enrollments in all courses in each type of institution (in thousands of students). Make a bar graph. *Graph* (2.6)

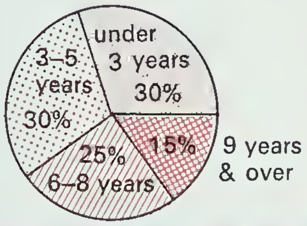
Total enrollment, 1300
School boards, 490
Correspondence courses, 67
Community colleges, 260
Universities, 483



Spindle
Gland Ring
Packing
Washer
Valve
Washer
Seating

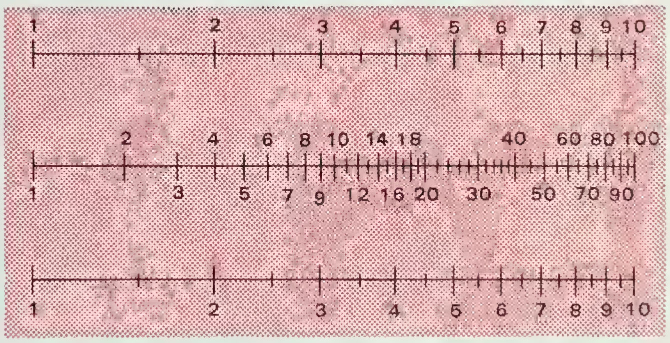
8. The circle graph shows that ___30___ out of every 100 cars and trucks in use are under 3 years old. (2.7)

Age of Cars and Trucks In Use



9. According to the schematic, which two parts are pressed together when the water faucet is turned off? (2.8)
Washer on valve, seating

10. What two arithmetic operations can be done using the nomograph? (2.9)
Multiplication, division



1 2 3 4 5 6 7 8 9 10
2 4 6 8 10 14 18 40 60 80 100
1 3 5 7 9 12 16 20 30 50 70 90
1 2 3 4 5 6 7 8 9 10

2 MAKING GRAPHS

Photo Research International

DRAFTING

PICTURE PREVIEW: Maps, graphs, and diagrams are used to show relationships in a simple and compact way. The pictures on these pages show some maps and diagrams in use. In this chapter you will read maps and some common kinds of diagrams. You will also make bar and line graphs from tables of information.

POLICE COMMUNICATIONS

Courtesy Chicago Police Department

Duryee/Monkmeyer Press Photo Service

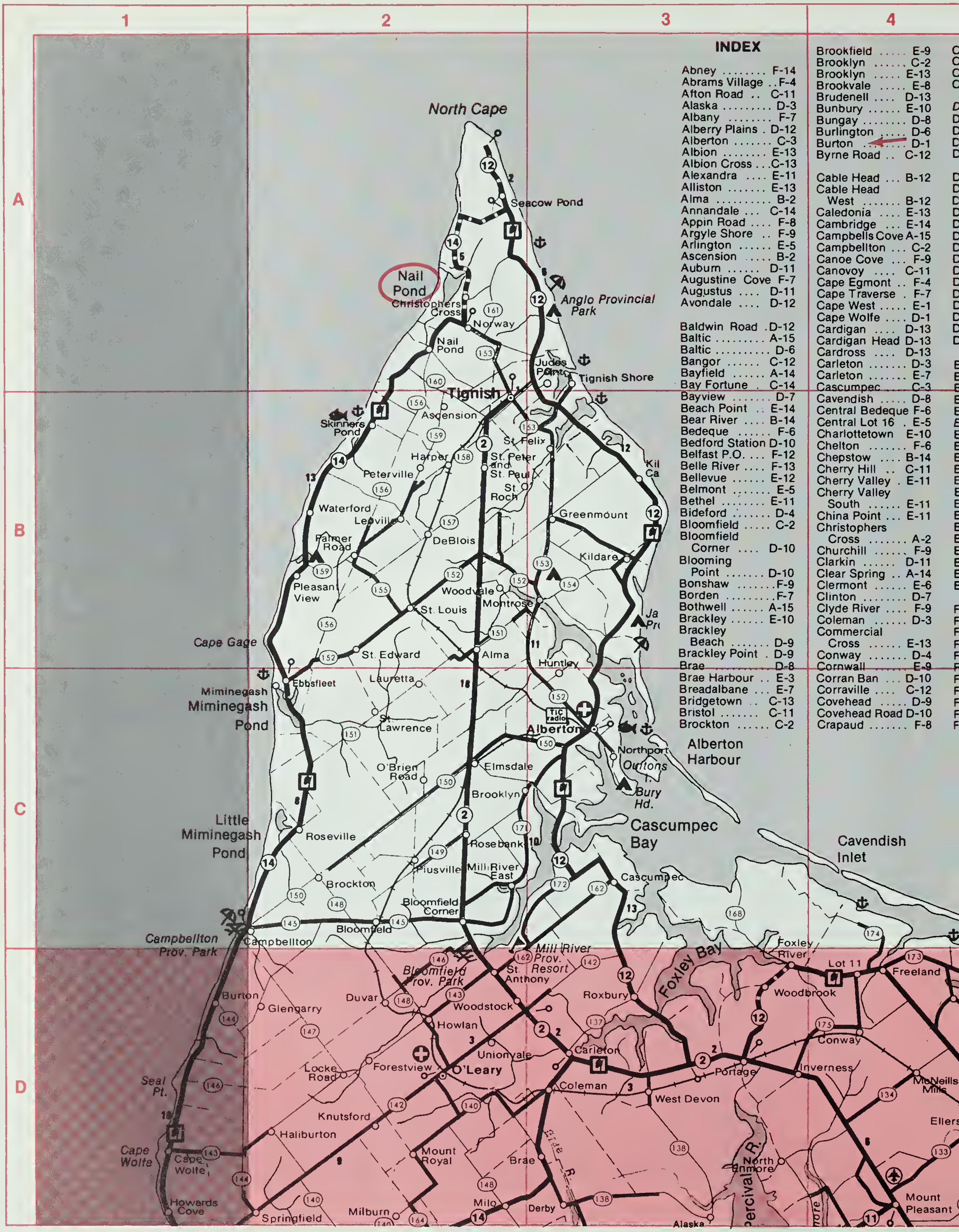
SKI PATROL

William Means/Van Cleve Photography

MACHINE OPERATING

CONSTRUCTION

Johnson/De Wys, Inc.



1
2
3
4
A
B
C
D
North Cape
Seacow Pond
Nail Pond
Christophers Cross
Norway
Anglo Provincial Park
Judes Point
Tignish Shore
Tignish
Skinners Pond
Ascension
St. Felix
Harper
St. Peter and St. Paul
St. Roch
Peterville
Waterford
Leoville
Greenmount
Palmer Road
DeBlois
Kildare
Pleasant View
Woodvale
Montrose
St. Louis
Cape Gage
St. Edward
Alma
Huntley
Miminegash
Miminegash Pond
Ebbsfleet
Lauretta
St. Lawrence
Alberton
Alberton Harbour
Northport
Ouftons
Bury Hd.
O'Brien Road
Elmsdale
Brooklyn
Little Miminegash Pond
Roseville
Rosebank
Cascumpec Bay
Cavendish Inlet
Piusville
Mill River East
Brockton
Cascumpec
Bloomfield Corner
Bloomfield
Campbellton
Campbellton Prov. Park
Mill River Prov. Resort
Bloomfield Prov. Park
St. Anthony
Foxley Bay
Foxley River
Lot 11
Freeland
Woodbrook
Burton
Glengarry
Duvar
Woodstock
Roxbury
Howlan
Conway
Unionvale
Carleton
Portage
Inverness
Locke Road
Forestview
O'Leary
Coleman
West Devon
McNeills Mills
Seal Pt.
Knutsford
Ellers
Haliburton
Cape Wolfe
Mount Royal
Brae
North Enmore
Howards Cove
Springfield
Milburn
Milo
Derby
Alaska
Percival R.
Mount Pleasant
Alberton R.

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Part of a map of the province of Prince Edward Island is shown on page 56. How could you find the village of Burton on this map?

Performance Objectives: The student can read and use coordinate names for regions on a map.

1st Look for Burton in the Index. (The index is alphabetical, like a dictionary.)
You should find Burton D-1

2nd Follow *across* the band labeled D and *down* the band labeled 1. The region where the bands cross is region D-1 on *this* map. It includes Burton, as well as other towns and villages. Search in this region to find Burton.

Near the top of the map, circled, is Nail Pond. How could you tell someone where to find the pond on this map? The pond is in the bands labeled A (at the left of the map) and 2 (at the top of the map). So the pond is in region A-2 on this map.

Exercises

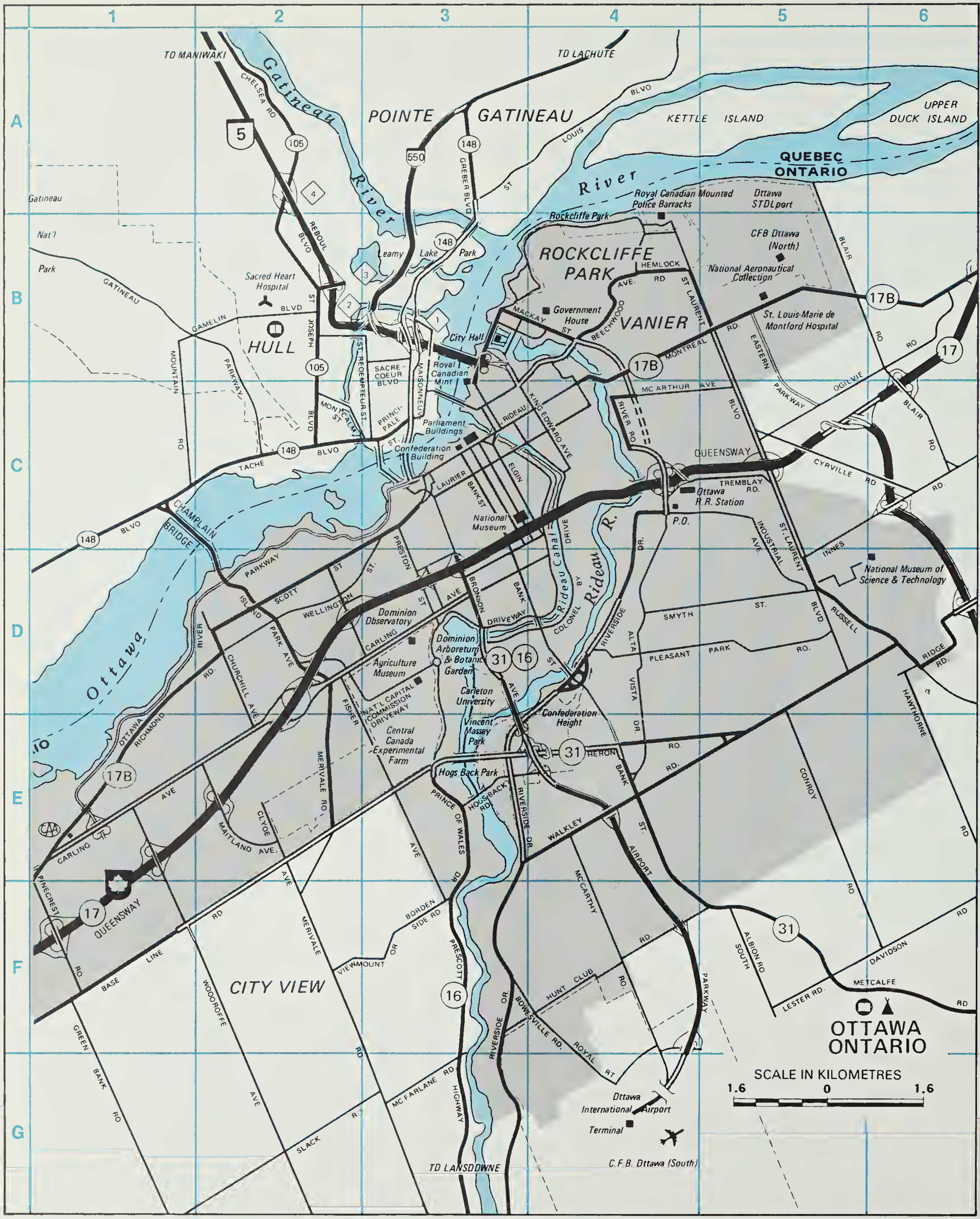
1–6. Refer to the map on page 56.

1. The name of each region is made up of a ____ and a ____. Letter, number
2. What other towns besides Burton are shown in region D-1? Cape Wolfe, Howards Cove
3. What towns would be labeled C-3 in the complete index? Huntley, Alberton, Northport, Cascumpec
4. How many *regions* are named B-4 on this map? One
5. How many names does any one region of the map have? One
6. Is there any reason why the region containing Burton should be labeled D-1 on other maps of Prince Edward Island? No

7–18. Locate each town on the map on page 56 by using the Index. Then tell in which region you found it.

7. Alaska D-3
8. Alma B-2
9. Carleton D-3
10. Conway D-4
11. Bloomfield C-2
12. Coleman D-3
13. Brockton C-2
14. Cape Wolfe D-1
15. Christopher's Cross A-2
16. Campbellton C-2
17. Ascension B-2
18. Brooklyn C-2

Courtesy of Department of Tourism, Parks and Conservation, Prince Edward Island


1
2
3
4
5
6
A
B
C
D
E
F
G
TO MANIWAKI
TO LACHUTE
CHELSEA RD
Gatineau River
5
105
POINTE GATINEAU
550
148
GREBER BLVD
BLVD
LOUIS ST
KETTLE ISLAND
UPPER DUCK ISLAND
QUEBEC
ONTARIO
River
Royal Canadian Mounted Police Barracks
Ottawa STOLport
Gatineau Nat'l Park
4
REBOUL ST
BLVD
Leamy Lake Park
3
2
1
Rockcliffe Park
ROCKCLIFFE PARK
HEMLOCK RD
CFB Ottawa (North)
National Aeronautical Collection
BLAIR
17B
GATINEAU
Sacred Heart Hospital
GAMELIN BLVD
JOSEPH
HULL
ST. REDEMPTEUR ST.
MAISONNEUVE
City Hall
MACKAY ST
Government House
BEECHWOOD AVE.
VANIER
ST. LAURENT
RD.
St. Louis-Marie de Montford Hospital
RO
17
MOUNTAIN RD
PARKWAY
105
SACRE-COEUR BLVD
Royal Canadian Mint
MONTREAL
17B
EASTERN PARKWAY
OGILVIE
MONTCALM ST
PRINCIPALE
BLVD
Parliament Buildings
RIDEAU
KING EDWARD AVE
MC ARTHUR AVE
BLVD
BLAIR RD
RIVER RD
148
TACHE BLVD
ST.
Confederation Building
QUEENSWAY
CYRVILLE RD
LAURIER
BANK ST
ELGIN
TREMBLAY RD.
Ottawa R.R. Station
P.O.
RD.
148
BLVD
CHAMPLAIN BRIDGE
National Museum
Rideau Canal
DRIVE
Rideau R.
DR.
INDUSTRIAL AVE.
ST. LAURENT
INNES
National Museum of Science & Technology
PRESTON ST.
PARKWAY
ST
SCOTT
WELLINGTON
AVE
BRONSON
BANK
COLONEL BY
RIVERSIDE
SMYTH ST.
BLVD.
RUSSELL
RIVER
ISLAND PARK AVE
Dominion Observatory
CARLING
DRIVEWAY
ALTA VISTA DR.
Dominion Arboretum & Botanic Garden
31
16
PLEASANT PARK RD.
RIDGE RD.
Ottawa
CHURCHILL AVE
RD.
Agriculture Museum
ST.
Carleton University
HAWTHORNE
OTTAWA
RICHMOND
FISHER
NAT'L CAPITAL COMMISSION DRIVEWAY
AVE.
Confederation Height
Vincent Massey Park
Central Canada Experimental Farm
31
HERON RD.
RD.
17B
MERIVALE RD
Hogs Back Park
BANK
CONROY
AVE
CLYDE
PRINCE OF WALES DR
HOGS BACK RD
RIVERSIDE DR.
ST.
MAITLAND AVE.
AVE
WALKLEY RD
AIRPORT
RD
CARLING
PINECREST RD
AVE
MC CARTHY RD.
17
QUEENSWAY
RD
MERIVALE
BORDEN SIDE RD
VIEWMOUNT DR
PRESCOTT
ALBION RD SOUTH
31
DAVIDSON RD
BASE LINE
CITY VIEW
WOODROFFE
16
HUNT CLUB RD.
PARKWAY
METCALFE
LESTER RD.
RD
GREEN BANK RD
AVE
RIVERSIDE DR
BOWESVILLE RD.
ROYAL RT
RD
OTTAWA
ONTARIO
SCALE IN KILOMETRES
1.6
0
1.6
MC FARLANE RD
HIGHWAY
Ottawa International Airport
Terminal
SLACK RD
TO LANSDOWNE
C.F.B. Ottawa (South)

19—24. In which region of the map on page 56 can each place be found? (These places are not in the part of the index shown.)

19. Tignish B-2 20. Haliburton D-2 21. Roxbury D-3
22. West Devon D-3 23. Freeland D-4 24. Waterford B-2

25—38. In which region of the map on page 58 is each of these located?

25. Parliament Buildings C-3
26. CFB Ottawa (North) B-5
27. Government House B-4
28. Vanier B-4
29. Royal Canadian Mint C-3
30. Ottawa International Airport G-4
31. Intersection of Trans Canada Highway 17 and St. Laurent Blvd. C-5
32. Intersection of Highway 5 and Highway 550 B-3
33. Ottawa R.R. Station C-4
34. Rockcliffe Park B-4
35. Carleton University D-3
36. Confederation Building C-3
37. Ottawa City Hall B-3
38. National Aeronautical Collection B-5

39—42. Use the map on page 58 to find what river or canal runs through the region listed.

39. C-4 Rideau River, Rideau Canal
40. A-3 Gatineau River
41. D-1 Ottawa River
42. D-4 Rideau River Rideau Canal

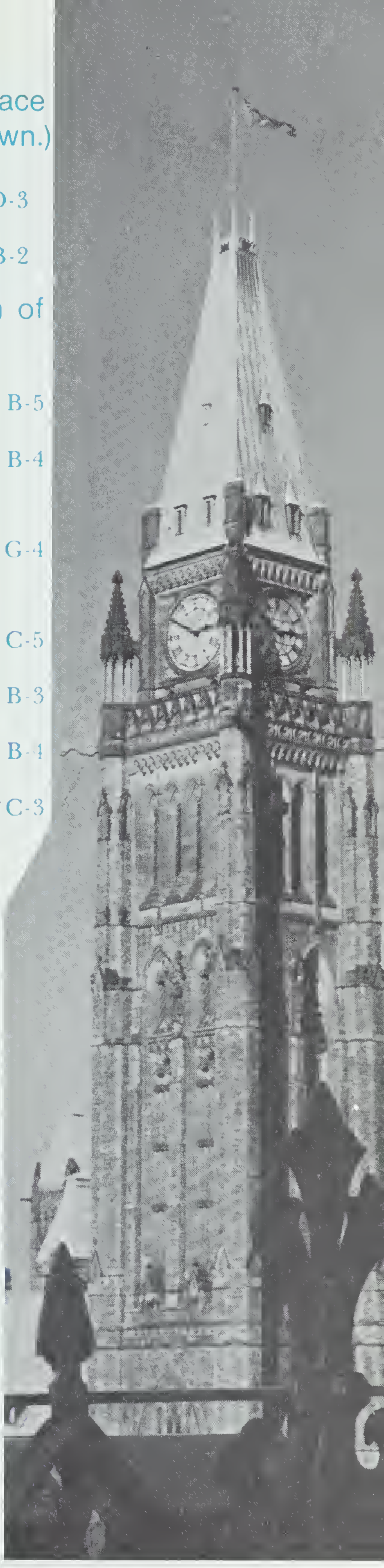
43—46. Refer to the map on page 58.

43. In which region does Bank St. cross the Rideau Canal? D-4

44. In which region does Highway 16 cross the Rideau River? E-3

45. Locate the Queensway. Use region names to tell where to find it on the map. Start from F-1; cross regions E-1 and E-2, D-2 and D-3, C-3 through C-6; cross into B-6.

46. Locate the Ontario-Quebec border. Use region names to tell where to find it on the map. Start from E-1; cross regions D-1, C-2, C-3, B-3, B-4 and A-4 through A-6.



Miller Services

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NUMBERS ON THE MAP

Have you ever heard of Zero Rock, B.C., or Fourcorner Creek, Ontario? There are many place names that are made up partly or entirely of numbers. Some of them are listed below. But you will have to do some figuring, and then spell out your results, to find out what they are!

For example:

(18 ÷ 3) – 1 Pines Lake, Ont. is Five Pines Lake, Ont.,

and [one] 12.5 x 8 Mile House, B.C., is Hundred Mile House, B.C.

1. 9 + 13 – 18 Ears Island, Nfld.
Four Ears Island, Nfld.
2. (90 ÷ 9) Shilling Creek, Man.
Ten Shilling Creek, Man
3. 6 x 2 ÷ 4 Rock Cove, Nfld.
Three Rock Cove, Nfld.
4. 1.7 + 2.3 Coins, N.B. [French]
Quatre Coins, N.B.
5. (5.6 ÷ 8) – 0.7 Lake, Ont.
Zero Lake, Ont.
6. (91 ÷ 13) Isles, P.Q. [French]
Sept Isles, P.Q.
7. (5 + 3 – 6) Guns, Alta.
Two Guns, Alta.
8. 5 x 5 x 2 Acre Island, N.S.
Fifty Acre Island, N.S.
9. (30.7 + 19.3) x 3 Mile House, B.C.
150 Mile House, B.C.
10. (0 + 2) Hills, Alta.
Two Hills, Alta.
11. 105 ÷ 21 Houses, N.S.
Five Houses, N.S.
12. (1.5 x 2) Pistoles, P.Q. [French]
Trois Pistoles, P.Q.
13. (5 + 1) x 2 O'Clock Harbour, Nfld.
Twelve O'Clock Harbour, Nfld.
14. (35 x 2) – 1 Corners, Ont.
Sixty-nine Corners, Ont.
15. (12 – 3) x 3 Mile Brook, N.B.
Twenty-seven Mile Brook, N.B.
16. 9.6 – 3.6 Mile Hill, Nfld.
Six Mile Hill, Nfld.
17. $\frac{[one]}{51 - 49}$ way Cove, N.S.
Halfway Cove, N.S.
18. 0.18 x 100 Mile Bay, Ont.
Eighteen Mile Bay, Ont.

19. (90 ÷ 2) + 3 Road, P.E.I.
48 Road, P.E.I.
20. (10 x 5) – 5 Island, Ont.
Fortyfive Island, Ont.
21. 22.4 – 19.4 Legged Lake, Ont.
Three Legged Lake, Ont.
22. 203 ÷ 29 Persons, Alta.
Seven Persons, Alta.
23. (6 x 12) –63 Mile Creek, P.E.I.
Nine Mile Creek, P.E.I.
24. 15 x 2 ÷ 10 Rivières, P.Q. [French]
Trois Rivières, P.Q.
25. 7.5 ÷ 1.5 Islands, N.S.
Five Islands, N.S.
26. (48 – 6) ÷ 3s River, Man.
Fourteens River, Man.
27. 120 ÷ 12 miles House, P.E.I.
Tenmile House, P.E.I.
28. [one] (999 + 1) Islands, Ont.
Thousand Islands, Ont.
29. (3.7 + 7.3) Foot Rock, Ont.
Eleven Foot Rock, Ont.
30. 2.3 + 3.6 – 1.9 Portages, Sask.
Four Portages, Sask.
31. (36 + 15) ÷ 51 Arrow, Sask
One Arrow, Sask.
32. [one] 25 000 x 40, Man.
Million, Man.
33. (52 ÷ 13) + 3 Sisters Falls, Man.
Seven Sisters Falls, Man.
34. $\frac{[one]}{16.2 \div 8.1}$ moon Bay, Ont.
Halfmoon Bay, Ont.
35. (1 + 4) x 5-10 Mile Butte, Alta.
Fifteen Mile Butte, Alta.
36. (28 ÷ 2) x 5 Mile House, B.C.
70 Mile House, B.C.

Points on the Plane

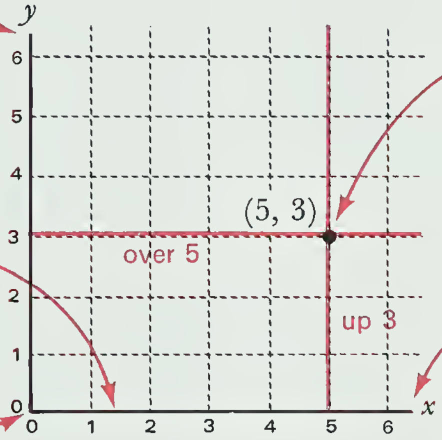
2.2

On the map on page 56, B-3 names a region that includes several towns and cities. But in a theater, B3 names a region that contains only *one* seat.

SEAT
B·3
ADMIT ONE
SAT.OCT.1
SEAT
A·7
SAT.OCT.1
SEAT
5
SAT.OCT.1

Performance Objective: The student can use coordinates to graph or identify a point.

Points on a plane are also named so that each name refers to exactly *one* point. Instead of a letter and a number, the name consists of two numbers, called the **coordinates** of the point.



This line is the **y-axis**.

This line is the **x-axis**.

This point is called the **origin**.

The name of this point is (5, 3).

from *x*-axis

from *y*-axis

Each axis can be drawn as long as you wish.

Because both parts of the name of a point are numbers, you have to know which number comes first.

The number on the *x*-axis is *first*; it tells how far *over* the point is.

The number on the *y*-axis is *last*; it tells how far *up* the point is.

So the coordinates are *always* given in the order (*x*, *y*).

Exercises

1. How would you locate the point (3, 5)? Is it different from (5, 3)? 3 units over, 5 units up; yes

2. What are the coordinates of the origin? (0, 0)

3. How far does the y-axis really go? How much of it can be drawn? Indefinitely; only part

4. How many **axes** (plural of *axis*) are used to locate points in a plane? 2

5–16. On graph paper, draw axes like those shown on page 61. Make the axes 12 units long. Then draw a dot for each of these points and label each point with its coordinates.

5–16. *Graph*

5. (4, 8) **6.** (5, 10) **7.** (6, 0) **8.** (9, 0)

9. (0, 7) **10.** (0, 3) **11.** (12, 2) **12.** (10, 1)

13. (4, 4) **14.** (11, 11) **15.** (8, 10) **16.** (9, 12)

17–28. What are the coordinates of each point?

17. *A* (0, 7) **18.** *B* (5, 7)

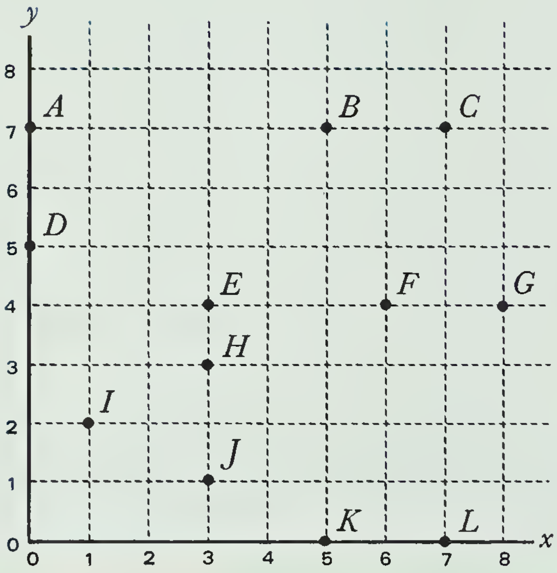
19. *C* (7, 7) **20.** *D* (0, 5)

21. *E* (3, 4) **22.** *F* (6, 4)

23. *G* (8, 4) **24.** *H* (3, 3)

25. *I* (1, 2) **26.** *J* (3, 1)

27. *K* (5, 0) **28.** *L* (7, 0)



Quick Quiz

Check these skills:

- *writing words for fractions*
- *writing equivalent decimals and fractions*

1–4. Write in words.

1. $\frac{1}{2}$ One-half

2. $\frac{2}{3}$ Two-thirds

3. $\frac{4}{5}$ Four-fifths

4. $\frac{3}{7}$ Three-sevenths

5–8. Write a fraction equal to the decimal.

Ex. 5—12: Typical answers:

5. 0.3 $\frac{3}{10}$

6. 0.13 $\frac{13}{100}$

7. 0.5 $\frac{5}{10}$ or $\frac{1}{2}$

8. 0.25 $\frac{25}{100}$ or $\frac{1}{4}$

9–12. Write a decimal equal to the fraction.

9. $\frac{7}{10}$ 0.7

10. $\frac{1}{2}$ 0.5

11. $\frac{3}{4}$ 0.75

12. $\frac{1}{5}$ 0.2

FISHY GRAPHS

Players: Two (The instructions are the same for both players.)

Materials for each player: Graph paper, pencil

Mark off two sets of axes, each 10 units by 10 units, on graph paper.

One graph is your *sea*, and the other is your *map*. Do not let the other player see your *sea* after this (place a hand or a book in front of it).

There are fish in the *sea*, and each fish covers a certain number of points.

shark: 5 points

swordfish: 4 points

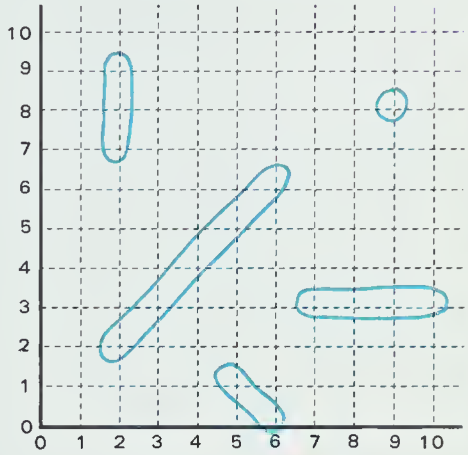
tuna: 3 points

mackerel: 2 points

sardine: 1 point

Place one fish of each kind anywhere in your *sea*. Do this by circling vertical, horizontal, or diagonal rows. (See example.)

Example of a Sea:

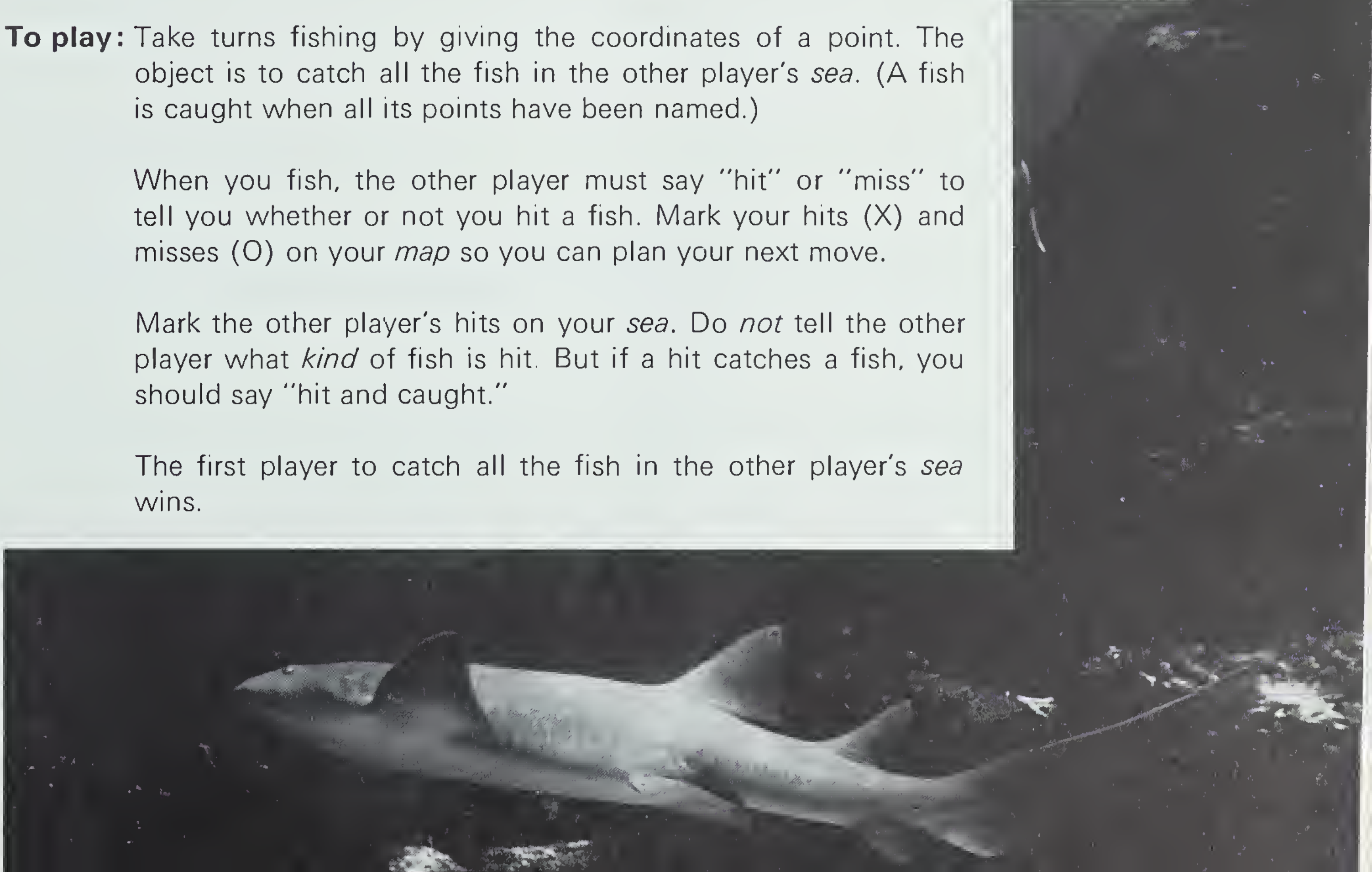


To play: Take turns fishing by giving the coordinates of a point. The object is to catch all the fish in the other player's *sea*. (A fish is caught when all its points have been named.)

When you fish, the other player must say "hit" or "miss" to tell you whether or not you hit a fish. Mark your hits (X) and misses (O) on your *map* so you can plan your next move.

Mark the other player's hits on your *sea*. Do *not* tell the other player what *kind* of fish is hit. But if a hit catches a fish, you should say "hit and caught."

The first player to catch all the fish in the other player's *sea* wins.



Grant Heilman

2.3 Graphing Points from Tables

Performance Objective:
The student can
- graph a set of points from a table of coordinates.
- choose an appropriate scale for a graph.

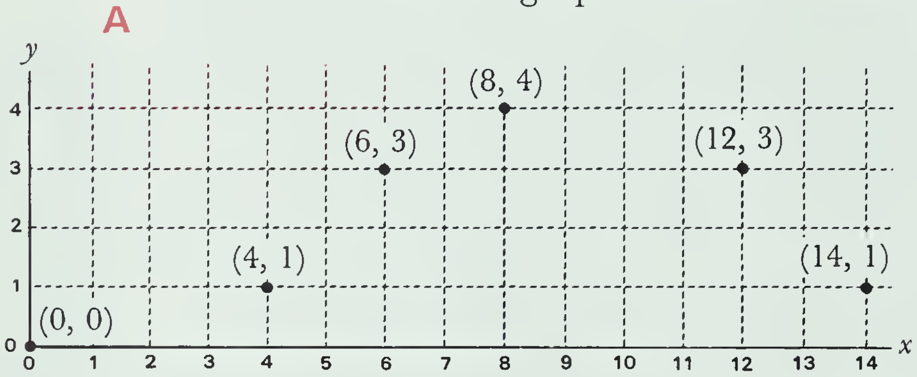
Sometimes a list of coordinates is given in table form.

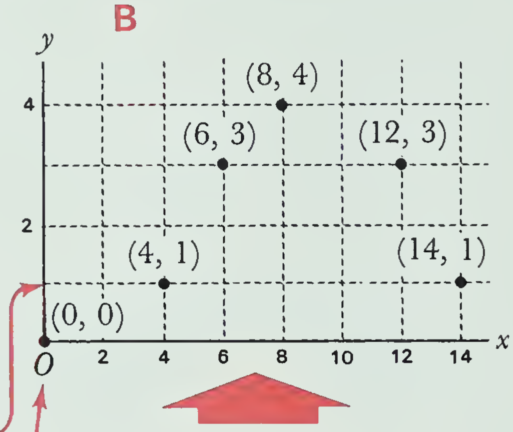
Coordinates: (0, 0), (4, 1), (6, 3), (8, 4), (12, 3), (14, 1)

Table:

x	0	4	6	8	12	14
y	0	1	3	4	3	1

You can see why, if you were graphing these points, it might be easier to keep track of the coordinates using the table. Here are two graphs of these coordinates.



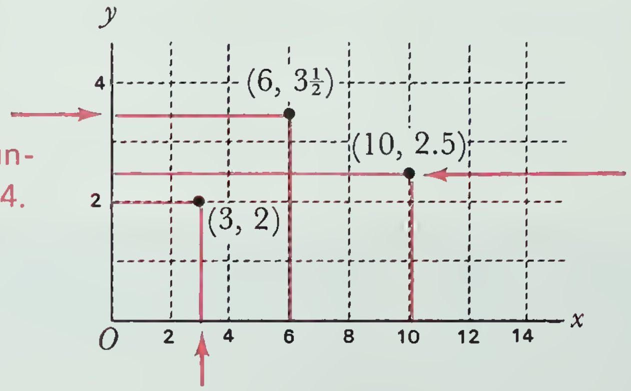


Every mark on the axes does not have to be labeled.

The origin is sometimes labeled O.

Each small part of the x-axis stands for 2.
Each small part of the y-axis stands for 1.
They have different scale values.

Suppose the list included coordinates like $(6, 3\frac{1}{2})$, $(10, 2.5)$ and $(3, 2)$. How could you graph these points?



$3\frac{1}{2}$ is midway between 3 (unlabeled) and 4.

2.5 is midway between 2 and 3.

3 is midway between 2 and 4.

Exercises

1–7. Use graphs A and B on page 64 to answer these questions.

1. In graph A, does each small part of the x-axis stand for 1? Yes

2. In graph A, does each small part of the y-axis stand for 1? Yes

3. In graph B, does each small part of the x-axis stand for 1? No

4. In graph B, does each small part of the y-axis stand for 1? Yes

5. Does the x-axis of a graph have to have the same scale value as the y-axis? No

6. Tell how to find (9, 2) using the scale values on graph B. 9 is midway between 8 and 10; go right 9, up 2.

7. Tell how to find $(8, 1\frac{1}{2})$ using the scale values on graph B. $1\frac{1}{2}$ is midway between 1 (not labeled) and 2; go right 8, up $1\frac{1}{2}$.

8. Rewrite in the form (x, y) the coordinates in Exercise **9** below. (1, 0), (2, 5), (3, 3), (4, 4), (5, 1), (6, 2)

9–20. Make a separate graph for each table. (In **13–18** it may be convenient to use different scale values on the axes.)

9–20. *Graphs*

9.

x	1	2	3	4	5	6
y	0	5	3	4	1	2

10.

x	1	2	3	4	5	6
y	4	0	3	5	2	1

11.

x	2	4	6	8	10	12
y	9	7	5	3	1	0

12.

x	1	3	5	7	9	11
y	10	8	6	4	2	0

13.

x	0	5	11	18	26	35
y	1	2	3	4	5	6

14.

x	1	4	9	16	25	36
y	1	2	3	4	5	6

15.

x	1	2	3	4	5	6
y	4	8	12	16	24	28

16.

x	1	2	3	4	5	6
y	5	10	15	20	25	30

17.

x	0	5	10	15	20	25
y	0	10	30	70	85	125

18.

x	3	6	9	12	18	21
y	12	24	60	96	66	84

19.

x	2	0	$4\frac{1}{2}$	$3\frac{1}{2}$
y	$3\frac{1}{2}$	$\frac{1}{2}$	6	$3\frac{1}{2}$

20.

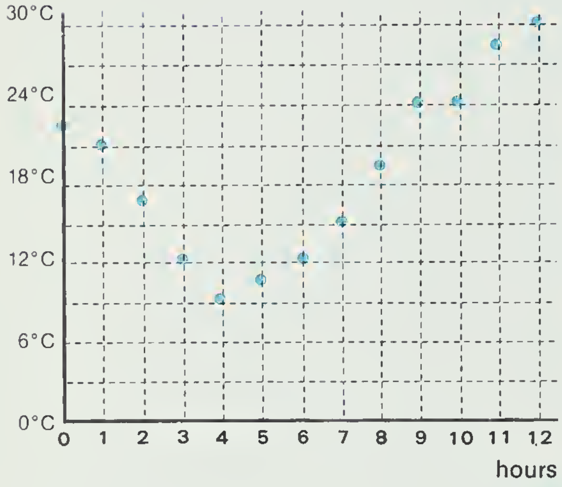
x	1.5	0	0.5	4.5
y	2	3.5	7	2.5

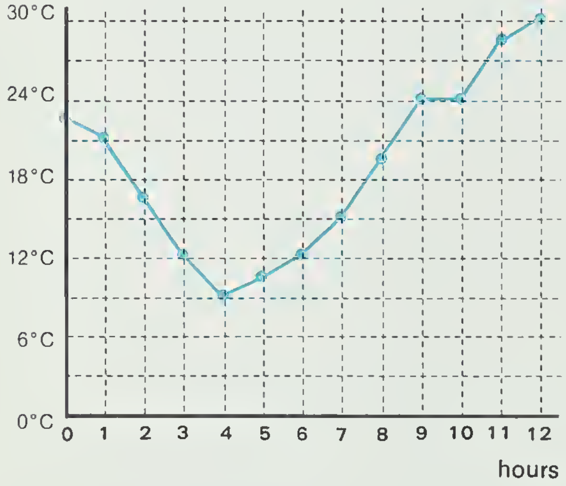
2.4 Making Line Graphs

Performance Objective: The student can
- use a table of information as a table of coordinates.
- make a line graph from such a table.

The *x* and *y* labels in a table of coordinates may be replaced with labels which tell what information is listed.

Time (hours)	0	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°C)	22	20	17	12	9	10	12	15	19	24	24	28	30





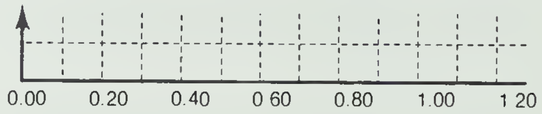
Steps for graphing this information:

1. Draw the axes.
2. Look at the table to choose convenient values for each scale.
3. Graph the points.

Often it is easier to see what is happening (the trend) if the points are joined by lines. The result is a **line graph**.

If there are no low values to graph on one axis or on both axes, you can show the origin and "break" the axes (as on page 26).

If a table shows amounts in dollars and cents, one or both axes may be marked in hundredths.



Exercises

1–4. Look at the second graph above.

1. Does each small part of the *x*-axis stand for 1? Yes
2. Does each small part of the *y*-axis stand for 1? No
3. How do you know the labels along the *x*-axis stand for hours? From the label "hours"
4. How do you know the labels along the *y*-axis stand for degrees? From the symbol °C

5. When you make a line graph, must you use the same scale value on the x-axis as you use on the y-axis? No

6. How should you label each axis (besides with numbers) to show what real-life information is being graphed?

With name of units

7–8. What scale values would you use on the axes to graph the given information?

7–8. *Typical answers:*

7.

Time	0	0.1	0.2	0.3	0.4
Temp. (°C)	25	20	10	5	1

x-axis: 0.1, y-axis: 5°C

8.

Time	0	1	2	3	4
Temp. (°C)	0.25	0.2	0.1	0.05	0.01

x-axis: 1, y-axis: 0.05°C

9–12. Match each table with the correct line graph.

9.

x	0	1	2	3
y	0	2	4	6

c

10.

x	0	2	4	6
y	0	1	2	3

b

11.

x	0	2	4	6
y	0	4	6	8

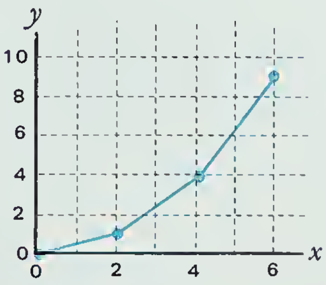
d

12.

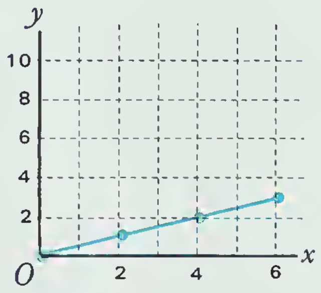
x	0	2	4	6
y	0	1	4	9

a

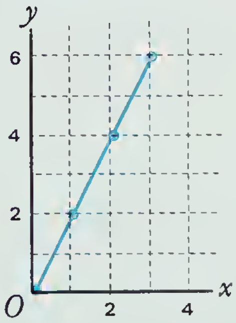
a.



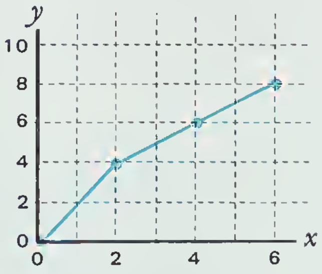
b.



c.



d.



13–18. Make a separate line graph for each table.

13–26. *Graphs*

13.

x	50	55	60	65	70	75	80
y	0	1	2	3	4	5	6

14.

x	40	50	60	70	80	90	100
y	0	10	15	20	25	30	35

15.

amt.	1	2	3	4	5	6
cost	0	0.10	0.14	0.17	0.20	0.23

16.

amt.	2	4	6	8	10	12
price ea.	1.20	0.60	0.30	0.15	0.10	0.05

17.

quantity	0	1	2	3	4
mass	0	3.3	6.6	9.8	13.1

18.

number	0	1	2	3	4
value	.0	3.8	7.6	11.3	15.1

19—20. Make a separate line graph for each table.

19. Gasoline consumption depends on many things, including speed. According to one carmaker, you can expect gas consumption at these rates from their new model.

Speed (km/h)	30	50	70	90	110	130	150	170	190
Litres per hundred kilometres (L/100km)	9.8	10	10.5	11.5	12.8	15.3	18.7	23.1	31.9

20. The faster you drive, the farther your car travels after you "hit the brakes." This table shows how much farther.

Driving Speed (km/h)	10	20	30	40	50	60	70	80	90	100
Braking Distance (metres)	1	3	5	7	9	12	22	35	55	80

21—22. The telephone company charges for both distance and time. If you were to dial a call yourself on a Sunday, this is how you might be charged. Make a separate line graph for each table.

21. For a call to a friend 30 km away but in another province

Length of call (minutes)	1	2	3	4	5	6	7	8	9	10
Charge (cents)	20	25	30	35	40	45	50	55	60	65

22. For a call to a friend 16 km away but in another province

Length of call (minutes)	1	2	3	4	5	6	7	8	9	10
Charge (cents)	14	18	22	26	30	34	38	42	46	50

23–24. The tables below give the amount of lumber that can be cut from various sizes of logs. Make a line graph from each table.

Lavine/De Wys, Inc

23. Distance across small end: 38 cm

Length of log (cm)	180	210	240	270	300	330	360	390	420
Cubic metres of lumber	0.35	0.42	0.49	0.56	0.63	0.7	0.77	0.84	0.84

24. Distance across small end: 35 cm

Length of log (cm)	180	210	240	270	300	330	360	390	420
Cubic metres of lumber	0.28	0.35	0.42	0.42	0.49	0.56	0.63	0.63	0.7

25–26. It is hard to tell from the price of raw meat what an average serving (100 g) of cooked meat will cost. Each table below gives the price per kilogram of a cut of beef and the cost of one serving, in dollars. Make a line graph from each table.

25. Ground beef, lean

Price, 1 kg	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
Cost, 1 serving	0.25	0.29	0.32	0.33	0.37	0.38	0.42	0.45	0.47	0.50	0.52

26. Round, bone in

Price, 1 kg	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
Cost, 1 serving	0.33	0.37	0.40	0.43	0.47	0.50	0.54	0.57	0.60	0.64	0.67

2.5

Graph Pictures

Performance Objective: The student practises graphing a set of points and joining them in order.
A further source of picture graphs is "Palatable Plotting", available from Setsco Educational Ltd., 567 Clarke Road, Coquitlam B.C. V3J 3X4.

Graph paper can be used for playing games, drawing patterns and designs, and even drawing pictures.

Suppose you graph the points in table 1 below and then connect them in order. Next, using the same axes, do the same for table 2, table 3, and so on. (Of course the points could also be given as (0, 6), (1, 10), (3, 10), and so forth.)

1

x	0	1	3	4	3	2	2	3	3	1	1	0
y	6	10	10	8	8	7	6	6	5	4	1	0

2

x	2	3	3	4	5	6	6	8	8	9
y	7	7	6	5	5	6	5	4	1	0

3

x	2	3	3	4	5	6	6	7
y	0	1	2	3	3	2	1	0

4

x	1	2	2	3	3
y	0	1	3	4	5

5

x	5	6	7	6	6	7	7
y	8	8	7	7	6	6	7

6

x	5	6	7	7	8
y	3	4	3	1	0

7

x	9	9	6	5	4
y	7	10	9	8	8

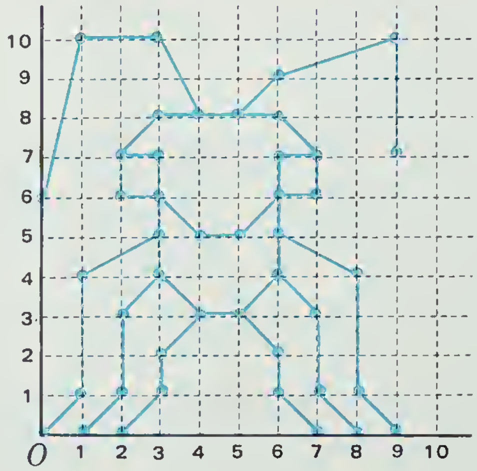
8

x	3	4
y	4	3

9

x	6	6
y	5	4

This creature is the result!



1–6. Refer to page 70.

1. What part of the creature is the graph of table 7? Left antenna (on right in picture)
2. What part of the creature is the graph of table 6? Left middle leg (on right in picture)
3. Which table gives the creature's right middle leg (on the left in the picture)? Table 4
4. Which table gives the creature's left eye (on the right in the picture)? Table 5
5. Which table gives the creature's right antenna? Table 1
6. Which table gives the creature's outer left leg? Table 2

7. For this picture you need graph paper 20 squares wide and 8 squares high. Graph all the points; then connect them in order. *Dachshund*

x	$5\frac{1}{2}$	4	3	4	4	3	1	$\frac{1}{2}$	2	2	3	4	6	18	20	18	$18\frac{1}{2}$	18	$18\frac{1}{2}$	18	17	$17\frac{1}{2}$	$16\frac{1}{2}$	$15\frac{1}{2}$	12	7	6	$5\frac{1}{2}$
y	$1\frac{1}{2}$	0	0	2	4	5	4	$4\frac{1}{2}$	$6\frac{1}{2}$	7	8	8	6	$5\frac{1}{2}$	7	5	3	2	$1\frac{1}{2}$	0	0	1	$1\frac{1}{2}$	$2\frac{1}{2}$	2	1	1	$1\frac{1}{2}$

Now graph (3, 6) and join it to (3, 8) and (4, 8).

8. For this picture you need graph paper 13 squares wide and 12 squares high. For each table, graph the points and connect them in order. *Stop sign*

x	7	6	6
y	4	5	$7\frac{1}{2}$

x	2	$2\frac{1}{2}$	4	4	2	2	3	4	4	6
y	$4\frac{1}{2}$	4	4	6	6	8	8	$7\frac{1}{2}$	8	8

x	9	11	11	9	9	7	8	8	7	6	6
y	6	6	8	8	4	4	5	$7\frac{1}{2}$	8	$7\frac{1}{2}$	8

x	5	5
y	4	8

x	0	4	9	13	13	9	4	0	0
y	$3\frac{1}{2}$	0	0	$3\frac{1}{2}$	$8\frac{1}{2}$	12	12	$8\frac{1}{2}$	$3\frac{1}{2}$

9–10. Reading the coordinates from left to right across the page, graph each set of points and connect them in order.

9. Use graph paper 10 squares wide and 13 squares high.

Sailboat

$(0, 0)\ (1, \frac{1}{2})\ (2, 0)\ (3, \frac{1}{2})\ (4, 0)\ (2, 12)$
$(1\frac{1}{2}, 13)\ (0, 12)\ (2, 12)\ (2, 7)\ (3, 3)\ (6, 2)$
$(2, 12)\ (8\frac{1}{2}, 2\frac{1}{2})\ (8\frac{1}{2}, 1)\ (8, 0)\ (9, \frac{1}{2})\ (10, 0)$

$(4, 7)\ (4, 4)\ (6, 2)\ (7, \frac{1}{2})\ (6, 0)$
$(5, \frac{1}{2})\ (4, 0)\ (5, 1)\ (3, 3)$

$(4, 4)\ (5, 1)\ (6, 2)\ (8\frac{1}{2}, 2\frac{1}{2})$

Now join $(7, \frac{1}{2})$ and $(8, 0)$.

10. Use graph paper 40 squares wide and 52 squares high.

Giraffe

$(0, 39)\ (0, 41)\ (5, 44)\ (5, 45)\ (6, 45)\ (7, 47)\ (11, 47)$
$(12, 45)\ (16, 45)\ (18, 43)\ (14, 43)\ (26, 30)\ (28, 30)\ (33, 27)$
$(35, 18\frac{1}{2})\ (37\frac{1}{2}, 15)\ (38, 13)\ (39, 2)\ (39, 1)\ (39\frac{1}{2}, 0)\ (37\frac{1}{2}, 0)$
$(38, 2)\ (37, 13)\ (36, 15)\ (32, 18)\ (31, 14)\ (22\frac{1}{2}, 0)\ (21, 0)$
$(22\frac{1}{2}, 2)\ (29\frac{1}{2}, 13\frac{1}{2})\ (29, 19)\ (27, 19)\ (25\frac{1}{2}, 14\frac{1}{2})\ (26, 1)$
$(25\frac{1}{2}, 0)\ (24\frac{1}{2}, 0)\ (25, 1)\ (24\frac{1}{2}, 18)\ (23\frac{1}{2}, 20)\ (23\frac{1}{2}, 1)\ (23, 0)$
$(22\frac{1}{2}, 0)\ (22\frac{1}{2}, 14)\ (21, 21\frac{1}{2})\ (10, 40)\ (8, 39)\ (5, 40)\ (0, 39)$

$(33, 27)\ (34, 26)\ (35, 23)\ (36, 21\frac{1}{2})\ (37, 21)$

$(11, 51)\ (10, 47)$ Make a large dot at $(11, 51)$.

$(13, 51)\ (11, 47)$ Make a large dot at $(13, 51)$.

Quick Quiz

Check these skills:

- *adding like fractions*
- *subtracting like fractions*

1–4. Add.

1. $\frac{2}{7} + \frac{3}{7}$ $\frac{5}{7}$ **2.** $\frac{2}{4} + \frac{1}{4}$ $\frac{3}{4}$ **3.** $\frac{5}{8} + \frac{2}{8}$ $\frac{7}{8}$ **4.** $\frac{1}{2} + \frac{1}{2}$ $\frac{2}{2}$ or 1

5–8. Subtract.

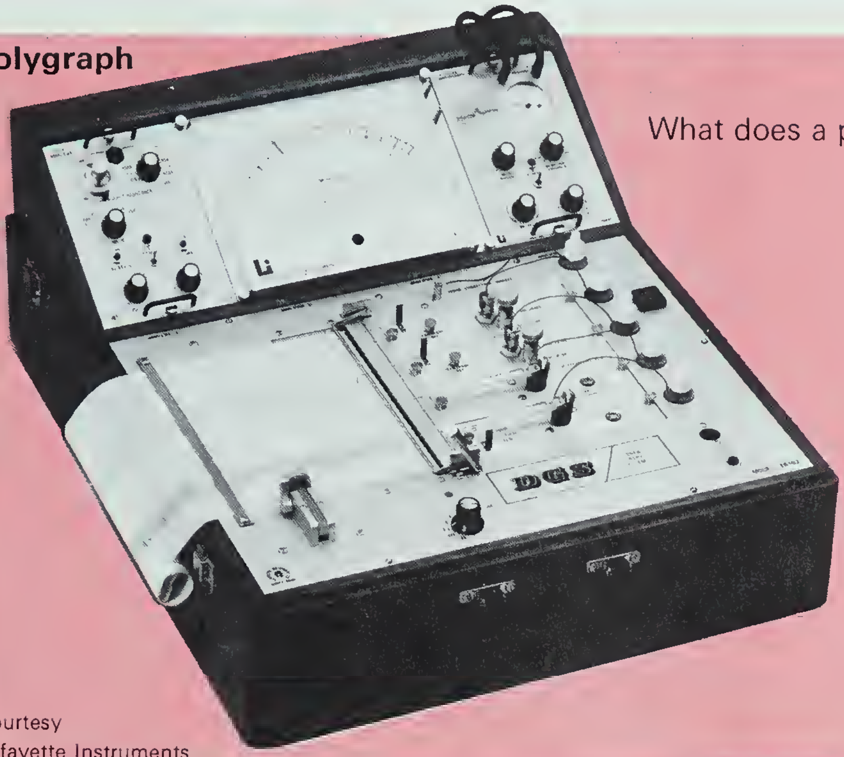
5. $\frac{3}{4} - \frac{2}{4}$ $\frac{1}{4}$ **6.** $\frac{5}{9} - \frac{1}{9}$ $\frac{4}{9}$ **7.** $\frac{3}{10} - \frac{3}{10}$ 0 **8.** $\frac{7}{12} - \frac{4}{12}$ $\frac{3}{12}$ or $\frac{1}{4}$

GRAPHS AND GRAMS

Do you collect **autographs**? Did you ever draw a **diagram**?

Words that end in *-graph* or *-gram* are related to the Greek word *graphein*, meaning to *write* or to *draw*.

polygraph



Courtesy
Lafayette Instruments

What does a polygraph show?

Shows changes in pulse and breathing

oscillograph

What does an oscillograph do?

Shows electrical waveforms

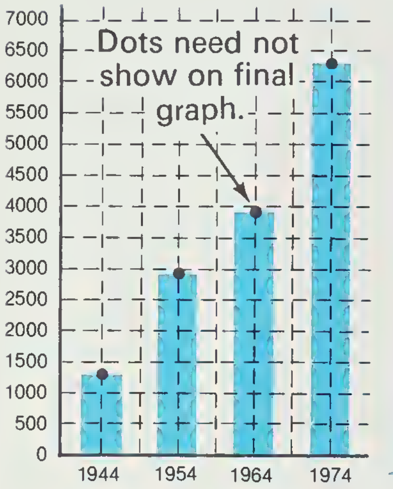
Here are some more *-graphs* and *-grams*. Find out what each of them is.
The suffix *-graph* refers to a recording instrument; *-gram* refers to the graph or record, the subject of which is listed.

cardiograph, cardiogram	heart motion
barograph, barogram	air pressure
seismograph, seismogram	vibrations of the earth
thermograph, thermogram	temperature
electrocardiograph, electrocardiogram	electrical impulses in the heart
hygrograph	changes in humidity

2.6 Making Bar Graphs

Performance Objective:
The student can make a vertical or horizontal bar graph from a table of information.

Sometimes you can make a bar graph by locating points on graph paper just as you do for a line graph. But you do not connect the points. Instead, you use them to tell you where the bars should end.



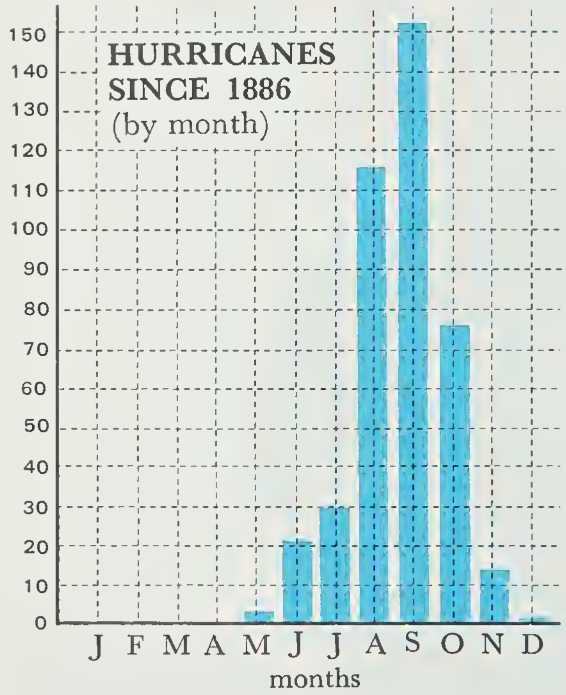
Source: National Safety Council

Label each bar.

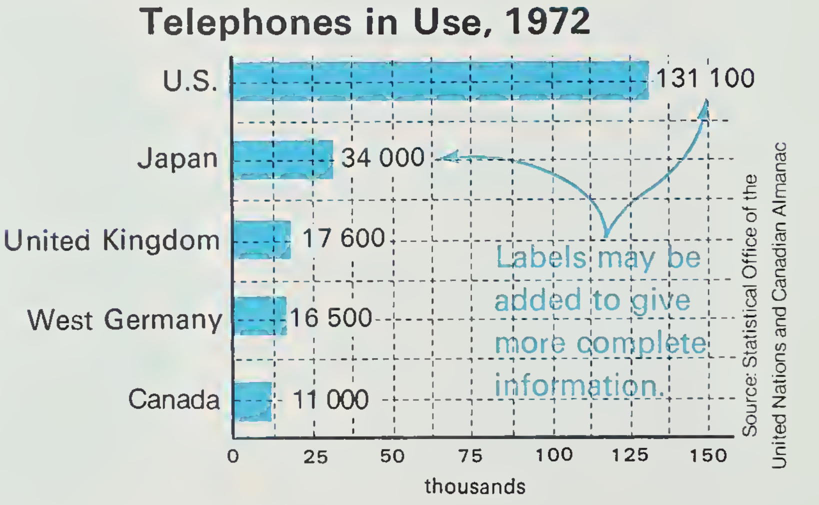
MOTOR-VEHICLE DEATHS

year	1944	1954	1964	1974
deaths	1372	2947	3943	6325

Choose convenient scale values.



The bars may go up (or down) to make a *vertical* bar graph, or they may go to the right (or left) to make a *horizontal* bar graph.



Source: Statistical Office of the United Nations and Canadian Almanac

Exercises

1–12. Look at the bar graphs above.

1. What do the lengths of the bars in the first graph show?
Number of motor-vehicle deaths each given year
2. What scale value is used on the y-axis in the first graph? (How much does each small part of the scale stand for?)
5000

3. How was the end of each bar in the first graph located? Point was found above the year, level with a point on the scale representing number of deaths for the given year.

4. In the first graph, are the labels on the *x*-axis located where they would be on a line graph? Yes

5. What scale value is used in the second graph? 10

6. What do the labels below the bars of the second graph stand for? Months

7. How can you use the second graph to estimate the number of hurricanes in any month? Use scale at left to estimate length of bar.

8. Which three months would be the "hurricane season"? Aug., Sept., Oct.

9. Which six months of the year have fewest hurricanes? Jan., Feb., Mar., Apr., May, Dec.

10. What scale value is used in the third graph? 12 500

11. What do the labels at the left in the third graph stand for? Nations

A. Devaney, Inc.

12. Why are the labels at the right of the bars in the third graph useful? They give more exact values than could be estimated using the scale.

13–14. Make a vertical bar graph from each table.

13–20. *Graphs*

13. National Parks in Canada

Year	1910	1930	1950	1970
Number	5	14	16	23

14. Housing Starts in Canada

Year	1940	1950	1960	1970
Thousand starts	53	93	109	191

15–16. Make a horizontal bar graph from each table.

15. Books Produced, 1972

Canada	17 030
France	24 497
Great Britain	33 109
Japan	31 074

16. Local Governments in Canada, 1975

Cities	184
Towns	815
Villages	1116
Rural Municipalities	2010

17—20. Make a bar graph from each table.

17. Number of positions required in clerical jobs in 1982 (Source: Manpower and Immigration)

Stenographers and secretaries	382 500
Bookkeeping workers	298 850
Cashiers	180 625
Typists	135 200
Stock clerks	80 800
Receptionists	75 100
Bank clerks	33 375
Computer operating personnel	31 125

18. Circulation of eight leading Canadian magazines in 1976.

Canadian Scene	1 500 000
Chatelaine	1 280 000
Reader's Digest	1 230 000
Midnight	901 000
MacLean's	775 000
Toronto Calendar	200 000
Saturday Night	100 000
Vancouver Calendar	100 000

19. Hourly rates of pay for plumbers (1976)

Montreal	$6.62
Vancouver	9.21
Edmonton	7.81
Halifax	6.58
Hamilton	6.67

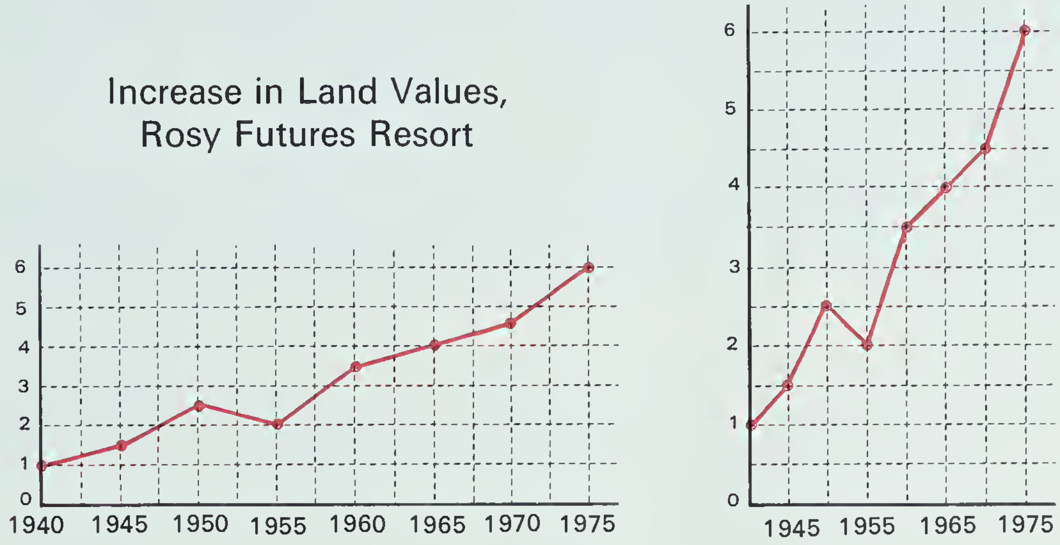
20. Hourly rates of pay for painters (1976)

Kingston	$6.02
Saint John	5.85
Calgary	7.22
Winnipeg	5.97
Victoria	8.84

SLANTING THE PICTURE

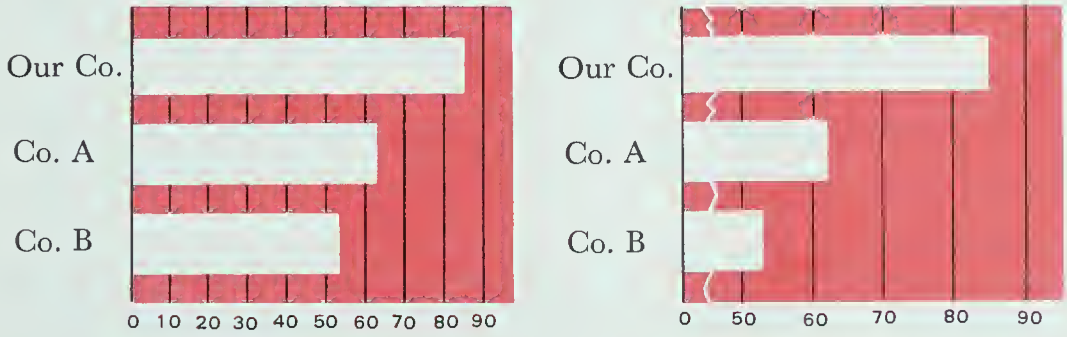
Even though a person knows how to read graphs, a cleverly drawn graph can give the wrong idea—it can appear to say something it doesn't say. Look at these examples. What has been done in each case to make the second graph look "better" than the first graph?

Increase in Land Values, Rosy Futures Resort



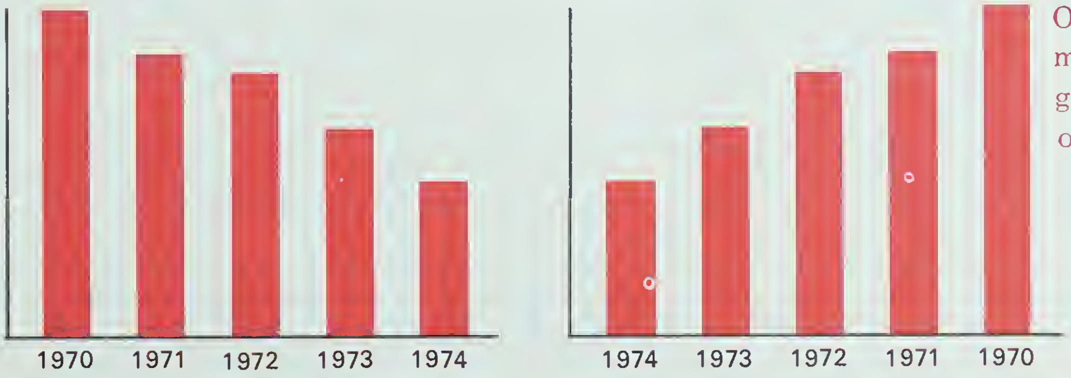
Vertical scale was stretched, horizontal scale condensed to make line rise more sharply.

Sales Comparison, Double-Rich Advertising Corp.



Scale was broken to shorten bars and make longer bar look twice as long as next one.

The Five-Year Profit Picture, Smart Investments Corp.

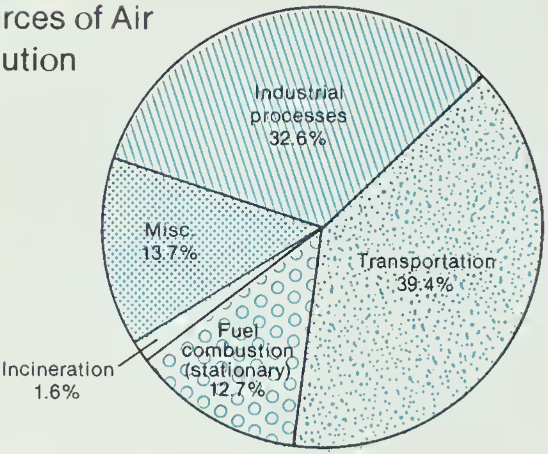


Order was rearranged to make left-to-right reading give incorrect impression of increased profit.

2.7 Reading Circle Graphs

Performance Objective:
The student can read and interpret circle graphs and other percent graphs.

Sources of Air Pollution



Source: Environment Canada

This graph, comparing sources of air pollution, is called a **circle graph**. Circle graphs are also called *pie charts*. You can see how amounts compare by looking at the size of each "slice of the pie."

14% means *14 out of every 100*. So 100% means *100 out of every 100*, or the entire amount. In a circle graph, the whole graph always stands for 100%.

So this graph shows that of all air pollution, 1.6 parts out of every 100 result from getting rid of garbage, probably by burning it. That is, garbage disposal causes

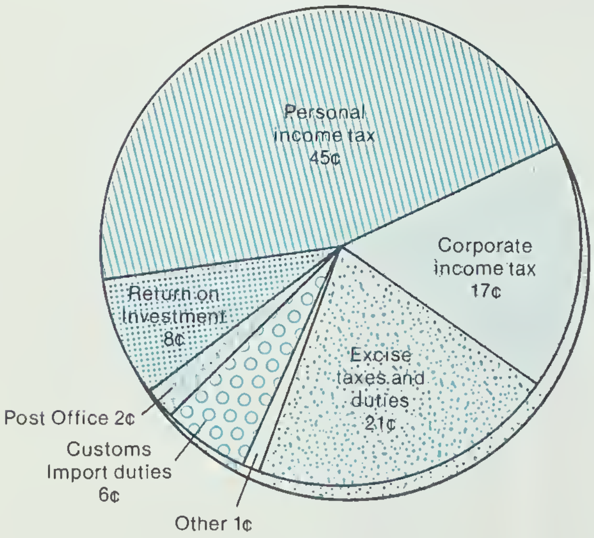
1.6 kg out of every 100 kg,

1.6 t out of every 100 t,

1.6 g out of every 100 g, and

1.6 mg out of every 100 mg of air pollution.

THE BUDGET DOLLAR



Source: Public Accounts of Canada, 1977

This graph shows where the federal government got its money in a recent year.

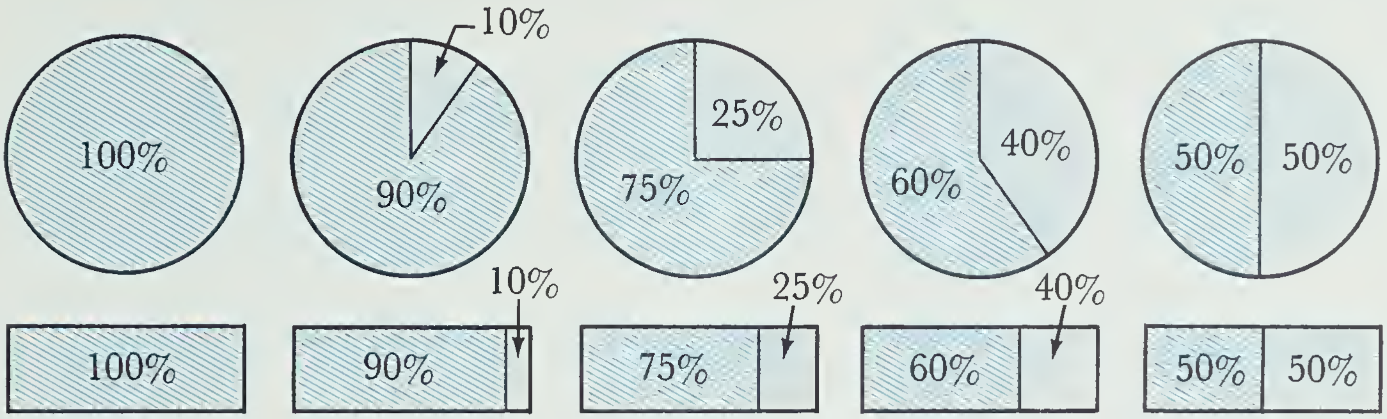
The whole graph stands for $1 or 100 cents. So 45¢ out of every 100¢ or 45% of every dollar, came from income taxes that we as individuals pay. That is, these taxes supplied

45¢ out of every 100¢,

$45 out of every $100, and

$45 000 out of every $100 000 that the government received.

Circle graphs are often used to show percents, but other shapes can also be "sliced" into percents. Here is how some percents relate to the size of the slice of a graph.



Exercises

1–8. Refer to the graphs on page 78.

1. How many tonnes of air pollution out of every 100 t are due to industrial process? 32.6 t

2. How many kilograms of air pollution out of every 100 kg are due to transportation? 39.4 kg

3. How would a slice representing 27.4% look compared to the slice shown for miscellaneous? Twice as big

4. How would a slice representing 19.7% look compared to the slice shown for transportation? Half as big

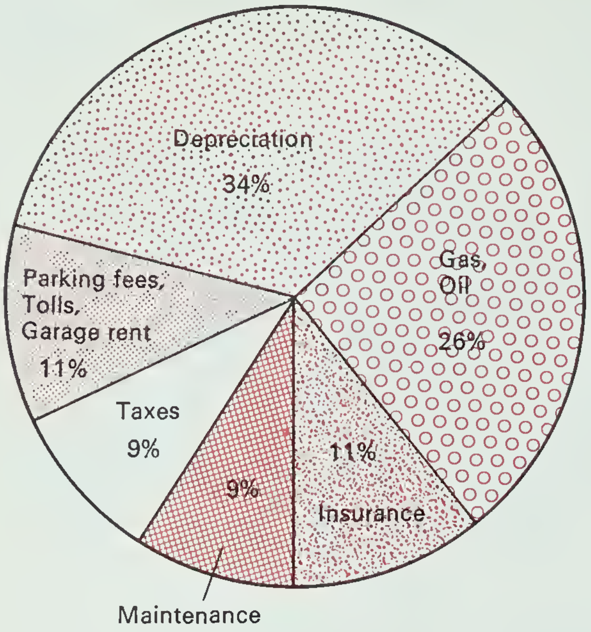
5. How many cents of every dollar does the government get from corporation income taxes? 17¢

6. How many cents of every dollar does the government get from return on investment? 8¢

7. How many dollars of every $100 does the government get from excise taxes? $21

8. How many thousand dollars out of every $100 000 does the government get from the post office? $2 thousand

Costs of Operating a Car



9–18. The graph shows a typical comparison of the costs of operating a one-year-old car.

9. How many cents of every dollar go for maintenance of the car? 9¢

10. How many cents of every dollar go for insurance? 11¢

11. How many dollars of every $100 are accounted for by depreciation? $34

12. How many dollars of every $100 go for gas and oil? $26

13. How many dollars of every $100 go into taxes? $9

14. How many dollars of every $100 go into tolls, parking fees, and garage rent for the car? $11

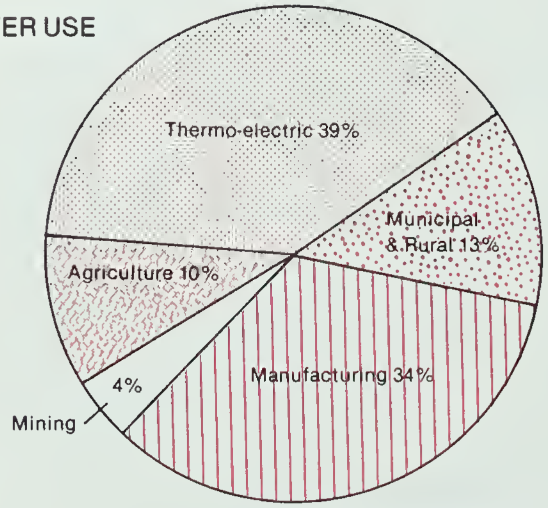
15. If the car costs $100 to operate, how much is spent for gas and oil? $26

16. If the car costs $100 to operate, how much is spent for insurance? $11

17. If the car costs $1000 to operate, how much is spent for gas and oil? $260

18. If the car costs $1000 to operate, how much is spent for insurance? $110

WATER USE



Source: Environment Canada

19–26. In recent years, the total use of water has been divided about as shown by the graph.

19. Which use takes the greatest amount of water? Thermo-electric

20. Which use takes the least amount of water? Mining

21. Which use takes three times as much water as the public uses? Thermo-electric

22. Which use takes about two and one half times as much water as the public uses? Manufacturing

23. Of every 100 L of water, how many are used for thermo-electricity 39 L

24. Of every 100 L of water, how many are used by manufacturing? 34 L

25. Of every 100 L of water, how many are used for agriculture? 10 L

26. Of every 100 L of water, how many go for public use? 13 L

Grant Heilman

27–34. Use the graph to answer these questions on energy consumption.

27. In 1951 which (form of energy) was used in greatest quantities? Coal

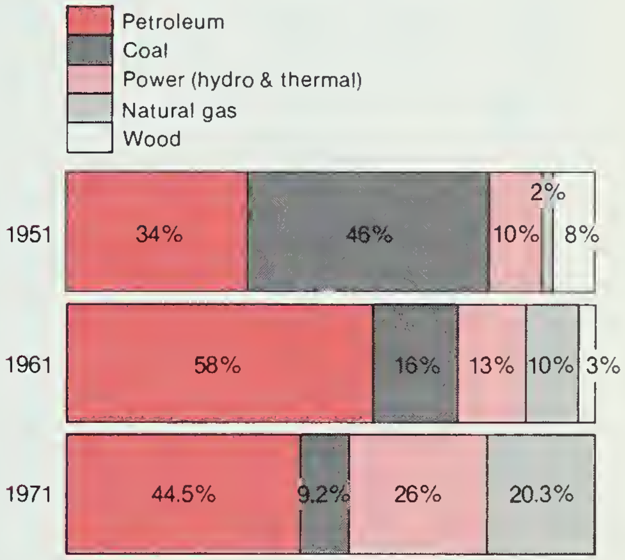
28. In 1951 which (form of energy) was used in smallest quantities? Natural gas

29. Which had a smaller usage in 1961 than in 1951? Coal, wood

30. Which increased the greatest amount from 1951 to 1961? Petroleum

31. Which increased the greatest amount from 1951 to 1971? Natural gas

Canadian Energy Consumption



Source: Statistics Canada

32. Which decreased the greatest amount from 1951 to 1971 Coal

33. In 1951, out of every 100 units of energy consumed, how many were petroleum? Coal? Hydro and thermal? 34; 46; 10

34. In 1971, out of every 100 units of energy consumed, how many were petroleum? Coal? Hydro and thermal? 44.5; 9.2; 26

2.8 Reading Schematics

Performance Objective:
The student can read and interpret simple schematics.

Have you ever taken a clock or a watch apart? It isn't hard to do! But when you tried to put it back together again, you may have wished you had a diagram to show where each part came from.

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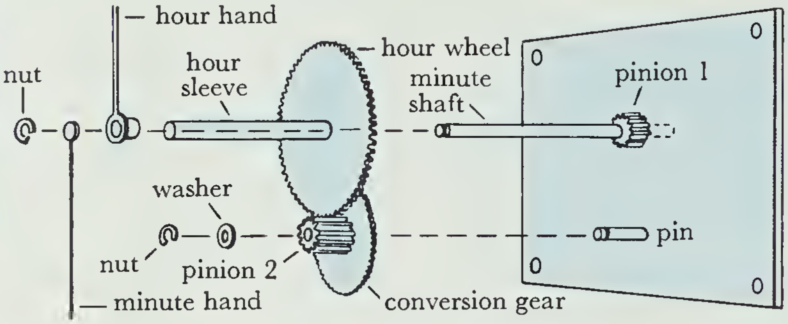
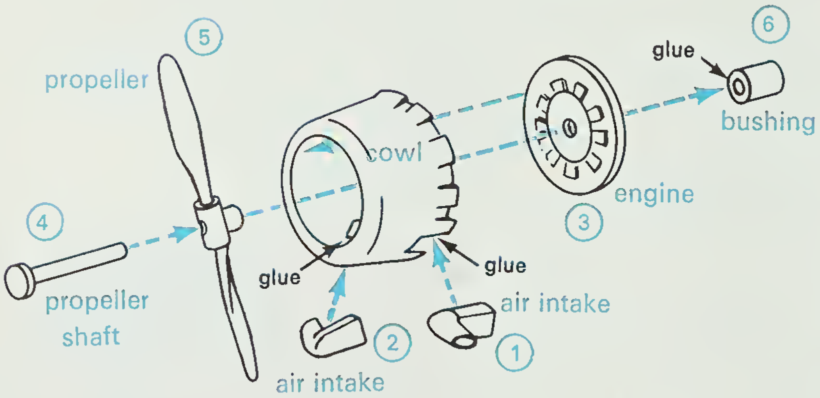


hour hand
nut
hour sleeve
hour wheel
minute shaft
pinion 1
0
0
washer
nut
pinion 2
minute hand
conversion gear
pin
0
0

A diagram that shows how the parts of a system fit together is called a **schematic**.



⑤
propeller
glue
⑥
bushing
cowl
engine
③
④
propeller shaft
glue
glue
air intake
②
①
air intake

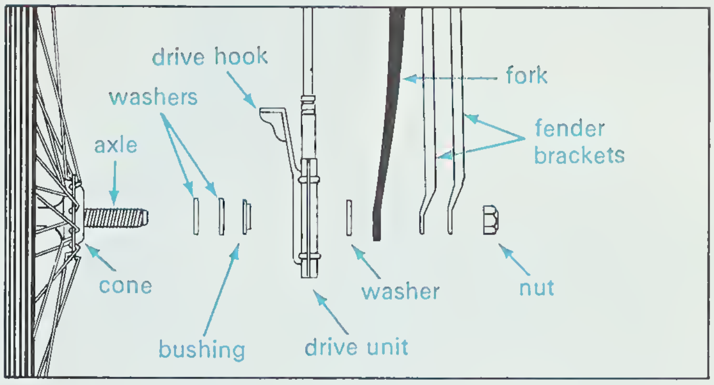
Here is a schematic for assembling some parts of a model airplane. The numbers tell in what order to assemble the parts and the arrows show where they go.

Exercises

1–6. Refer to the schematics above.

1. Which parts of the clock go on the minute shaft?
Hour wheel, hour sleeve, hour hand, minute hand, nut
2. Which parts go on the pin below the minute shaft?
Conversion gear, pinion 2, washer, nut
3. Does the conversion gear go on the pin before the hour wheel goes on the minute shaft, or after? Before
4. Which parts of the model airplane are put together first?
Air intakes, cowl
5. Which parts does the propeller shaft go through?
Propeller, cowl, engine, bushing
6. To which parts of the plane is glue applied? Cowl, bushing

7–12. Refer to the schematic below showing how to install the drive unit for a bicycle speedometer.



7. In what order are the fork and the fender brackets put on the axle?
Fork, then fender brackets

8. In what order are the fork and the drive unit put on the axle?
Drive unit first

9. How many washers are shown? 3

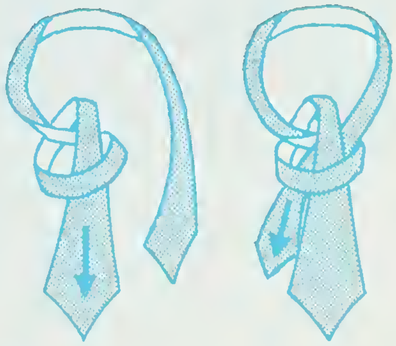
10. How many fender brackets are shown? 2

11. Which part of the wheel does the axle go through? Cone

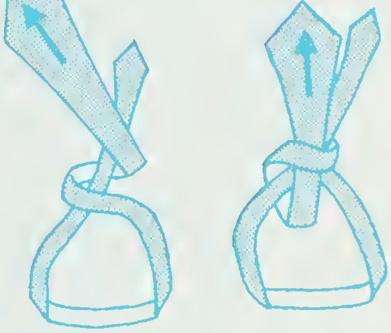
12. What goes on the axle last? Nut

13–20. Instructions for macrame and other types of knotting often use schematics. The ones shown here are for some popular necktie knots, as tied by a right-handed person.

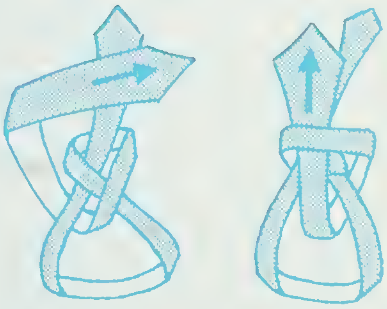
FOUR–IN–HAND
version A



version B



HALF–WINDSOR



13. How can you tell the back of the tie from the front in these schematics? Back is white, front blue.

14. How do the schematics show which end of the tie is held still and which is moved to make the knot?
Arrow indicates motion.

15. Which knots are shown from the wearer's point of view? Which are shown from the point of view of a person looking at the wearer?
Four-in-hand version B, Half-Windsor; four-in-hand version A

16. On which side of the wearer is the narrow end of the tie placed to start each knot, the right or the left? Left

17. On which side would a left-handed person probably want to start the narrow end of the tie? Why?
Right; so the wide end can be moved with the left hand

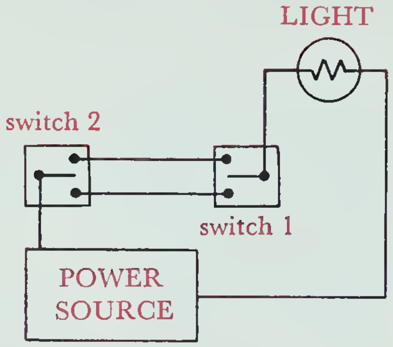
18. Tie each knot in a necktie or piece of cloth around your own neck. Which schematics were easiest to use? (Does point of view make a difference?)
Probable answer: four-in-hand version B, Half-Windsor; yes

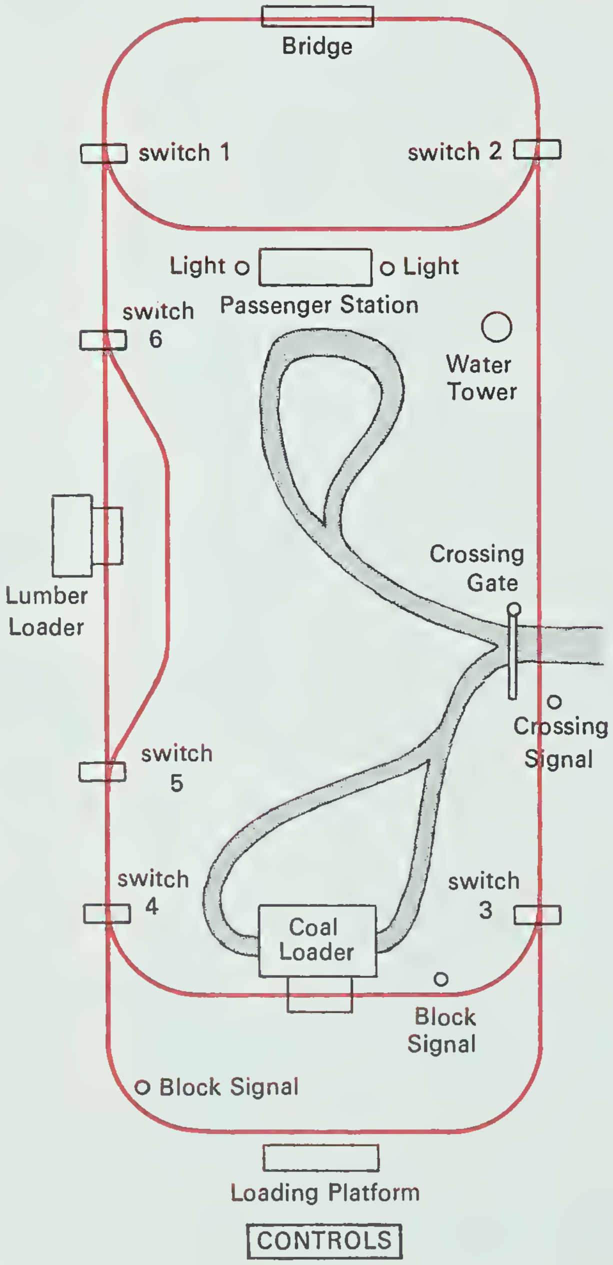
19. Which knot is started by tying a loop in the wide end of the tie? Four-in-hand version B

20. To start each knot, which end of the tie should be longer, the wide end or the narrow end? Wide end

21–24. The schematic below shows how two switches may be wired to control the same light. Each switch is actually always *up* or *down*—there is no middle position. The light is *on* only when the wires form a closed path from the power source, through the light, and back to the power source. Complete the chart.

	Switch 1	Switch 2	Light
21.	up	up	On
22.	up	down	Off
23.	down	Down	on
24.	down	Up	off


LIGHT
switch 2
switch 1
POWER SOURCE


Bridge
switch 1
switch 2
Light
Light
Passenger Station
switch 6
Water Tower
Lumber Loader
Crossing Gate
Crossing Signal
switch 5
switch 4
switch 3
Coal Loader
Block Signal
Block Signal
Loading Platform
CONTROLS

25–32. The schematic shows an electric-train system. A train leaves the passenger station going clockwise. It passes the water tower and then returns to the station as soon as possible without going in reverse.

25. How many switches does the train pass? 8 (6 different)

26. How many times does the train have a choice of paths? 4

27. Can the train go to both the coal loader and the lumber loader on this trip? Yes

28. Can the train go to both the coal loader and the loading platform on this trip? No

29. Must the train pass the crossing signal on this trip? Yes

30. Must the train pass a block signal on this trip? Yes

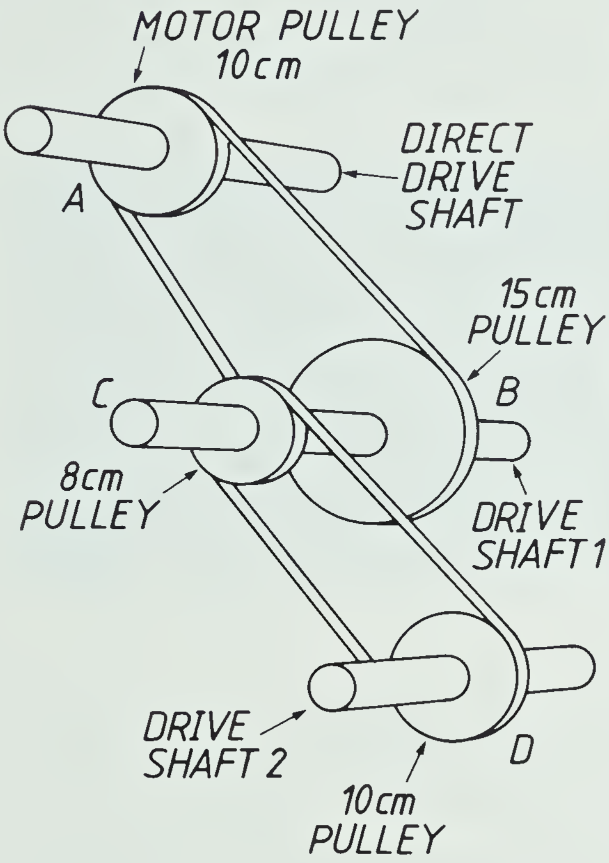
31. How many switches does the train pass twice? 2

32. How many times does the train cross the bridge? 2

CAREER CORNER

MATH IN FACTORY JOBS

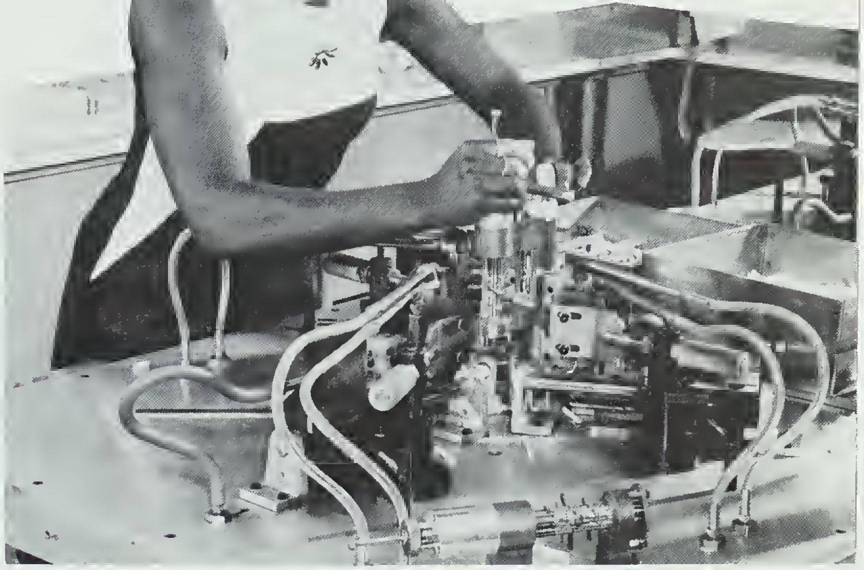
Machine operators and repairers, assemblers, inspectors, and so on, form a large and important part of industry. There are millions of such workers throughout the country, and reading schematics is a needed skill for many of them.


MOTOR PULLEY
10cm
A
DIRECT DRIVE SHAFT
15cm PULLEY
B
C
8cm PULLEY
DRIVE SHAFT 1
DRIVE SHAFT 2
D
10cm PULLEY

How many pulleys are involved in this setup? 4 How many belts? 2 How many shafts? 3 Which pulley "drives" the other pulleys? Motor pulley

For which of these purposes can a pulley system be used? All

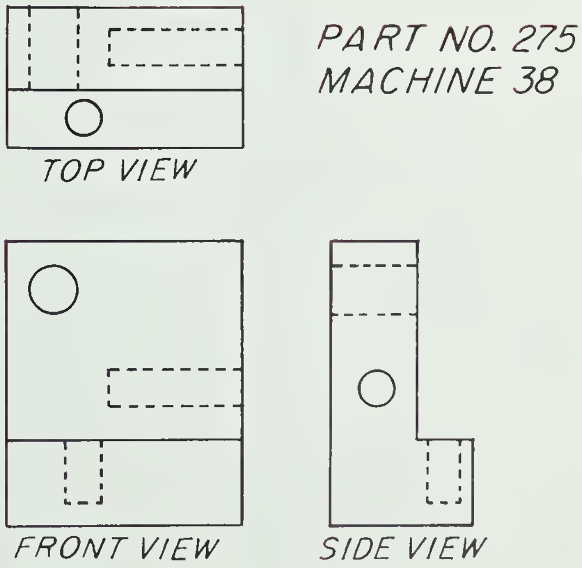
- to transfer power from one place to another
- to turn several shafts at different rates
- to transfer power around obstacles
- to use one motor to drive several machines



H. Armstrong Roberts

The dashed lines in the schematic below indicate edges that cannot be seen from that particular view.

How many holes are there in the piece? 3 How many of them are drilled from the front? 1 From a side? 1 From the bottom? 0 From the top? 1 Do any of the holes go all the way through the piece? Which ones? Yes; the one drilled from the front


PART NO. 275
MACHINE 38
TOP VIEW
FRONT VIEW
SIDE VIEW

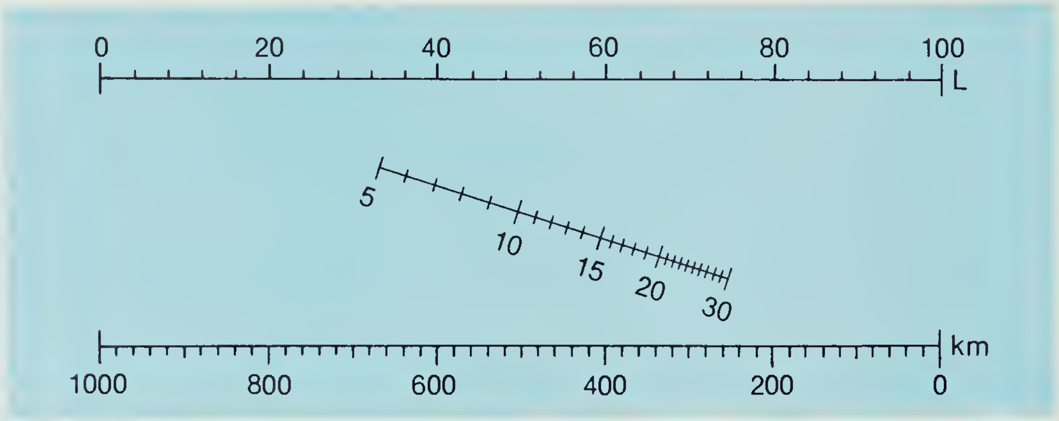
2.9 Using Nomographs

Performance Objective: The student can use simple nomographs such as those based on the four fundamental operations.

To find the rate of gas consumption (litres of gasoline your car uses for each 100 km), you can use this rule:

$$\text{l}/100\text{ km} = (\text{litres of gas used} \times 100) \div (\text{kilometres traveled})$$

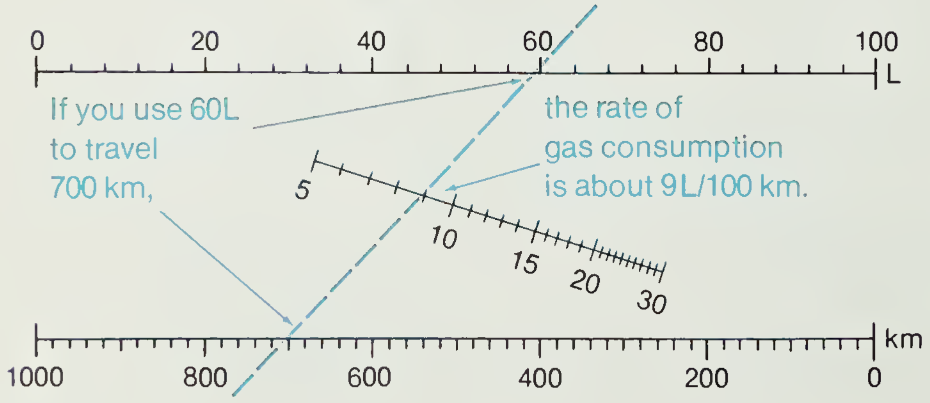
However, if you had to figure this very often, you could probably save time by using the **nomograph** shown below to *estimate* the rate of gas consumption (L/100 km). Another name for this kind of diagram is **nomogram**.



Steps for finding the rate of gas consumption (L/100 km):

1. On the top scale, find the number of litres of gas used.
2. On the bottom scale, find the number of kilometres traveled.
3. Lay any straight-edged object so that the edge goes through the points located in steps 1 and 2.
4. Read the litres per 100 km where the edge crosses the middle slanted scale.

Example:



If you use 60L to travel 700 km,

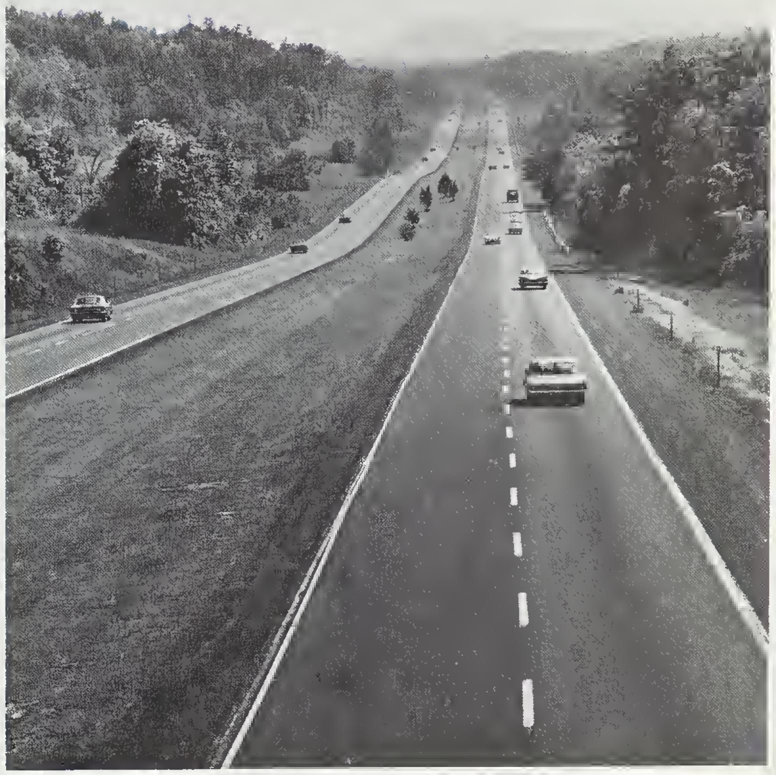
the rate of gas consumption is about 9L/100 km.

Of course, you can start with the middle scale and either the top or the bottom scale if you know the rate of gas consumption (L/100 km) and want to find the distance or the amount of gas.

1–2. Refer to the nomograph on page 86.

1. How many litres does each small part of the top scale represent? 5

2. How many kilometres does each small part of the bottom scale represent? 50



A. Devaney, Inc.

3–6. On the nomograph on page 86, how many litres per 100 km does each small part of the middle scale represent between the given marks?

3. 5 and 10 1

4. 10 and 20 1

5. 20 and 30 1

6. 30 and 40 2

7. Does a nomograph give an estimate or an exact amount? Estimate

8. Is an estimate "good enough" in this kind of problem? Yes

9–16. Use the nomograph to estimate the gas consumption (L/100 km) for each distance and amount of gas.

9. 500 km, 75 L 15

10. 400 km, 44 L 11

11. 240 km, 24 L 10

12. 500 km, 55 L 11

13. 225 km, 45 L 20

14. 425 km, 51 L 12

15. 625 km, 75 L 12

16. 525 km, 84 L 16

17–22. Use the nomograph to estimate how far you can drive for each rate of gas consumption and amount of gas.

17. 8 L/100 km, 40 L 500 km

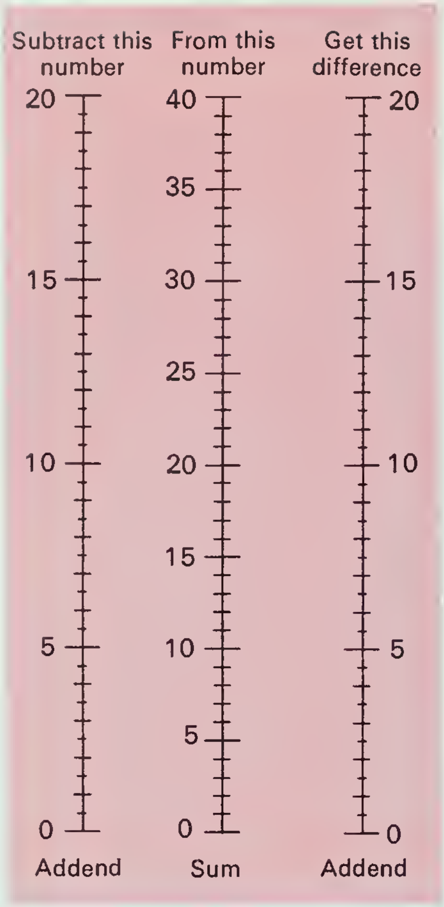
18. 10 L/100 km, 25 L 250 km

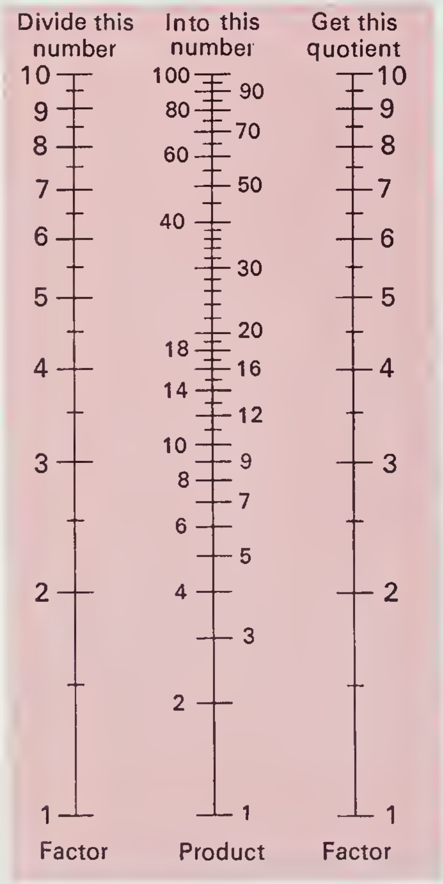
19. 12 L/100 km, 60 L 500 km

20. 8 L/100 km, 50 L 625 km

21. 9 L/100 km, 65 L 722 km

22. 13 L/100 km, 32 L 246 km


Subtract this number
From this number
Get this difference
20
40
20
35
15
30
15
25
10
20
10
15
5
10
5
5
0
0
0
Addend
Sum
Addend


Divide this number
Into this number
Get this quotient
10
100
90
10
9
80
70
9
8
60
8
7
50
7
6
40
6
5
30
5
20
18
16
4
14
4
12
10
9
3
8
7
3
6
5
2
4
2
3
2
1
1
1
Factor
Product
Factor

23–42. A nomograph is usually labeled for a specific use. But often what it is really doing is estimating sums and differences or products and quotients. Use the nomographs shown here to estimate each result.

23. $15 + 20$ 35

24. $20 + 10$ 30

25. $9 + 19$ 28

26. $16 + 19$ 35

27. $14.5 + 16.5$ 31

28. $17.5 + 12.5$ 30

29. $21 - 18$ 3

30. $23 - 19$ 4

31. $29 - 17.5$ 11.5

32. $32 - 18.5$ 13.5

33. 5×7 35

34. 7×4 28

35. 8×3.1 24.8

36. 7×4.3 30.1

37. 2.4×4.8 11.5

38. 3.3×6.2 20.5

39. $20 \div 6$ 3.3

40. $14 \div 3$ 4.7

41. $1.8 \div 1.4$ 1.3

42. $2.6 \div 1.8$ 1.4

43–46. Refer to the nomographs.

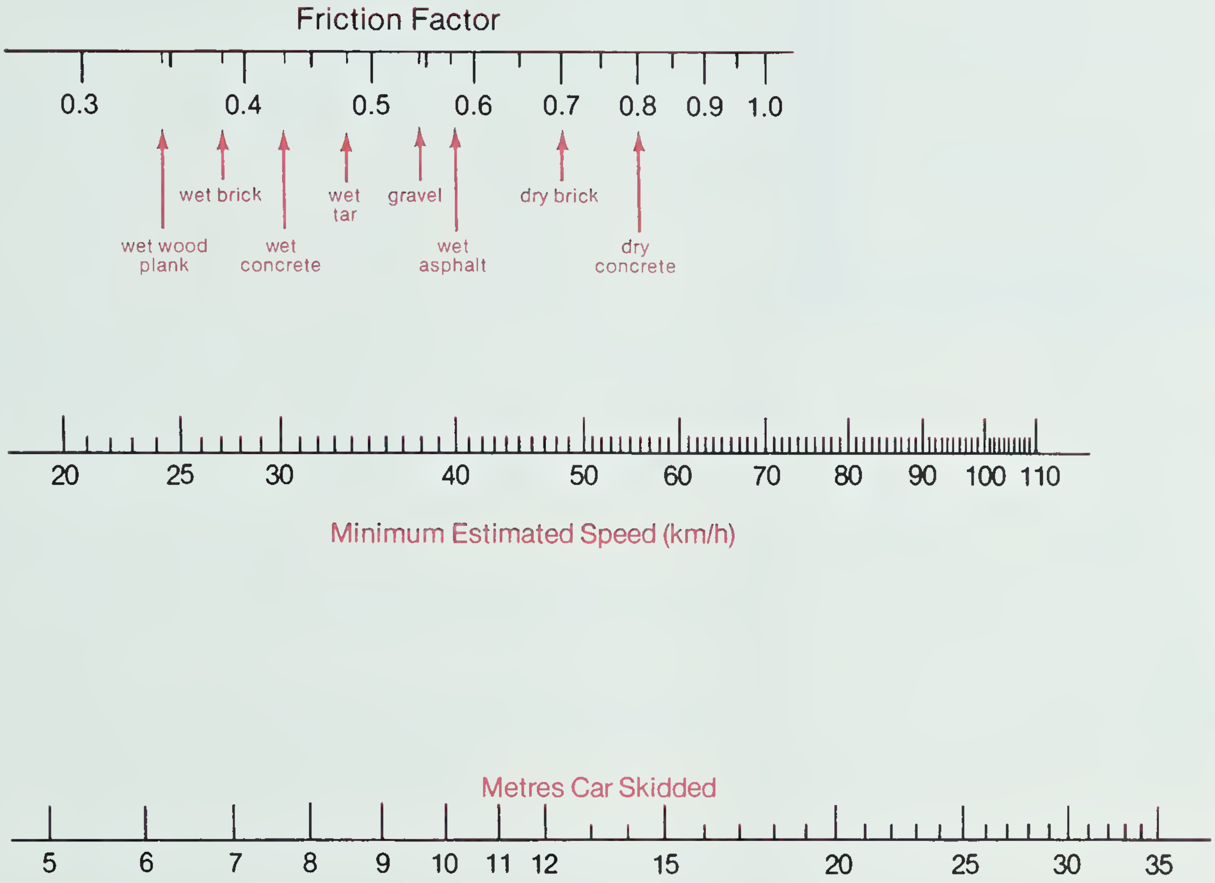
43. Suppose both addend scales were multiplied by 10. That is, suppose they were labeled 0, 50, 100, 150, 200. What would the numbers on the sum scale have to be multiplied by? 10

44. Suppose one addend scale was increased by 10. That is, suppose it was labeled 10, 15, 20, 25, 30. How would the sum scale have to be changed?
Increased by 10

45. Suppose one factor scale was multiplied by 10. That is, suppose it was labeled 10, 20, 30, 40, 50, 60, 70, 80, 90, 100. How would the product scale have to be changed? Multiplied by 10

46. Suppose both factor scales were multiplied by 10. How would the product scale have to be changed?
Multiplied by 100

47–54. What happens when a driver sees a hazard in the road ahead? Most drivers "hit the brakes" and skid to a stop! If all four tires skid and the car doesn't hit anything, the nomograph below can be used to estimate how fast the car was going from the length of the skid marks. (The effect of friction for each kind of road surface is given by the *friction factor*.)


Friction Factor
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
wet brick
wet tar
gravel
dry brick
wet wood plank
wet concrete
wet asphalt
dry concrete
20
25
30
40
50
60
70
80
90
100
110
Minimum Estimated Speed (km/h)
Metres Car Skidded
5
6
7
8
9
10
11
12
15
20
25
30
35

47–50. About how fast was a car going on dry concrete if it skidded the given distance?

47. 35 m 84 km/h

48. 30 m 78 km/h

49. 12 m 50 km/h

50. 9 m 43 km/h

51-54. About how fast was a car going if it skidded 30 m on a road with the given surface?

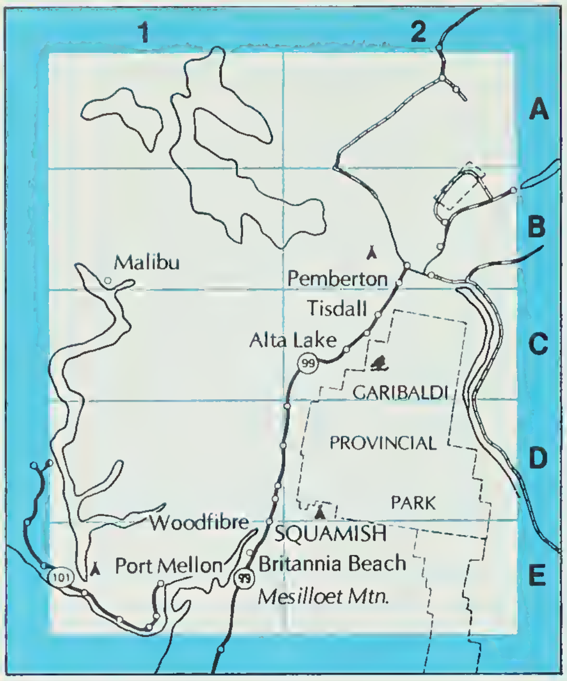
51. wet brick 55 km/h

52. gravel 67 km/h

53. wet asphalt 66 km/h

54. wet concrete 57 km/h

Test for Chapter 2



1
2
A
B
C
D
E
Malibu
Pemberton
Tisdall
Alta Lake
99
GARIBALDI
PROVINCIAL
PARK
Woodfibre
SQUAMISH
Port Mellon
Britannia Beach
101
99
Mesilloet Mtn.

Refer to the map. (2.1)

1. In what region is Malibu located? B-1

2. What mountain is located in region E-2? Mesilloet

Graph and label each point.

3-5. *Graphs*

3. (6, 3), (1, 5), (3, 3), (2, 0) (2.2)

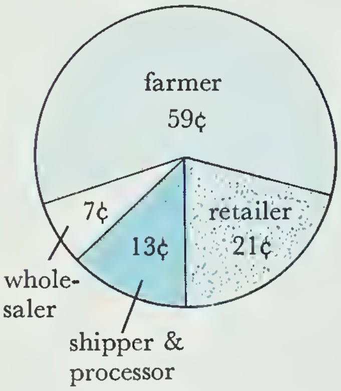
4. (2.3)

x	4	0	6	$2\frac{1}{2}$	5	2
y	1	3	8	5	$4\frac{1}{2}$	$\frac{1}{2}$

5. Make a line graph from this table of mean incomes. (2.4)

Year	1972	1973	1974	1975
Income $	5192	5539	6042	6403

YOUR CHICKEN DOLLAR



6. Graph the points. Join each set of points in order. (2.5)

Stork

x	1	1	2	$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	5	$5\frac{1}{2}$	$5\frac{1}{2}$	5	6	$4\frac{1}{2}$	$4\frac{1}{2}$	5	5
y	3	7	11	$11\frac{1}{2}$	11	13	14	14	$13\frac{1}{2}$	$12\frac{1}{2}$	12	8	12	11	$10\frac{1}{2}$	3

x	5	4	4	$3\frac{1}{2}$	$2\frac{1}{2}$	2	2	1	6	6	0	0	1
y	$2\frac{1}{2}$	0	2	1	1	2	0	$2\frac{1}{2}$	$2\frac{1}{2}$	3	3	$2\frac{1}{2}$	$2\frac{1}{2}$

Put a dot at (5, 13).

7. Below are normal masses (in kilograms) of the adult males of some animals. Make a bar graph. (2.6)

Graph

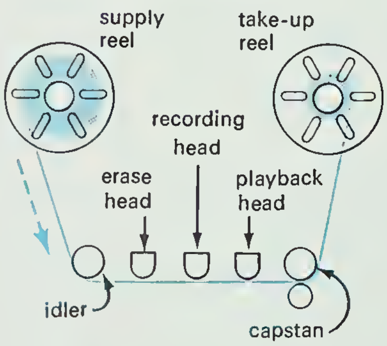
Kangaroo	72	Gorilla	180
Kodiak bear	495	Siberian tiger	260

RATE YOURSELF

Number Correct	
10	Superior
9	Excellent
8	Good
6–7	Fair
0–5	Poor

8. The circle graph shows that the retailer of chickens receives about three times as much as the Wholesaler. (2.7)

Tape Recorder



supply
reel
take-up
reel
recording
head
erase
head
playback
head
idler
capstan

9. According to the schematic, in what order does the tape pass the three heads? (2.8)
Erase head, recording head, playback head

10. A nomograph for adding two numbers needs how many scales? What other operation can it be used for? (2.9)
3; subtraction

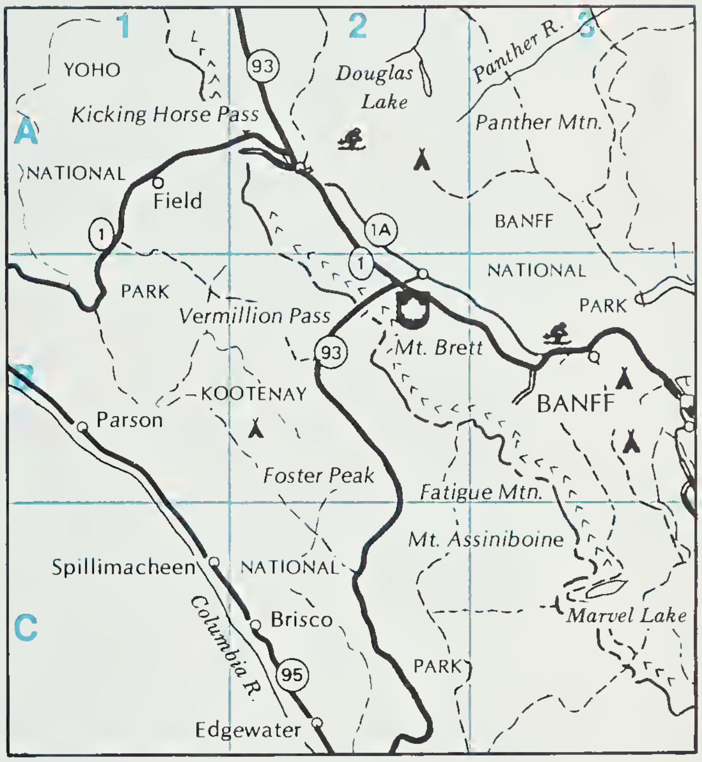
Chapter 2 Review

Refer to the map. (2.1)

1. In what region is each of these?

a.	Kicking Horse Pass	A-2
b.	Foster Peak	B-2
c.	Marvel Lake	C-3
d.	⛷	A-2, B-3

2. Which regions have camping areas (indicated by ⛺)
A-2, B-2, B-3



1
2
3
A
B
C
YOHO
Kicking Horse Pass
NATIONAL
Field
PARK
Douglas Lake
Panther R.
Panther Mtn.
BANFF
NATIONAL
PARK
93
1A
1
Vermillion Pass
Mt. Brett
BANFF
KOOTENAY
Parson
Foster Peak
Fatigue Mtn.
Mt. Assiniboine
Spillimacheen
NATIONAL
Columbia R.
Brisco
95
PARK
Marvel Lake
Edgewater

Graph and label each point.

3–5. *Graphs*

3. (6, 4), (5, 5), (0,3), (0, 0) (2.2)

4. (2.3)

x	6	3	4	5	$2\frac{1}{2}$	0
y	6	0	$2\frac{1}{2}$	2	4	1

5. Make a line graph from this table of cooking times for turkeys. (2.4)

Mass (kg)	4.5	5.5	6.5	7.5	8.5
Cooking time (h)	4	4.5	5	5.5	6

6. Graph the points. Join each set of points in order. (2.5)

Cicada

x	$1\frac{1}{2}$	$1\frac{1}{2}$	2	$2\frac{1}{2}$	2	4	$4\frac{1}{2}$	5	7	$6\frac{1}{2}$	$4\frac{1}{2}$	$2\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$	8	8
y	5	6	7	7	3	1	$4\frac{1}{2}$	1	3	8	$4\frac{1}{2}$	8	8	9	10	11

x	$2\frac{1}{2}$	$2\frac{1}{2}$	3	3	$2\frac{1}{2}$	$2\frac{1}{2}$	3	4	5	$6\frac{1}{2}$	$6\frac{1}{2}$	6	6	7	$7\frac{1}{2}$	8
y	7	8	8	9	9	$9\frac{1}{2}$	$9\frac{1}{2}$	10	10	$9\frac{1}{2}$	9	9	8	8	$7\frac{1}{2}$	6

x	$6\frac{1}{2}$	7	$7\frac{1}{2}$	$7\frac{1}{2}$
y	7	7	6	5

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	$1\frac{1}{2}$	1	1
y	6	$7\frac{1}{2}$	8	8	9	10	11

x	4	$4\frac{1}{2}$	5
y	11	10	11

Make a bar graph to show the given information. (2.6)

7–8. Graphs

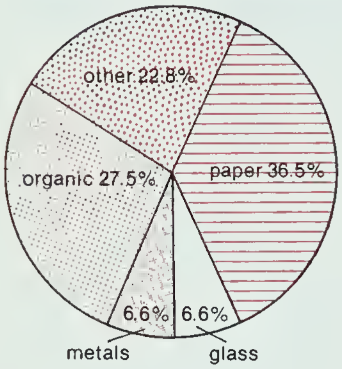
7. Life expectancy of females (in years)

Sweden	77.4	United States	75.2
Poland	73.7	Canada	75.2
India	40.5	Afghanistan	37.5

8. Species of wildlife that vanished from North America.

	From 1600 to 1850	From 1850 to 1973
Mammals	2	17
Birds	3	28
Fish	0	12

Residential Solid Waste



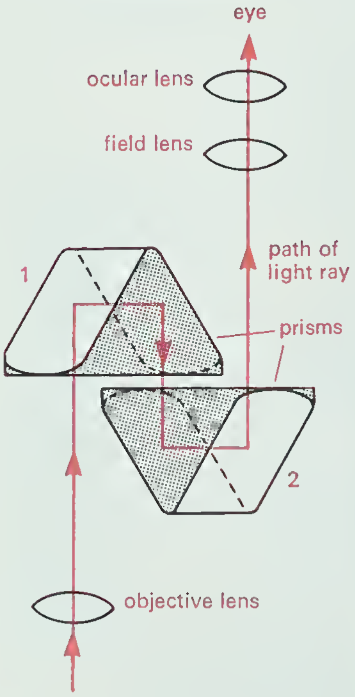
Refer to the circle graph. (2.7)

9. Out of every 100 t of residential solid waste, how many are organic materials like food and wood? 27.5

10. More than a third of all residential solid waste is Paper .

Refer to the schematic. (2.8)

Lenses & Prisms in Binoculars



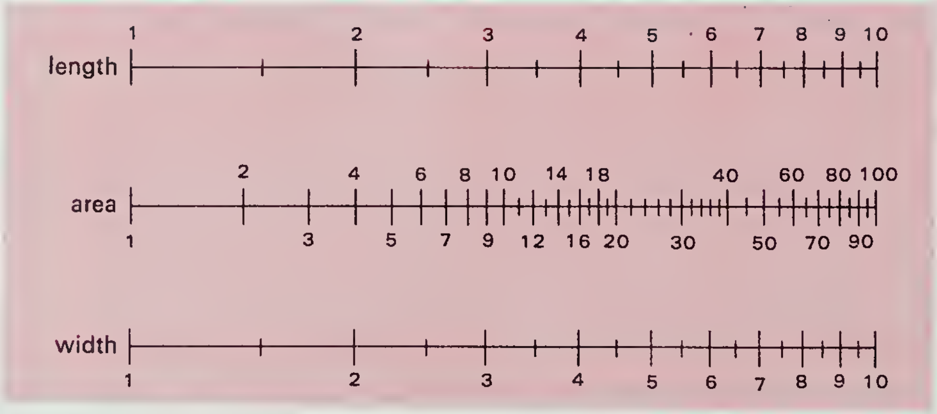
11. How many lenses are shown in the schematic? How many prisms? 3; 2

12. Name the parts of the binoculars in the order in which light passes through them to reach the eye.
Objective lens, prism 1, prism 2, field lens, ocular lens

Use the nomograph to estimate each area. (2.9)

13. length: 7, width: 8.5
60

14. length: 4.5, width: 3.5
15.5



Refer to the bar graph. (1.1)

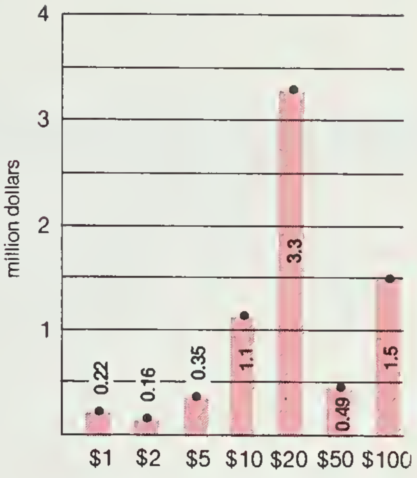
1. Which was greater, the value of the $50 bills or the value of the $5 bills in circulation in 1975? $50

2. The value of $10 bills in circulation in 1975 was about one sixth that of $20 bills.

3. Which bills in circulation in 1975 had the least total value? $1

4. About how many $1 bills were in circulation in 1975? 0.22 million

VALUE OF CAN. PAPER MONEY IN CIRCULATION, 1975



Source: Canada Yearbook

Refer to the line graph (1.2)

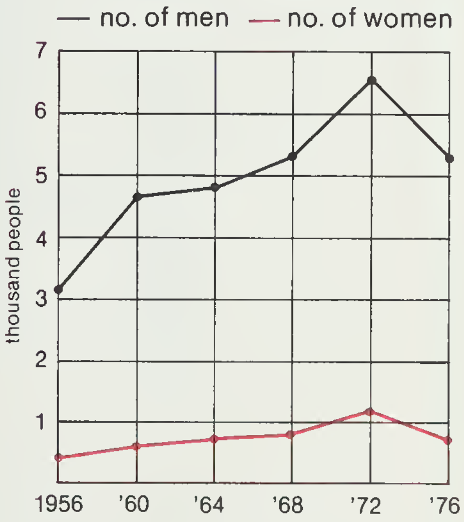
5. For what years are the Olympic Games shown? Every four years from 1956-1976

6. Did the number of women participants or men participants have the more gradual increase? Women

7. Compare participation for 1976 with that for 1956. Then pick the truest statement.

   a. Male participation had the biggest increase.
   b. Female participation had the biggest increase.
   c. Both increased about the same.

PARTICIPATION IN OLYMPICS



8. In which year(s) did the number of men and women participants decrease? 1976

Refer to the bus schedule. (1.3)

9. At what time does bus 530 leave Amherst? 12 15

10. At what time does bus 533 reach Amherst? 13 50

11. At what time is there a bus from Summerside to Amherst? 08 20, 11 10

12. Which buses make signal stops? 530, 531, 532, 533

AMHERST — CHARLOTTETOWN

* 532 Daily Quot.	* 530 Daily Quot	Atlantic Time Heure de l'Atlantique		* 531 Daily Quot.	* 533 Daily Quot.
19 10	12 15	Dp Amherst, N.S.	Ar	10 50	13 50
⑨	⑨	Port Elgin, N.B.		⑨	⑨
⑨	⑨	Cape Tormentine, N.B.		⑨	⑨
		} Ferry- Traversier			
⑨	⑨	Borden, P.E.I.		⑨	⑨
22 00	14 45	Summerside		08 20	11 10
⑨	⑨	Kensington		⑨	⑨
⑨	⑨	Hunter River		⑨	⑨
23 10	15 55	Ar Charlottetown, P.E.I.	Dp	07 10	10 00

* All coach or bus seats must be reserved
⑨Stops on signal

Refer to the graphs and table on page 93. (1.4)

13. Do any of the graphs use a percent scale? No

14. What does the ⌇ in the line graph show? Omission of part of the scale

15. From the line graph, which of these is the better guess?

a. Male participation in the 1980 Olympics changed significantly.
b. Female participation in the 1980 Olympics changed significantly.

16. If the bar graph were a pictograph and each picture stood for one million dollars, how many whole pictures would be used in showing the value of each of these? (1.5)

a. $20 bills 3
b. $5 bills 1
c. $1 bills 0
d. $50 bills 1

Using sets A and B shown at the right, find each mode, median, mean, and range.

A

10	9	8
10	8	11
12	11	10
11	10	

B

3	1	2
5	9	1
3	10	9
9	10	10

17. mode of set A 10
18. mode of set B 9, 10 (1.6)

19. median of set A 10
20. median of set B 7 (1.7)

21. mean of set A 10
22. mean of set B 6 (1.8)

23. range of set A 8–12 or 4
24. range of set B 1–10 or 9 (1.9)

25.

29	1
31	1
32	4
33	1
35	1
37	2
38	1
40	1
41	2

Use this set of numbers. (1.10)

32, 29, 31, 32, 37, 41, 33, 32, 37, 35, 38, 40, 32, 41

25. Make a frequency table. *Answer at left.*

26. Find (**a**) the range, (**b**) the mode, (**c**) the median, and (**d**) the mean. **a.** 29–41 or 12 **b.** 32 **c.** 34 **d.** 35

27. The normal monthly temperatures (in degrees Celsius) for Winnipeg are given. Find the mean monthly temperature and the median monthly temperature. (1.11)

-17.8, -11.6, -9.9, 5.9, 11.8,
18.2, 19.7, 19.3, 13.3, 3.1,
-5.8, -18.5

Median: 4.5°C; mean: 2.3°C

Refer to the map. (2.1)

28. In what region is each of these?

a. Animaland C-2

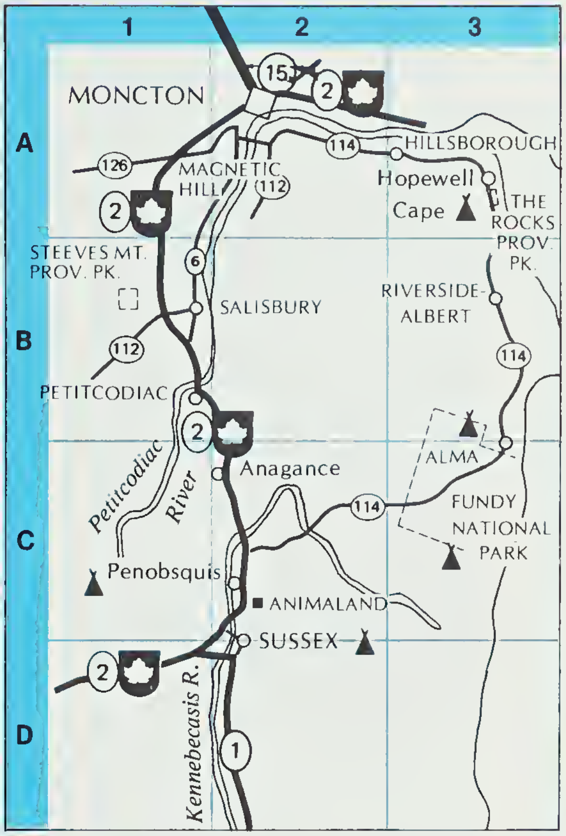
b. Magnetic Hill A-1

c. Airport A-2

d. Fundy National Park C-3

29. What provincial park is in region B-1?

Steeves Mountain


1
2
3
A
B
C
D
MONCTON
15
2
126
MAGNETIC HILL
112
114
HILLSBOROUGH
Hopewell
Cape
THE ROCKS PROV. PK.
STEEVES MT. PROV. PK.
6
SALISBURY
RIVERSIDE-ALBERT
112
114
PETITCODIAC
2
Anagance
ALMA
Petitcodiac River
114
FUNDY NATIONAL PARK
Penobsquis
ANIMALAND
SUSSEX
2
Kennebecasis R.
1

30–32. Graphs

30. Graph and label each point. (2.2)
(3, 0), (1, 4), (5, 5), (4, 1), (0, 2)

31. Graph and label each point. (2.3)

x	2	3	$4\frac{1}{2}$	6	0	3
y	4	3	1	$2\frac{1}{2}$	5	0

32. Make a line graph of this information. (2.4)

Thousands of Kilometres of Paved Highway Open to Traffic

Year	1966	1967	1968	1969	1970	1971	1972
Thousand kilometres	106.9	106.1	116	117.8	120.6	125.4	187.2

33. Graph the points. Join each set of points in order. (2.5)

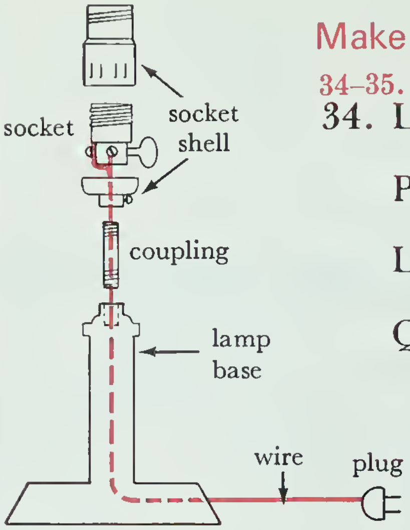
Praying mantis

x	15	16	$15\frac{1}{2}$	16	$13\frac{1}{2}$	$13\frac{1}{2}$	$14\frac{1}{2}$	15	15	$14\frac{1}{2}$	20	21
y	4	5	$5\frac{1}{2}$	7	5	$7\frac{1}{2}$	$8\frac{1}{2}$	8	9	$9\frac{1}{2}$	12	12

x	20	19	17	14	$14\frac{1}{2}$	14	$9\frac{1}{2}$	4	3	4	7	9	13	13	$14\frac{1}{2}$	16	16
y	13	13	12	9	$8\frac{1}{2}$	$8\frac{1}{2}$	4	3	$2\frac{1}{2}$	2	2	3	7	$4\frac{1}{2}$	5	7	5

x	11	$10\frac{1}{2}$	9	8	7	5	3	$2\frac{1}{2}$
y	0	0	5	$2\frac{1}{2}$	$1\frac{1}{2}$	5	0	0

x	4	$1\frac{1}{2}$	2	3	5	7
y	2	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$



Make a bar graph to show the given information. (2.6)

34–35. Graphs

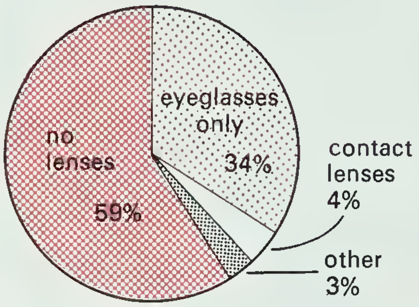
34. Length of Canadian bridges, in metres

Pierre Laporte	66.75	Thousand Islands	51.76
Lions Gate	47.24	Ambassador Bridge	56.39
Quebec Bridge	54.86	Angus L. MacDonald	44.10

35. Consumer Price Index

	1972	1973	1974	1975	1976
Canada	104.8	112.7	125.0	138.5	148.9
U.S.	97.6	103.7	114.5	125.5	132.8

Use of Corrective Eye Lenses Among Persons 17–24 Years Old (1971)



Refer to the circle graph. (2.7)

36. Out of every 100 persons 17 to 24 years old, how many do not wear corrective eye lenses? 59

37. How many people out of every 100 in this age group wear contact lenses? 4

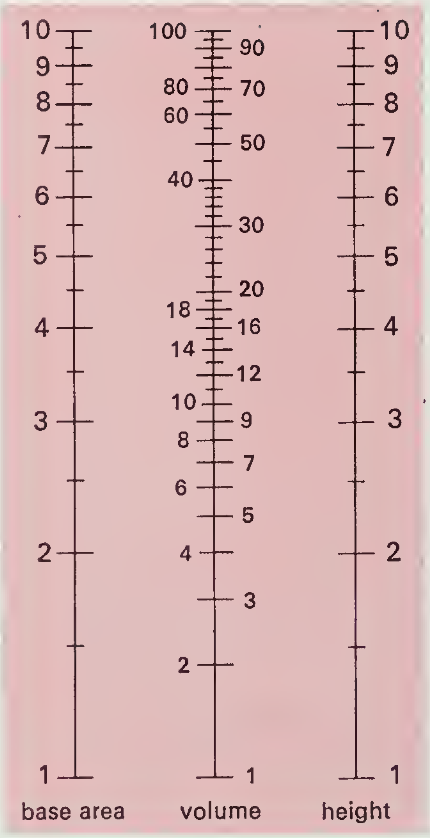
Refer to the schematic of a lamp. (2.8)

38. What two parts are connected by the wire? Plug, socket

39. When the lamp is put together, what covers the socket? Socket shell

40. What two parts does the coupling connect? Lamp base, socket shell

Use the nomograph to estimate each missing number. (2.9)



	base area	height	volume
41.	3.5	6.3	22
42.	7.4	4	30

Preview Test for Chapter 3

Tell if each sentence is true, false, or open. (3.1)

1. $3 + 5 = 2 + 6$ True **2.** $x + 8 = 10$ Open

3. $63 = 7 \times 8$ False **4.** $5n = 20$ Open

Find each answer when x is replaced with 6. (3.2)

5. $3x$ 18 **6.** $2(x - 3)$ 6 **7.** $x(5 - 3)$ 12

Find each answer. (3.3)

8. $12 + 8 - 4$ 16 **9.** $9 + 18 \div 3$ 15 **10.** $3(4) - 2(5)$ 2

Use the given formula to solve. (3.4)

11. A car travels at a rate of 90 km/h for 4 h. What distance will it go? ($d = rt$) 360 km

12. A rectangle is 5 m wide and 8 m long. Find its perimeter. ($P = 2\ell + 2w$) 26 m

Solve and check.

13. $w - 8 = 10$ 18 **14.** $14 = x + 6$ 8 (3.5)

15. $3n = 18$ 6 **16.** $9 = \frac{z}{5}$ 45 (3.6)

17. $3a + 2 = 14$ 4 **18.** $2 = \frac{x}{3} - 4$ 18 (3.7)

19. $4y = 7$ $\frac{7}{4}$ **20.** $7 = 5b + 4$ $\frac{3}{5}$ (3.8)

21. $\frac{4}{5}n = 20$ 25 **22.** $\frac{5}{3}c + 2 = 12$ 6 (3.9)

Use the given formula. (3.10)

$m = \frac{5}{4}h - 150$

m: mass (kg)

h: height (cm)

23. By the formula, what is the mass of someone who is 164 cm tall? 55 kg

$P = tn - c$

P: profit

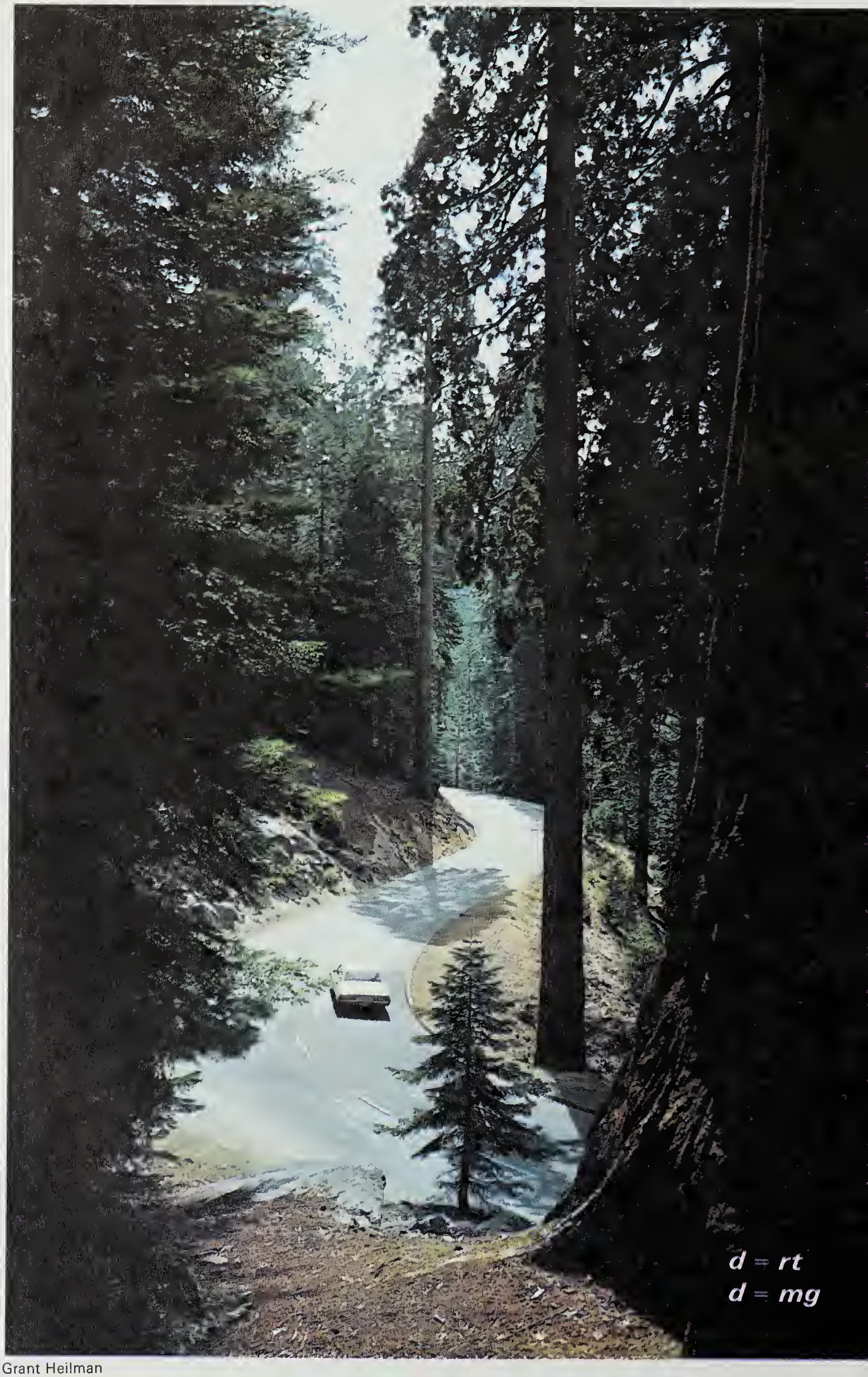
t: price per ticket

n: number of tickets

c: daily cost

24. A theater charges \$2 per ticket. The daily cost of running the theater is \$300. How many tickets must be sold each day to make a profit of \$200? 250

3 Equations and Formulas



$d = rt$

$d = mg$

Grant Heilman

PICTURE PREVIEW: In whatever you do, it is likely you will run into problems that can be solved with equations or formulas. What are equations and formulas? How do you work with them? These are the questions this chapter will answer. The pictures here take a brief look at the many uses for formulas that are coming up.

Artstreet

Photo Research International

THE RECORD SHOP
$C = cn + t$

Means/Van Cleve Photography

3.1 Open Sentences

In both English and mathematics, a sentence can be true, false, or open.

Performance Objective:
The student can
- identify an English or mathematical sentence as true, false, or open.
- find solutions to open sentences by trial and error.

Examples:

	English sentences		Mathematical sentences
1.	Sir John A. MacDonald was the first Prime Minister of Canada.	◀ True ▶	$3 + 7 = 10$
2.	Montreal is in Ontario.	◀ False ▶	$11 - 2 = 4$
3.	pronoun She was the first woman to fly solo over the Atlantic.	◀ Open ▶ (neither true nor false)	variable $x - 4 = 18$
	The pronoun can be replaced with a name.		The variable can be replaced with a number.
4.	Amelia Earhart was the first woman to fly solo over the Atlantic.	◀ True ▶	$22 - 4 = 18$
5.	Billie Jean King was the first woman to fly solo over the Atlantic.	◀ False ▶	$10 - 4 = 18$

Frank Kuechmann

Wide World Photos

Photri

Exercises

1. An Open sentence is neither true nor false.

2. A variable can be replaced with a Number.

3–14. Tell if each sentence is true, false, or open.

3. Lake Erie is one of the Great Lakes. True

4. He flew a kite to discover electricity. Open

5. Babe Ruth hit many home runs. True

6. Australia is the largest continent. False

7. One metre is shorter than one centimetre. False

8. It is red and white on the outside, gray on the inside. Open

9. $7 \times 9 = 63$ True

10. $5 + x = 8$ Open

11. $6 + 13 = 13 + 6$ True

12. $48 \div 8 = 2 \times 6$ False

13. $2 \times a = 12$ Open

14. $24 = 4 + y$ Open

15–20. Copy and complete the chart.

Open sentence	Variable	Replace variable with 3	Replace variable with 5
Example: $n + 5 = 8$	n	$3 + 5 = 8$, True	$5 + 5 = 8$, False
15. $4 + x = 9$	x	$4 + 3 = 9$, False	$4 + 5 = 9$, True
16. $8 = 13 - a$	a	$8 = 13 - 3$, False	$8 = 13 - 5$, True
17. $12 = 4 \times c$	c	$12 = 4 \times 3$, True	$12 = 4 \times 5$, False
18. $w - 2 = 3$	w	$3 - 2 = 3$, False	$5 - 2 = 3$, True
19. $10 = p + 7$	p	$10 = 3 + 7$, True	$10 = 5 + 7$, False
20. $5 \times b = 15$	b	$5 \times 3 = 15$, True	$5 \times 5 = 15$, False

21–30. Find a replacement to make each sentence true.

21. He is prime minister of Canada right now. *Answers will vary.*

22. It is known as the King of Beasts. Lion

23. It is the smallest province in Canada. Prince Edward Island.

24. It is a river running through Egypt. Nile

25. $4 \times y = 16$ 4

26. $15 = a + 7$ 8

27. $9 = n - 2$ 11

28. $b = 3 \times 4$ 12

29. $7 + x = 10$ 3

30. $4 \times w = 24$ 6

3.2 Parentheses

Performance Objective: The student can simplify expressions containing parentheses.

One use of parentheses ( ) is to indicate which operation to do first.

Example 1: Find each answer.

a. $(2 + 3) \times 4$
5×4 Add first.
20 Then multiply.

b. $2 + (3 \times 4)$
$2 + 12$ Multiply first.
14 Then add.

A second use for parentheses is to indicate multiplication.

Example 2: Find each answer.

a. $4(7)$ means 4×7.
$4(7) = 28$

b. $2(7 - 3)$
$2(4)$ Subtract first.
8 Then multiply.

When there is a variable, parentheses are sometimes left out.

Example 3: Replace x with 2 and y with 6. Then find each answer.

a. $5x$ means $5(x)$ or $5 \times x$.
$5(2)$ Replace x.
10 Multiply.

b. xy means $x(y)$ or $x \times y$.
$2(6)$ Replace x and y.
12 Multiply.

Note: You *cannot* leave out the parentheses when there are only numbers. $4(7) = 28$, but 47 is forty-seven.

Exercises

1. Which *two* things are parentheses used to indicate?
   a. forty-seven **b.** which operation to do first
   c. multiplication **d.** that a variable is left out
2. Which *two* of the following mean $5 \times x$?
   a. $5(x)$ **b.** $5 \div x$ **c.** $\frac{5}{x}$ **d.** $5x$

3–6. Place ( ) in each so that the answer is 32.

3. $4 \quad 8$ $4(8)$

4. $6 - 2 \times 8$ $(6 - 2) \times 8$

5. $2 \quad 1 \quad 6$ $2(16)$

6. $4 + 4 \times 7$ $4 + (4 \times 7)$

7–24. Find each answer.

7. $(2 + 3) \times 4$ 20

8. $2 + (3 \times 4)$ 14

9. $2(7)$ 14

10. $3(4)$ 12

11. $5 + (2 \times 3)$ 11

12. $(4 + 1) \times 2$ 10

13. $(7 - 2) \times 3$ 15

14. $7 - (2 \times 3)$ 1

15. $(8 - 4) \div 2$ 2

16. $6 - (4 \div 2)$ 4

17. $2(5 + 3)$ 16

18. $3(6 - 4)$ 6

19. $4(7 + 5)$ 48

20. $5(6 + 7)$ 65

21. $7 - (3 + 2)$ 2

22. $(7 - 3) + 2$ 6

23. $12 - (4 + 5)$ 3

24. $(12 - 4) + 5$ 13

25–36. Replace *n* with 2 and *x* with 5. Then find each answer.

25. $5n$ 10

26. $4x$ 20

27. nx 10

28. xn 10

29. $5(n + 2)$ 20

30. $3(n + 4)$ 18

31. $4(x - 3)$ 8

32. $2(x + 4)$ 18

33. $x(5 - 3)$ 10

34. $n(4 + 1)$ 10

35. $3(n + x)$ 21

36. $4(x - n)$ 12

37–42. Substitute 7 for *c* in each open sentence. Then tell if the sentence is true or false.

Examples:

a. $4c = 28$

$4(7) = 28$ Replace *c* with 7.

$28 = 28$ Multiply.

True

b. $(c - 2) + 3 = 4$

$(7 - 2) + 3 = 4$ Replace *c* with 7.

$5 + 3 = 4$ Subtract first.

$8 = 4$ Then add.

False

37. $3c = 21$
$21 = 21$, True

38. $4c = 11$
$28 = 11$, False

39. $2(c - 2) = 7$
$10 = 7$, False

40. $3(9 - c) = 6$
$6 = 6$, True

41. $(c - 3) + 5 = 9$
$9 = 9$, True

42. $(c - 5) + 3 = 5$
$5 = 5$, True

3.3 Order of Operations

Performance Objective: The student can simplify expressions by using the rule for order of operations.

What is the answer to $2 + 3 \times 4$?
There are no parentheses to tell what to do first.

If you add first,

$$\underbrace{2 + 3} \times 4$$
$$\underbrace{5 \times 4}$$
$$20$$

If you multiply first,

$$2 + \underbrace{3 \times 4}$$
$$\underbrace{2 + 12}$$
$$14$$

You get different answers!

When there are no parentheses, we agree to the following rule:

Order of operations

First multiply and divide in order from left to right.

Then add and subtract in order from left to right.

Then everyone will get the same answer.
14 is the right answer for $2 + 3 \times 4$ above.

Example 1: Find each answer.

a. $\underbrace{9 - 4} + 2$

$\underbrace{5 + 2}$ Add or subtract from left to right.

7

b. $12 - \underbrace{9 \div 3}$

$\underbrace{12 - 3}$ Divide first.

9 Then subtract.

Example 2: Substitute 8 for x and 6 for y. Then find each answer.

a. $2x + 3$

$2(8) + 3$ Replace x with 8.

$16 + 3$ Multiply first.

19 Then add.

b. $\frac{y}{2} - 1$

$\frac{6}{2} - 1$ Replace y with 6.

$3 - 1$ Divide first.

2 Then subtract.

Exercises

1–6. Tell what should be done first to find each answer.

1. $2 + 3 \times 4$ Mult.
2. $9 - 4 + 2$ Subt.
3. $12 - 9 \div 3$ Div.
4. $\frac{12}{3} \times 2$ Div.
5. $8 + 2 \times 3$ Mult.
6. $8 - 2(3)$ Mult.

7–24. Find each answer.

7. $18 - 7 + 5$ 16
8. $23 - 2 + 4$ 25
9. $5 \times 4 + 7$ 27
10. $3 + 6 \times 4$ 27
11. $16 \div 2 \times 4$ 32
12. $12 \div 2 \times 3$ 18
13. $8 + \frac{12}{4}$ 11
14. $\frac{15}{3} + 2$ 7
15. $8 - 3(2)$ 2
16. $10 - 2(3)$ 4
17. $4(5) + 8$ 28
18. $5(2) + 3$ 13
19. $5(4) - 2$ 18
20. $2(5) - 3$ 7
21. $8 - 4 - 2$ 2
22. $9 - 5 - 3$ 1
23. $2(3) + 4 \div 2$ 8
24. $8 \div 2 + 3(2)$ 10

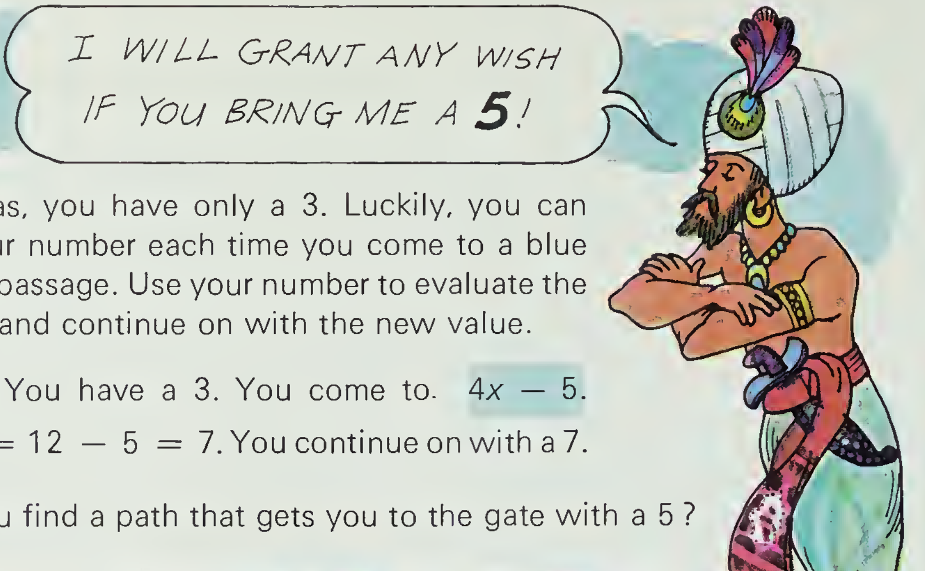
25–36. Substitute 8 for x and 6 for y. Then find each answer.

25. $2x + 1$ 17
26. $5 + 3y$ 23
27. $5y - 7$ 23
28. $4x - 5$ 27
29. $\frac{y}{3} + 9$ 11
30. $5 + \frac{x}{2}$ 9
31. $2x + 3y$ 34
32. $4y - x$ 16
33. $xy + 2$ 50
34. $50 - xy$ 2
35. $9 - \frac{24}{x}$ 6
36. $\frac{18}{y} - 3$ 0

37–44. Substitute 5 for n in each open sentence. Then tell if the sentence is true or false.

37. $3n + 2 = 17$ $17 = 17$, True
38. $4n + 2 = 27$ $22 \neq 27$, False
39. $6n - 4 = 34$ $26 \neq 34$, False
40. $5n - 3 = 22$ $22 = 22$, True
41. $21 = 3(n + 2)$ $21 = 21$, True
42. $16 = 4(n - 1)$ $16 = 16$, True
43. $25 = 2n + 3n$ $25 = 25$, True
44. $25 = 4n - 2n$ $25 \neq 10$, False

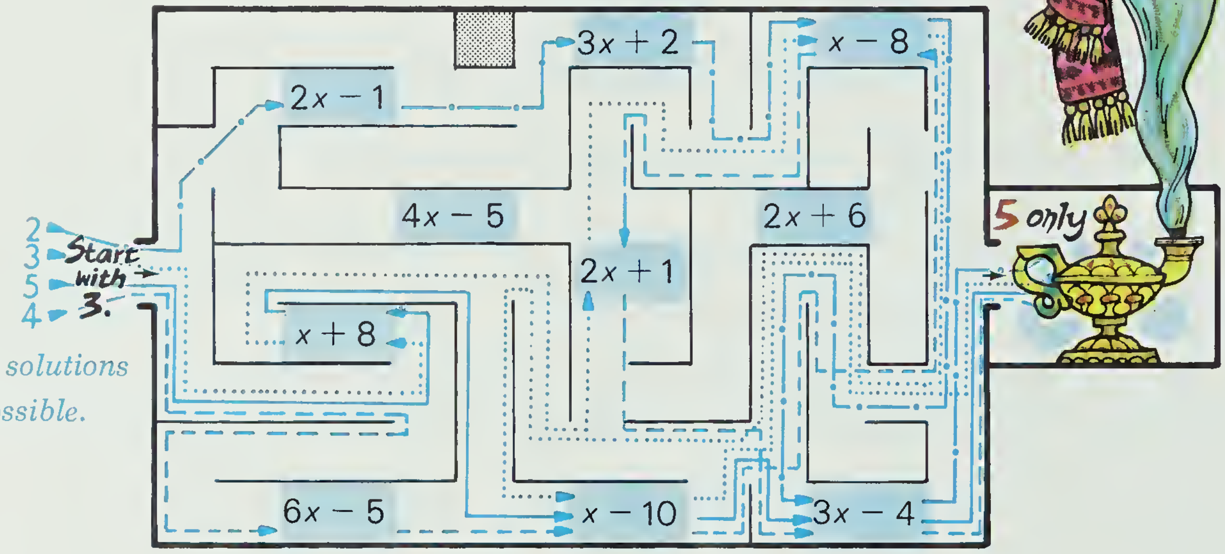
INGENIOUS GENIE


I WILL GRANT ANY WISH IF YOU BRING ME A 5!

But, alas, you have only a 3. Luckily, you can change your number each time you come to a blue expression passage. Use your number to evaluate the expression and continue on with the new value.

Example: You have a 3. You come to $4x - 5$. $4(3) - 5 = 12 - 5 = 7$. You continue on with a 7.

Can you find a path that gets you to the gate with a 5?



Other solutions are possible.

If you started with a 2, a 4, or a 5, you could also reach the end with a 5. Can you find those paths?

PUZZLE = 13,628,160

Give each letter of the alphabet the value of its position: *a* is 1, *b* is 2, and so on, up to *z* is 26. Since writing letters next to each other means multiply, you can find the "value" of a word.

Example: *and* $= 1(14)(4)$
$= 14(4)$
$= 56$ The value of *and* is 56.

1. Find the value of each of these: 336; 1620; 140; 341 250

up *for* *bean* *money*

2–3. Answers will vary.

2. Find the value of your first name.

3. Who has the most "value-able" name in class?

4. Check to see if the title "Puzzle = 13 628 160" is correct.
Title is correct.

Performance Objective: The student can substitute known values in a formula and simplify to find the value of the variable.

Using Formulas 3.4

A **formula** is a rule or a principle written as a mathematical sentence.

Principle

The distance an object travels is its rate of speed multiplied by its time of travel.

⬇

Formula

$$\underset{\text{distance}}{d} = \underset{\text{rate}}{r}\underset{\text{time}}{t}$$

Muench/Van Cleve Photography

Example 1: How far will a car go in 3 h at 90 km/h?

$d = rt$ Write the formula.

$d = 90(3)$ Substitute the given values.
The rate r is 90. The time t is 3.

$d = 270$ Multiply.

The car will go 270 km.

Example 2:

A person's *approximate* normal blood pressure can be found by dividing the person's age by 2 and adding 110.

Formula

$$\overset{\substack{\text{blood}\\ \text{pressure}\\ \downarrow}}{P} = \frac{\overset{\swarrow\ \text{age}}{a}}{2} + 110$$

Find the blood pressure of an average 16-year-old.

$P = \frac{a}{2} + 110$	Write the formula.
$P = \frac{16}{2} + 110$	Substitute the given values. The age a is 16.
$P = 8 + 110$	Divide first.
$P = 118$	Then add.

A 16-year-old's blood pressure is about 118.

Exercises

1. A formula is a rule or a principle written as a ______.
Mathematical sentence

2. Tell what each formula is used for.

a. Finding distance for a given rate and time

a. $d = rt$

b. $P = \frac{a}{2} + 110$

b. Finding blood pressure for a given age

3–6. Use the formula $P = \frac{a}{2} + 110$. Find the blood pressure P for a person with the given age a.

3. 14 years old 117

4. 24 years old 122

5. 30 years old 125

6. 60 years old 140

7–10. Use the formula $d = rt$. Find the distance d for the given rate r and time t.

7. Car: 60 km/h
2 h
120 km

8. Car: 70 km/h
3 h
210 km

9. Jet: 1000 km/h
4 h
4000 km

10. Turtle: 5 cm/min
10 min
50 cm

11–14. Use the formula $x = ab + c$. Find x for each a, b, and c.

Example: a is 5, b is 4, and c is 9.

$x = ab + c$	Write formula.
$x = 5(4) + 9$	Substitute.
$x = 20 + 9$	Multiply.
$x = 29$	Add.

	a	b	c	
11.	2	9	7	25
12.	4	5	8	28
13.	3	9	1	28
14.	6	7	14	56

15–16. Use the formula $d = 333t$ for the distance to a storm.

$d = 333t$

d: distance (in metres)

t: time (in seconds)

15. You hear thunder three seconds after you see lightning. How far away is the storm? 999 m

16. How far is the storm if there are five seconds between the lightning and the thunder? 1665 m

17–18. Find the mover's packing fees.

f = n x c

f = packing fee

n = number of cartons

c = cost per carton

17. If a mover packs a carton of books, the fee is \$6.05/carton. What would be the charges for 11 cartons? \$66.55

18. A mover charges \$16.45/carton to pack dishes. What would be the fee for 8 cartons of dishes? \$131.60

19–20. Use the perimeter formula.

19. Find the perimeter of a room 6 m wide and 9 m long. 30 m

20. How much fence is needed for a lot 9 m wide and 15 m long? 48 m

Quick Quiz

Check these skills:

- *adding whole numbers or decimals*
- *subtracting whole numbers or decimals*

1–4. Add.

1. $\begin{array}{r} 24 \\ +39 \\ \hline 63 \end{array}$ **2.** $\begin{array}{r} 327 \\ +408 \\ \hline 735 \end{array}$ **3.** $\begin{array}{r} 7.50 \\ +6.27 \\ \hline 13.77 \end{array}$ **4.** $\begin{array}{r} 14.95 \\ +6.26 \\ \hline 21.21 \end{array}$

5–8. Subtract.

5. $\begin{array}{r} 32 \\ -19 \\ \hline 13 \end{array}$ **6.** $\begin{array}{r} 519 \\ -324 \\ \hline 195 \end{array}$ **7.** $\begin{array}{r} 8.25 \\ -1.99 \\ \hline 6.26 \end{array}$ **8.** $\begin{array}{r} 34.25 \\ -9.08 \\ \hline 25.17 \end{array}$

3.5 Equations with Addition or Subtraction

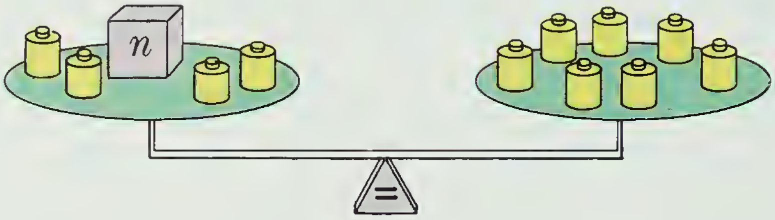
Performance Objective: The student can solve equations involving addition or subtraction.

A mathematical sentence containing an equal sign $=$ is called an **equation**.

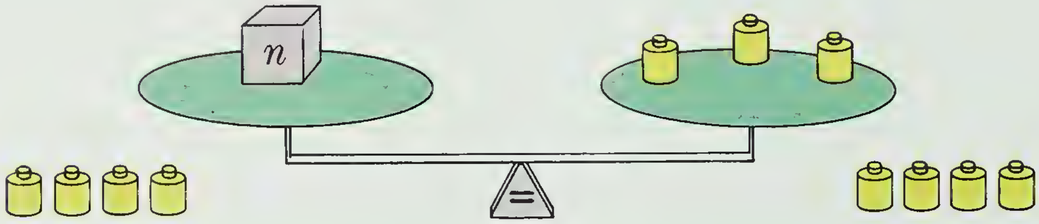
Example 1: Solve $n + 4 = 7$

To **solve** means to find that number which when substituted for the variable makes the equation true. When you solve an equation, you must always keep it in *balance*.

$n + 4 = 7$



4 weights have been added to the side with n. To find n, take away 4 weights. But to keep the balance, 4 weights must be taken away from *each* side.



The solution can be written like this.

$$\begin{array}{rcl} n + 4 &=& 7 \\ -4 && -4 \\ \hline n &=& 3 \end{array}$$

Write the equation.

Subtract 4 from each side.

It is always a good idea to check.

Check: $n + 4 = 7$ Write the equation.

$3 + 4 = 7$ Replace n with 3.

$7 = 7$ True (It checks.)

Example 2: Solve $23 = x + 17$

Notice 17 is added to x.
To *undo* this, subtract 17.

$$23 = x + 17$$

$$\underline{-17 \quad\quad -17}$$

Subtract 17 from each side.

$$6 = x$$

Check:

$$23 = x + 17$$

$$23 = 6 + 17$$

$$23 = 23$$

True (It checks.)

Example 3: Solve $y - 8 = 12$

Notice 8 is subtracted from y.
To *undo* this, add 8.

This step does not have to be written.

$$y - 8 = 12$$

$$\underline{+8 \quad\quad +8}$$

Add 8 to each side.

$$y = 20$$

Check:

$$y - 8 = 12$$

$$20 - 8 = 12$$

$$12 = 12$$

True (It checks.)

Exercises

1–6. What number should be added to or subtracted from each side?

1. $x + 9 = 13$ Subt. 9

2. $16 = y - 8$ Add 8

3. $w - 9 = 4$ Add 9

4. $a + 2 = 27$ Subt. 2

5. $19 = b + 2$ Subt. 2

6. $8 + t = 18$ Subt. 8

7–18. Solve and check each equation.

7. $x + 15 = 22$ 7

8. $8 = y + 6$ 2

9. $a - 9 = 4$ 13

10. $w - 4 = 11$ 15

11. $36 = x + 16$ 20

12. $b - 14 = 49$ 63

13. $27 = n - 13$ 40

14. $p - 22 = 48$ 70

15. $34 = n + 11$ 23

16. $56 = d + 25$ 31

17. $c - 56 = 4$ 60

18. $85 = z + 83$ 2

19–24. Solve and check each equation.

19. $w - 156 = 39$ 195

20. $234 = c + 83$ 151

21. $x + 32 = 142$ 110

22. $29 = s - 189$ 218

23. $145 = m - 399$ 544

24. $a + 279 = 486$ 207

Equations with decimals can be solved the same way.

Example:

$$\begin{array}{rcl} 8.4 & = & x - 5.5 \\ \underline{+5.5} & & \underline{+\ 5.5} \\ 13.9 & = & x \end{array}$$

Add 5.5 to each side.

25–30. Solve and check.

25. $7.9 = y + 2.8$ 5.1

26. $c + 2.5 = 7.7$ 5.2

27. $p - 6.50 = 9.95$ 16.45

28. $7.66 = b - 2.33$ 9.99

29. $w + 3.55 = 7.00$ 3.45

30. $8.45 = x + 4.55$ 3.9

31–38. For each exercise, substitute the given values into the formula. Then solve to find the variable not given.

Example: $a = b + c$, a is 28, c is 13

$$\begin{array}{rcl} a & = & b + c \\ 28 & = & b + 13 \\ \underline{-13} & & \underline{-\ 13} \\ 15 & = & b \end{array}$$

Write formula.

Substitute given values.

Subtract 13 from each side.

So b is 15.

	$a = b + c$		
	a	b	c
31.	93	17	76
32.	49	37	12
33.	37	28	9
34.	10.25	7.4	2.85

	$x = y - z$		
	x	y	z
35.	6	14	8
36.	54	76	22
37.	87	111	24
38.	10.25	12.50	2.25

39–44. Use the given formula to solve.

retail price = wholesale price + markup

$p = w + m$

39. A salesclerk must put a \$3 markup on a sweater that costs \$12 wholesale. What should the retail price be? \$15

40. A jacket has a retail price of \$22. The store's markup is \$6. What is the wholesale price? \$16

41. A tennis racket has a retail price of \$16 and a wholesale price of \$12. What is the markup? \$4

sale price = regular price − discount (markdown)

$s = p - d$

42. An LP that regularly sells for \$4.98 is marked down by \$1.49. What is the sale price? \$3.49

43. A rug is on sale for \$12.50. The discount from the regular price is \$2.60. Find the regular price. \$15.10

amount = principal + interest

$a = p + i$

44. An account opened two years ago with a principal of \$240 now has an amount of \$271.68. Find the interest. \$31.68

45–46. Write an equation and solve.

45. If you add 15 to x, you get 34. What is x? $x + 15 = 34$, 19

46. If you subtract 18 from y, you get 26. What is y? $y - 18 = 26$, 44

Quick Quiz

Check these skills:

- multiplying whole numbers or decimals
- dividing whole numbers

1–4. Multiply.

1. 24×17 408

2. 4.5×3.2 14.4

3. $12(254)$ 3048

4. $6.7(1.25)$ 8.375

5–8. Divide.

5. $96 \div 8$ 12

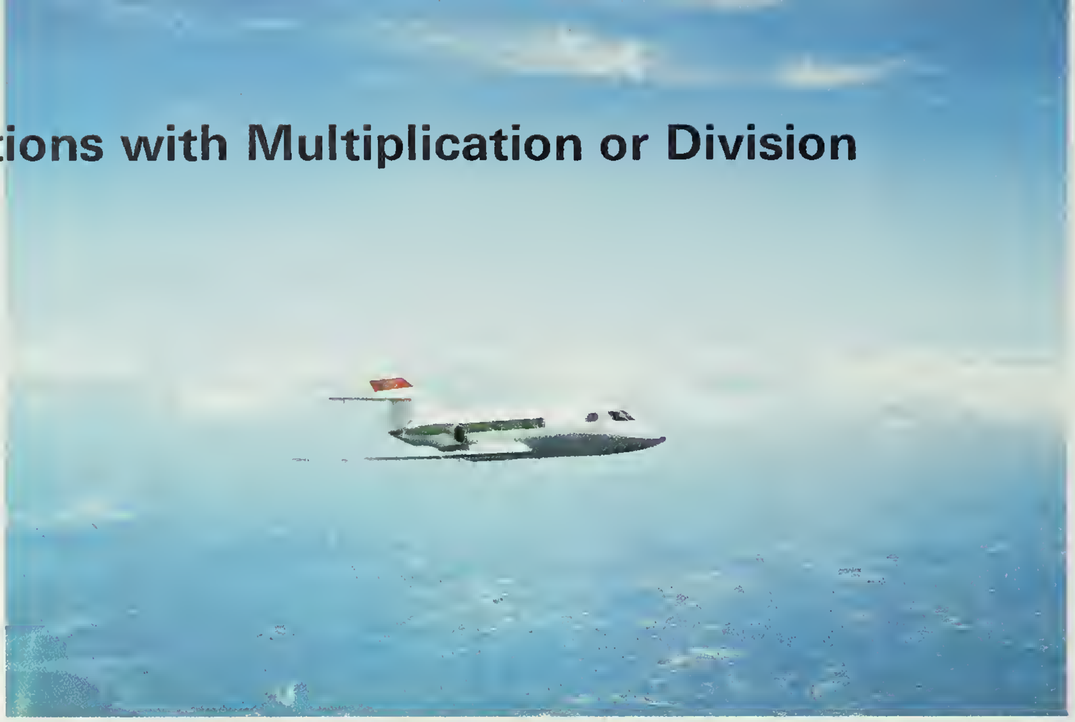
6. $312 \div 13$ 24

7. $\frac{135}{9}$ 15

8. $\frac{560}{35}$ 16

3.6 Equations with Multiplication or Division

Performance Objective: The student can solve equations involving multiplication or division.



A. Devaney, Inc.

A plane flies 1700 km from Montreal to Thunder Bay in 2 h. What is its average rate of speed?

To solve this, we can use the formula $d = rt$.

Write the formula. $d = rt$

The distance d is 1700 km. The time t is 2 h.

Substitute the given values into the formula. $1700 = r(2)$

In writing the product of a variable and a number, we usually write the number first.

Change the order of the multiplication. $1700 = 2r$

Now solve the equation.

Example 1: Solve $1700 = 2r$

Notice r is multiplied by 2. To *undo* this, divide by 2.

$$1700 = 2r$$

$$\frac{1700}{2} = \frac{2r}{2}$$ Divide each side by 2.

$$850 = r$$ The plane's speed is 850 km/h.

Check: $1700 = 2r$

$1700 = 2(850)$

$1700 = 1700$ True (It checks.)

Example 2: Solve $\frac{a}{3} = 15$

Notice *a* is divided by 3.
To *undo* this, multiply by 3.

$$\frac{a}{3} = 15$$

$$3\left(\frac{a}{3}\right) = 3(15) \qquad \text{Multiply each side by 3.}$$

$$a = 45$$

Check:

$$\frac{a}{3} = 15$$

$$\frac{45}{3} = 15$$

$$15 = 15 \qquad \text{True}$$

Exercises

1–6. By what number should each side be multiplied or divided?

1. $4y = 96$ Div. 4 **2.** $36 = 12b$ Div. 12 **3.** $14 = \frac{x}{4}$ Mult. 4

4. $\frac{c}{9} = 7$ Mult. 9 **5.** $23 = \frac{w}{6}$ Mult. 6 **6.** $5t = 65$ Div. 5

7–18. Solve and check each equation.

7. $3x = 18$ 6 **8.** $4a = 96$ 24 **9.** $50 = 10w$ 5

10. $2y = 128$ 64 **11.** $\frac{b}{4} = 10$ 40 **12.** $17 = \frac{y}{3}$ 51

13. $6d = 126$ 21 **14.** $245 = 7p$ 35 **15.** $17m = 289$ 17

16. $256 = 16n$ 16 **17.** $8 = \frac{a}{24}$ 192 **18.** $\frac{x}{13} = 9$ 117

19–26. For each exercise, substitute the given values into the formula. Then solve to find the variable not given.

	$a = bc$		
	a	b	c
19.	92	4	23
20.	104	8	13
21.	126	9	14
22.	324	27	12

	$\frac{x}{y} = z$		
	x	y	z
23.	102	6	17
24.	84	7	12
25.	133	19	7
26.	72	8	9

Equations with decimals can be solved in the same way

Example:

$$3x = 1.74$$

$$\frac{3x}{3} = \frac{1.74}{3}$$

$$x = 0.58$$

$$\begin{array}{r} .58 \\ 3\overline{)1.74} \\ \underline{1\,5} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

27–32. Solve and check each equation.

27. $3y = 4.23$ 1.41

28. $4n = 9.60$ 2.4

29. $8.46 = 6w$ 1.41

30. $24.95 = 5p$ 4.99

31. $\frac{a}{4} = 12.50$ 50

32. $4.75 = \frac{b}{12}$ 57

33–40. Use the given formula to solve.

33. If a car goes 248 km in four hours, what is its average rate of speed? 62 km/h

34. A car can keep up a rate of 76 km/h on a certain road. How much time will a 304 km trip take? 4 h

distance = rate × time

$$d = rt$$

Sunrise Photos

machine's output = machine's rate × running time

$$M = r \times t$$

35. A machine produces metal parts at the rate of 16/min. How long will it take the machine to produce 400 parts? 25 min

36. A postage meter can stamp and seal envelopes at the rate of 8/s. How long will it take to do 2400 envelopes? Give the answer in minutes. (min = s ÷ 60) 5 min

A light plane will glide about 1 km for every 100 m of altitude.

distance (in km) → $d = \frac{a}{100}$ ← altitude (in metres)

37. How far will a plane glide from an altitude of 2000 m? 20 km

38. A plane has an engine failure 30 km from the closest airport. How high would it have to be in order to glide safely to the airport? 3000 m

total wages = hourly pay × time

$W = p \times t$

39. For an 8 h. day, Denise's wages totaled \$37.20. What is her hourly rate? \$4.65/h

40. At \$4/h, how many hours would you have to work to make \$50? $12\frac{1}{2}$ h

41–44. Write an equation and solve.

41. 7 times a number n is 105. Find the number. $7n = 105$, 15

42. If you divide y by 6, you get 14. What is y? $\frac{y}{6} = 14$, 84

43. A number b divided by 12 is 9. Find the number. $\frac{b}{12} = 9$, 108

44. The product of 4 and a number a is 396. Find a. $4a = 396$, 99

Slippery Sam

A frog fell into a 20 m well. It started climbing up the wall to escape. It climbed 5 m the first day, but slipped back 4 m during the night. Each day it climbed up 5 m and slipped back 4 m during the night. How many days will it take the frog to climb out of the well? (**HINT**: the answer is not 20.)

Rue/Van Cleve

16 d

3.7 Two-Step Equations

Performance Objective: The student can solve equations involving addition or subtraction *and* multiplication or division.

Example 1: Solve $3x - 2 = 19$

If the x in $3x - 2$ is replaced with a number, you should *first* multiply by 3 and then subtract 2. To solve the equation, you should undo these operations in the *reverse* order.

$$\begin{aligned}
3x - 2 &= 19 && \text{First \textit{undo} ``subtract 2.''}\\
\underline{+2} &\;\; \underline{+2} && \text{Add 2 to each side.}\\
3x &= 21 && \text{Then \textit{undo} ``multiply by 3.''}\\
\frac{3x}{3} &= \frac{21}{3} && \text{Divide each side by 3.}\\
x &= 7
\end{aligned}$$

Check:

$$\begin{aligned}
3x - 2 &= 19\\
3(7) - 2 &= 19\\
21 - 2 &= 19\\
19 &= 19 && \text{True}
\end{aligned}$$

Notice that in the check the operations are done in the usual order.

Example 2: Solve $8 = \frac{c}{4} + 5$

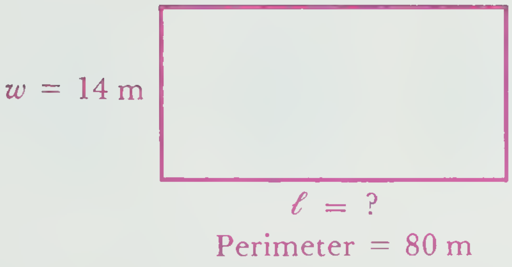
$$\begin{aligned}
8 &= \frac{c}{4} + 5\\
\underline{-5} &\;\; \underline{-5} && \text{Subtract 5 from each side.}\\
3 &= \frac{c}{4}\\
4(3) &= 4\left(\frac{c}{4}\right) && \text{Multiply each side by 4.}\\
12 &= c
\end{aligned}$$

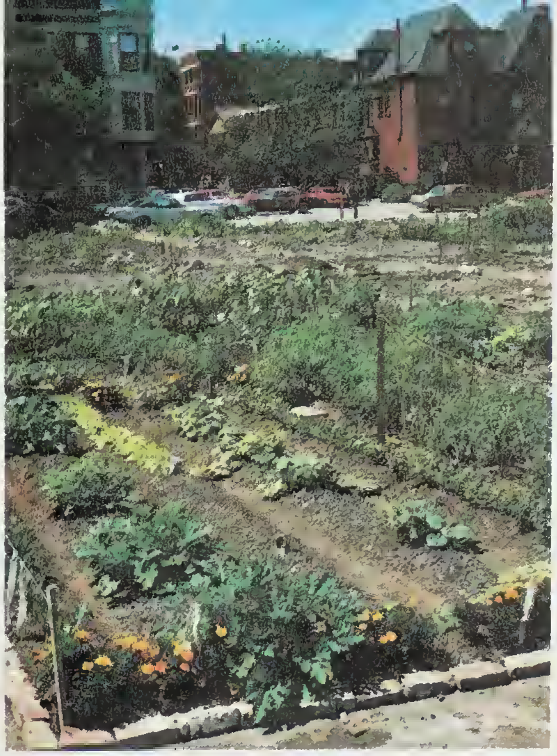
Check:

$$\begin{aligned}
8 &= \frac{c}{4} + 5\\
8 &= \frac{12}{4} + 5\\
8 &= 3 + 5\\
8 &= 8 && \text{True}
\end{aligned}$$

Example 3:

You have 80 m of fence to enclose part of an empty lot for a vegetable garden. If you make it 14 m wide, how long will it be?




Artstreet

$$P = 2\ell + 2w$$ Write the formula.

$$80 = 2\ell + 2(14)$$ Substitute the given values.

$$80 = 2\ell + 28$$

$$\underline{-28 \qquad\qquad -28}$$ Subtract 28 from each side.

$$52 = 2\ell$$

$$\frac{52}{2} = \frac{2\ell}{2}$$ Divide each side by 2.

$$26 = \ell$$ The garden will be 26 m long.

Exercises

1–10. Tell: **a.** what number you would first add or subtract and **b.** by what number you would then multiply or divide.

1. $6x + 5 = 17$ Subt. 5 Div. 6
2. $5y - 7 = 13$ Add 7 Div. 5
3. $\frac{w}{4} - 2 = 9$ Add 2 Mult. 4
4. $\frac{h}{2} + 8 = 12$ Subt. 8 Mult. 2
5. $16 = 3a - 2$ Add 2 Div. 3
6. $17 = 2t + 3$ Subt. 3 Div. 2
7. $20 = \frac{c}{9} + 6$ Subt. 6 Mult. 9
8. $2 = \frac{m}{7} - 8$ Add 8 Mult. 7
9. $40 = 2\ell + 2(9)$ Subt. 18 Div. 2
10. $4p + 2(6) = 32$ Subt. 12 Div. 4

11–26. Solve and check each equation.

11. $2x + 3 = 21$ 9

12. $5b + 4 = 19$ 3

13. $4a - 6 = 22$ 7

14. $3w - 9 = 6$ 5

15. $35 = 8t - 5$ 5

16. $20 = 6z + 2$ 3

17. $5 = \frac{w}{3} - 7$ 36

18. $14 = \frac{s}{5} + 6$ 40

19. $14 = \frac{n}{4} + 10$ 16

20. $16 = \frac{t}{7} - 3$ 133

21. $\frac{m}{13} - 2 = 8$ 130

22. $\frac{r}{9} + 4 = 10$ 54

23. $2d - 9 = 19$ 14

24. $12x + 12 = 12$ 0

25. $48 = 4x + 3(4)$ 9

26. $31 = 3m + 2(8)$ 5

27–36. For each exercise, substitute the given values into the formula. Then solve to find the variable not given.

	$ab + c = d$			
	a	b	c	d
27.	5	12	7	67
28.	3	8	4	28
29.	6	3	8	26
30.	2	7	26	40
31.	4	2	5	13

	$w = xy - z$			
	w	x	y	z
32.	17	3	7	4
33.	17	4	8	15
34.	25	9	4	11
35.	22	5	6	8
36.	11	2	9	7

37–43. Use the given formula to solve.

perimeter = twice length + twice width

$P = 2\ell + 2w$

37. The perimeter of a rectangle is 48 m. The width is 10 m. Find the length 14 m

38. Mike put up a fence around his backyard. He used 36 m of fencing. If his yard is 7 m wide, how long is it? 11 m

39. Find about how old a person is if that person's blood pressure is 122. 24 years

40. Theresa's blood pressure is 134. According to the formula, how old is she? 48 years

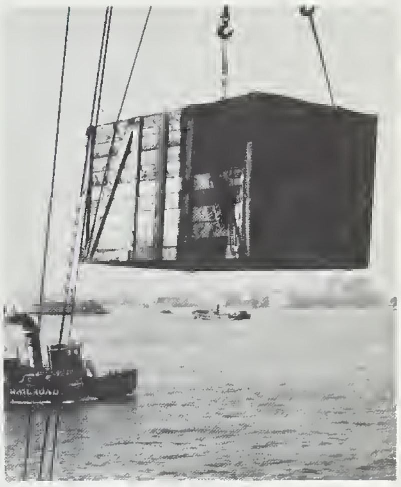
$$\overset{\text{blood pressure}}{P} = \frac{\overset{\text{age}}{a}}{2} + 110$$

For any container filled with objects of equal mass:

full mass = **mass of each object** × **number of objects** + **mass of empty container**

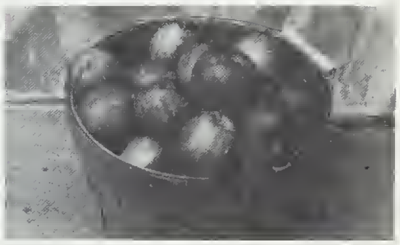
$M = mn + c$

41. Metal parts having a mass of 3 kg apiece are packed in a crate that has a mass of 14 kg empty. A shipping clerk finds the full crate has a mass of 80 kg. How many parts are in the crate? 22



H. Armstrong Roberts

42. Suppose an apple has a mass of 54 g and an empty basket a mass of 250 g. How many apples are in a full basket having a mass of 1600 g? 25



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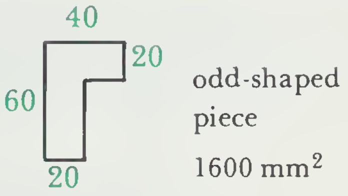
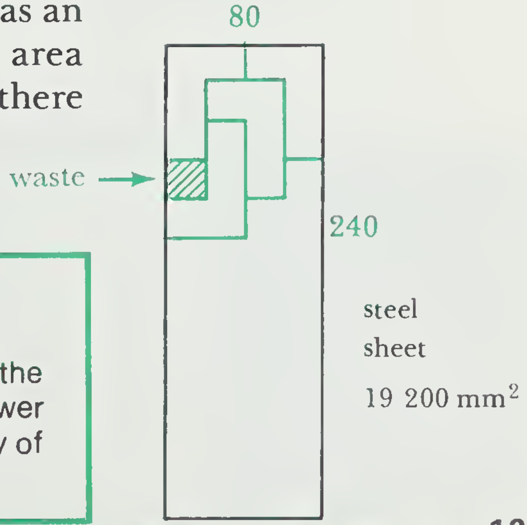
For any area cut into equal-sized pieces:

whole area = **area of each piece** × **number of pieces**

$A = an$

43. A sheet-metal worker is cutting copies of an odd-shaped piece from a steel sheet. The piece has an area of 1600 mm². The whole sheet has an area of 19 200 mm². How many pieces should there be? 12

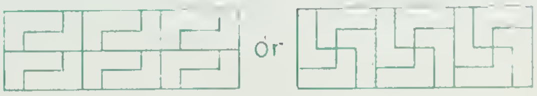




After 43

1. Can you show the areas really are 1600 and 19 200?
2. The way the sheet-metal worker has started to cut the pieces, there is already some waste. There will be fewer pieces than your answer to Ex. 43. Can you find a way of arranging the pieces to get your answer?

Typical answers: 1. $20 \times 20 + 20 \times 60 = 1600$ mm²
$240 \times 80 = 19\ 200$ mm²



EQUATION CROSS-NUMBER

^{1}1	^{2}2	■	^{3}9	■	^{4}3	^{5}6
^{6}3	5	■	^{7}9	^{8}4	■	^{9}5
^{10}2	■	^{11}1	9	■	^{12}4	0
■	^{13}6	7	■	^{14}3	2	■
^{15}5	4	■	^{16}1	6	■	^{17}4
^{18}7	■	^{19}2	2	■	^{20}2	6
^{21}6	^{22}3	■	^{23}5	■	^{24}4	0

Copy the puzzle onto graph paper and use the equations to solve it.

Across

1. $3x = 36$

3. $40 = 4a + 4$

4. $40 = b + 4$

6. $2x - 3 = 67$

7. $n + 6 = 100$

9. $17 = 4b - 3$

10. $7m + 6 = 20$

11. $40 = 2y + 2$

12. $6t = 240$

13. $w + 3 = 70$

14. $4 = \frac{n}{8}$

15. $30 = \frac{x}{2} + 3$

16. $2b - 16 = 16$

17. $30 = 7x + 2$

18. $30 = 4x + 2$

19. $\frac{c}{2} = 11$

20. $r - 6 = 20$

21. $\frac{a}{7} = 9$

23. $17 = 4b - 3$

24. $100 = 2y + 20$

Down

1. $\frac{n}{12} = 11$

2. $100 = 4p$

3. $w + 1 = 1000$

4. $30 = 8t + 6$

5. $\frac{z}{13} = 50$

8. $20 = 6y - 4$

11. $2d + 3 = 37$

12. $\frac{w}{6} + 3 = 10$

13. $\frac{m}{8} = 8$

14. $75 = 2d + 3$

15. $36 = \frac{x}{16}$

16. $1000 = 8a$

17. $\frac{r}{2} + 20 = 250$

19. $13 = 5x + 3$

20. $3\ell + 3 = 75$

22. $4q + 4 = 16$

Quick Quiz

Check these skills:

- *changing fractions to mixed numbers*
- *multiplying whole numbers by fractions*

1–4. Write as mixed numbers.

1. $\frac{24}{5}$ $4\frac{4}{5}$ **2.** $\frac{37}{8}$ $4\frac{5}{8}$ **3.** $\frac{21}{9}$ $2\frac{1}{3}$ **4.** $\frac{26}{8}$ $3\frac{1}{4}$

5–8. Multiply.

5. $3(\frac{5}{3})$ 5 **6.** $6(\frac{5}{6})$ 5 **7.** $4(\frac{5}{2})$ 10 **8.** $8(\frac{7}{4})$ 14

Equations with Fractional Solutions

3.8

Most of the equations you have solved so far have had whole number solutions. But many real problems do not work out so nicely. Many have fractional or decimal answers.

Performance Objective: The student can solve equations in which the solution is to be expressed in fractional or decimal form.

Example 1: Solve $5a + 4 = 13$

$$\begin{aligned} 5a + 4 &= 13 \\ -4 &\;\; -4 \qquad \text{Subtract 4 from each side.} \\ 5a &= 9 \\ \frac{5a}{5} &= \frac{9}{5} \qquad \text{Divide each side by 5.} \\ a &= \tfrac{9}{5} \text{ or } 1.8 \end{aligned}$$

The check can be done with either fractions or decimals.

Check using fractions:

$$\begin{aligned} 5a + 4 &= 13 \\ 5(\tfrac{9}{5}) + 4 &= 13 \\ \tfrac{45}{5} + 4 &= 13 \\ 9 + 4 &= 13 \\ 13 &= 13 \qquad \text{True} \end{aligned}$$

Check using decimals:

$$\begin{aligned} 5a + 4 &= 13 \\ 5(1.8) + 4 &= 13 \\ 9.0 + 4 &= 13 \\ 9 + 4 &= 13 \\ 13 &= 13 \qquad \text{True} \end{aligned}$$

Example 2: Solve $3x + 1.65 = 6$

Notice decimal point and zeros are included for the subtraction.

$$\begin{aligned} 3x + 1.65 &= 6.00 \\ -1.65 &\;\; -1.65 \qquad \text{Subtract 1.65 from each side.} \\ 3x &= 4.35 \\ \frac{3x}{3} &= \frac{4.35}{3} \qquad \text{Divide each side by 3.} \\ x &= 1.45 \end{aligned}$$

Check:

$$\begin{aligned} 3x + 1.65 &= 6 \\ 3(1.45) + 1.65 &= 6 \\ 4.35 + 1.65 &= 6 \\ 6.00 &= 6 \qquad \text{True} \end{aligned}$$

Exercises

1–8. Solve and check each one-step equation.

1. $4x = 9$ $\frac{9}{4}$

2. $8n = 13$ $\frac{13}{8}$

3. $14 = 3y$ $\frac{14}{3}$

4. $7 = 9b$ $\frac{7}{9}$

5. $w + 2.9 = 3.8$ .9

6. $3.2 = c - 4.6$ 7.8

7. $5.50 = p - 4.45$ 9.95

8. $x + 3.45 = 6.75$ 3.3

9–28. Solve and check each equation.

9. $5x - 12 = 16$ $\frac{28}{5}$

10. $25 = 3y - 8$ 11

11. $7n + 4 = 12$ $\frac{8}{7}$

12. $15 = 5w + 11$ $\frac{4}{5}$

13. $21 = 3b + 1$ $\frac{20}{3}$

14. $7r + 8 = 10$ $\frac{2}{7}$

15. $5 = 5a - 2$ $\frac{7}{5}$

16. $4p + 3 = 12$ $\frac{9}{4}$

17. $5x + 11 = 15$ $\frac{4}{5}$

18. $24n - 6 = 0$ $\frac{1}{4}$

19. $3z - 1.1 = 1.3$ .8

20. $3r + 2.1 = 6.3$ 1.4

21. $2.45 = 4b - 2.47$ 1.23

22. $2.26 = 2y - 6.12$ 4.19

23. $7 = 2n - 2.8$ 4.9

24. $10 = 3c + 1.9$ 2.7

25. $20 = 6x + 5.6$ 2.4

26. $20 = 4w - 2.4$ 5.6

27. $3a + 2.14 = 7$ 1.62

28. $2b - 2.8 = 7$ 4.9

29–36. For each exercise substitute the given values into the formula. Then solve for the variable not given.

	$r = st + v$			
	r	s	t	v
29.	77.6	8	9.2	4
30.	19.4	4.1	4	3
31.	5	3	1.2	1.4
32.	28	2	8	12

	$cd - e = f$			
	c	d	e	f
33.	2	3.5	4	3
34.	2.8	3	2.4	6
35.	5	2.4	2	10
36.	2	5.6	1.3	9.9

37–44. Use the given formula to solve.

37. How long will it take a car to travel 216 km at a rate of 48 km/h? ($d = rt$) 4.5 h

38. At a certain speed, a car can travel 5 min for every litre of gas. If the car traveled for 90 min, how much gas was burned? 18 L

39. Zina drove for 30 min in traffic and used 2.5 L of gas. How long did it take her car to burn 1 L? 12 min

$t = Lm$

t: time

L: number of litres

m: minutes per litre

40. A rectangle has a perimeter of 81 m. If it is 14 m wide, how long is it? ($P = 2\ell + 2w$) 26.5 m

total cost = cost per unit × number of units + additional charge

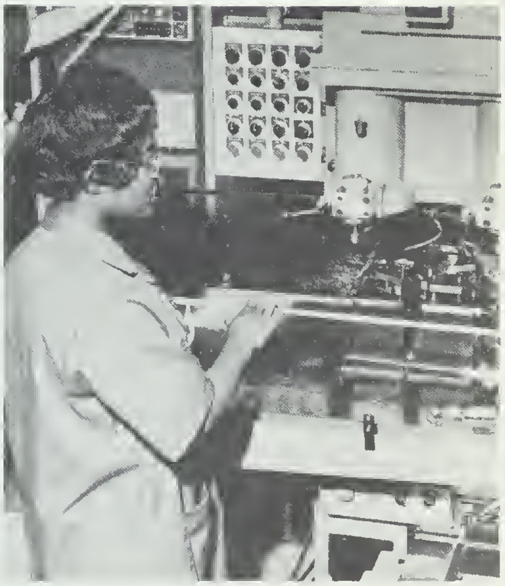
$C = cn + a$

41. A record club charges $4 per record plus $1.30 for shipping and handling. How many records did Rosa get if her total cost was $37.30? 9 records

42. Rick bought 5 T-shirts. The tax on his purchase was $.69. If his total cost was $14.44, what was the price of each T-shirt? $2.75

total time = time per unit × number of units + additional time

$T = tn + a$



Courtesy Olin Corporation

43. It takes 7 min to jog around a track. You need 2 min altogether to change clothes. How many laps can you make if you have exactly half an hour? 4 laps

44. A machine-tool operator knows her machine can produce a finished part in 8 s. She can load and prepare her machine in 28 s. How many parts can she expect to make in 5 min (300 s)? 34 parts

3.9 Equations Containing Fractions

Example 1: Solve $\frac{2}{3}x = 18$

Performance Objective: The student can solve equations in which the variable is multiplied by a fraction.

In this sentence x is multiplied by $\frac{2}{3}$. That is the same as multiplying x by 2 and dividing by 3. To find x, undo each operation one step at a time.

$$\frac{2}{3}x = 18$$

$$\frac{2}{3}x = 18$$ Multiply each side by 3.

$$2x = 54$$

$$\frac{2x}{} = \frac{54}{}$$ Divide each side by 2.

$$x = 27$$

Check: $$\frac{2}{3}x = 18$$

$$\frac{2}{3}(27) = 18$$

$$\frac{54}{3} = 18$$

$$18 = 18$$ True

Example 2: Solve $17 = \frac{5}{8}y - 3$

$$17 = \frac{5}{8}y - 3$$

First add 3.

$$20 = \frac{5}{8}y$$

$$20 = \frac{5}{8}y$$ Then multiply by 8.

$$160 = 5y$$

$$\frac{160}{} = \frac{5y}{}$$ Finally divide by 5.

$$32 = y$$

Check: $17 = \frac{5}{8}y - 3$

$17 = \frac{5}{8}(32) - 3$

$17 = \frac{160}{8} - 3$

$17 = 20 - 3$

$17 = 17$ True

Exercises

1–6. Tell: **a.** by what number you would first multiply each side and **b.** by what number you would then divide each side.

1. $\frac{2}{3}x = 18$ Mult. 3 Div. 2

2. $\frac{3}{4}n = 30$ Mult. 4 Div. 3

3. $16 = \frac{4}{5}a$ Mult. 5 Div. 4

4. $\frac{7}{2}w = 42$ Mult. 2 Div. 7

5. $\frac{8}{3}t = 56$ Mult. 3 Div. 8

6. $45 = \frac{5}{7}r$ Mult. 7 Div. 5

7–28. Solve and check each equation.

7. $\frac{2}{3}x = 24$ 36

8. $27 = \frac{3}{4}b$ 36

9. $28 = r(\frac{4}{3})$ 21

10. $\frac{2}{9}n = 14$ 63

11. $56 = \frac{4}{13}c$ 182

12. $\frac{9}{5}z = 72$ 40

13. $\frac{7}{3}t = 105$ 45

14. $32 = r(\frac{4}{7})$ 56

15. $4 = \frac{3}{4}x - 5$ 12

16. $37 = \frac{7}{3}w + 2$ 15

17. $12 = \frac{2}{3}z + 2$ 15

18. $13 = \frac{4}{5}a - 3$ 20

19. $\frac{5}{4}c - 3 = 17$ 16

20. $\frac{3}{4}w + 4 = 10$ 8

21. $\frac{4}{9}m + 22 = 34$ 27

22. $\frac{8}{3}d - 3 = 45$ 18

23. $\frac{3}{4}x = 1.5$ 2

24. $\frac{2}{3}n = 7.2$ 10.8

25. $3.72 = \frac{4}{5}b$ 4.65

26. $5.43 = \frac{3}{11}c$ 19.91

27. $4.2 = \frac{2}{5}x + 2.4$ 4.5

28. $13.8 = \frac{4}{3}a - 2.6$ 12.3

29–34. Use the given formula to solve.

total cost = amount (no. of units) × cost per unit

$C = np$

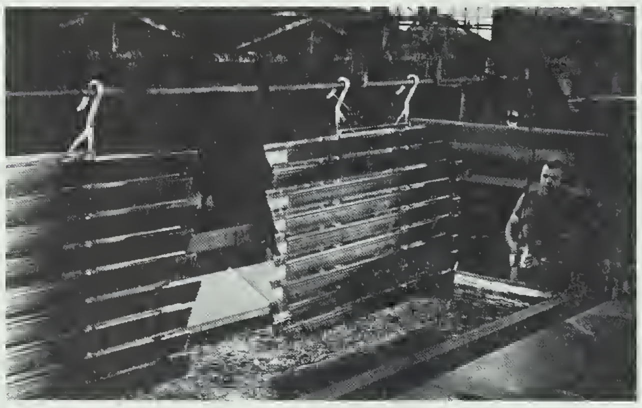
29. If 0.6 kg of hamburger costs \$1.50, what is the cost per kilogram? \$2.50

30. A 50 mL bottle of perfume costs \$15. Find the cost per mL. \$0.30

31. A car went a distance of 63 km in 3/4 h. Find its rate of speed. ($d = rt$) 84 km/h

32. Suppose a snail moves at a rate of 2.5 cm/h. How long will it take to crawl a distance of 60 cm? ($d = rt$) 24 h

33. Many people believe a rectangle has the most pleasing shape when the width is $\frac{5}{8}$ of the length ($w = \frac{5}{8}\ell$). Such a rectangle is called a *golden rectangle*. If a window 90 cm wide is a golden rectangle, how long is it? 144 cm



A. Devaney, Inc.

This photograph of an electroplater is in the shape of the golden rectangle. Do you find the shape to be a pleasing one?

34. An electroplater's instructions call for this solution: amount of copper equals $\frac{4}{7}$ of amount of nickel ($C = \frac{4}{7}N$). For 76 g of copper, how much nickel should be used? 133 g

35–36. A person's height (h) in centimetres and mass (m) in kilograms are approximately related by this formula:

$$m = \frac{5}{4}h - 150$$

35. About how tall is a 60 kg person? 168 cm

36. According to the formula, what is the height of someone who has a mass of 80 kg? For what kind of people do you suppose the formula gives the least-correct answers?
184 cm; people whose mass or height vary from the average.

37–40. Find the equivalent Canadian value of travelers' cheques in a foreign currency.

37. $100 Can $116.25 **38.** $1000 Can $1151.25

39. $150 Can $173.75 **40.** $350 Can $403.75

41–44. Find the equivalent foreign value of travelers' cheques in Canadian dollars.

41. $231.25 $200 **42.** $2301.25 $2000

43. $288.75 $250 **44.** $863.75 $750

$F = 1.15C + 1.25$

F: foreign value

C: Canadian value

45–48. Use the given rule or principle to write a formula. (Choose your own letters.) Then use the formula to solve the problems.

45. A 14 m boat burns 13 L every hour. How long will the boat be able to cruise if it has a 520 L tank? 40 h

time = number of litres ÷ litres per hour

46. A 10 m boat cruised for 35 h without refueling. If it took 227.5 L to fill the tank, how many litres did the boat burn every hour? 6.5 L/h

47. Gasoline has a mass of about 0.7 kg/L. How many litres are in a tank containing 84 kg of gasoline? (Use $\frac{7}{10}$ for 0.7.) 120 L

total mass = mass per unit × number of units

48. To find out how much wire was left in a coil of wire without unrolling it, an electrician found the mass of the coil. It had a mass of 3750 g. If the wire's mass is 60 g/m, how many metres of wire are left? 62.5 m

Kuechmann/Bedford Photographic

The electrician found a clever way to measure how much wire was left. Can you think of any other clever uses for this formula?

3.10 Solving Problems with Formulas

Performance Objective: The student can apply formulas to solve verbal problems.

Camerique

Example 1: The prom committee decided to raise money by giving a rock concert. They charged \$2 a ticket and their expenses were \$300. How many tickets do they have to sell to make a \$400 profit?

Here is a rule that can be used:

$$\underbrace{\text{Profit}}_{P}\ \underset{=}{\underset{\blacktriangledown}{\text{is}}}\ \underbrace{\text{price per ticket}}_{t}\ \underset{\times}{\underset{\blacktriangledown}{\text{times}}}\ \underbrace{\text{number of tickets}}_{n}\ \underset{-}{\underset{\blacktriangledown}{\text{less}}}\ \underbrace{\text{expenses.}}_{e}$$

$$\begin{array}{rcll} P & = & tn - e & \text{Write the formula.} \\ 400 & = & 2n - 300 & \text{Substitute the given values.} \\ \underline{+300} & & \underline{+300} & \text{Add 300.} \\ 700 & = & 2n & \\ \dfrac{700}{2} & = & \dfrac{2n}{2} & \text{Divide by 2.} \\ 350 & = & n & \end{array}$$

They need to sell 350 tickets to make a \$400 profit.

Example 2: Suppose only 200 people are expected to attend the concert. How much should each ticket cost to keep the profit at $400?

$P = tn - e$ Write the formula.

$400 = t(200) - 300$ Substitute the given values.

$400 = 200t - 300$ Change the order of the multiplication.

$+300 \qquad\qquad +300$ Add 300.

$700 = 200t$

$\frac{700}{200} = \frac{200t}{200}$ Divide by 200.

$3.5 = t$

Each ticket would have to cost $3.50 to make the same profit.

Exercises

1–6. Use the following problem and formula:

Problem: A movie theater charges $3 for all tickets. It costs $510 a day to run the theater. How many tickets must be sold each day to make a daily profit of $150?

Formula: $P = tn - e$

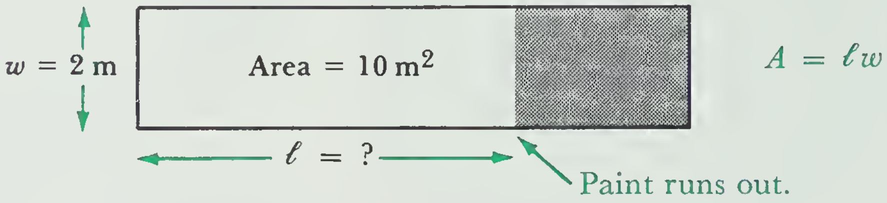
1. Which variable has the value $3? t
2. Which variable has the value $510? e
3. Which variable has the value $150? P
4. Which variable is to be found? n
5. Solve the problem. 220
6. Suppose the theater has 400 seats. If the theater is filled every night, how much could the manager charge per ticket with the same profit and expenses? $1.65

Artstreet

7–24. Use the given formula to solve.

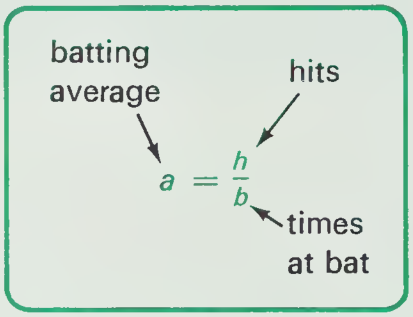
7. At a rate of 75 km/h, how long will it take a car to go 450 km? ($d = rt$) 6 h

8. A litre of paint covers an area of about 10 m^2. How long a section of a 2 m high fence can be painted with a litre? 5 m



9. Suppose you make 8 hits in 20 times at bat. What is your batting average? 0.400

10. How many hits did someone whose batting average is 0.450 get in 40 times at bat? 18 hits



$$a = \frac{h}{b}$$

A cricket will chirp faster as the temperature goes up.

Grant Heilman

11. How fast will a cricket chirp at 26°C? 172 chirps/min

12. Find the temperature when the cricket chirps 120 times/ min. 13°C

approximate formula

$$T = \frac{c}{4} - 17$$

T: temperature (°C)
c: chirps per minute

An ant will run faster as the temperature goes up.

Covington/Tom Stack and Associates

13. Find the temperature when an ant moves at 10 cm/min. 28°C

14. How fast is the ant at 23°C? 7.5 cm/min

15. If crickets chirp 112 times/min, how fast does an ant run? (HINT: Use two formulas.) 1.5 cm/min

approximate formula

$$T = 2s + 8$$

T: temperature (°C)
s: speed (cm/min)

$P = bn - e$

P: profit
b: price per bracelet
n: number of bracelets
e: expenses

16. Lydia decided to earn some money by making and selling bracelets. She spent \$43 on supplies to make 24 bracelets. For what price should she sell them to make a profit of \$65? \$4.50

The longer an object falls, the farther it goes.

17. Jean dropped a stone into a well. It hit bottom in 2 s. How deep is the well? 19.6 m

18. A life raft dropped from a helicopter hit the water in 6 s. How high was the helicopter? 176.4 m

$d = 4.9t^2$

t^2 means $t \times t$

d: distance (metres)
t: time (seconds)

19. A rock falls 490 m into a canyon. Estimate the time it will fall by trying values for t. 10s

The faster a car is moving, the farther it takes to stop.

20. How many metres does it take to stop at 70 km/h? 29.4 m

21. Find the required stopping distance at 100 km/h. 60 m

22. The skid marks left by a stopping car measured 9.6 m. Estimate the speed it was going when it started to brake by trying values for s. 40 km/h

stopping distance on dry concrete road *after* the brakes are applied

$d = 0.006\,s^2$

s^2 means $s \times s$

d: distance (metres)
s: speed (km/h)

23. Light travels 300 000 km/s. How many seconds does it take light to get from the sun to the earth, 150 million km away? ($d = rt$) 500 s

24. How many minutes does it take the sun's light to reach the earth? (Use your answer to Exercise 23.) $8\frac{1}{3}$ min

There are two important electrical formulas.

Grant Heilman

$E = IR$

$W = EI$

E:	volts V	amount of *pressure* "pushing" the electricity through the wire
I:	amperes A	*rate* of flow of the current
R:	ohms Ω	*resistance* of the wire to the flow of current
W:	watts W	unit of *power*

Switch on your brain and be bright about choosing the right formula for each of these.

25. How many ohms of resistance does a copper wire have if 5 A of current flows through it at 110 V? 22 Ω

26. Find the number of amps used by a light bulb with a resistance of 50 Ω on a 110 V line. 2.2 A

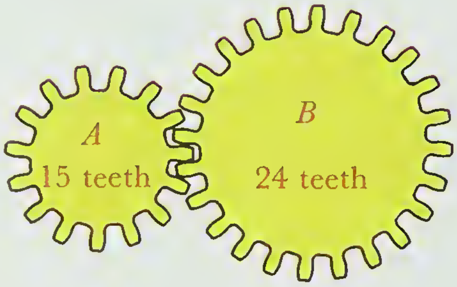
27. How many amps does a 60 W bulb use on a 120 V line? 0.5 A

28. How many ohms of resistance does a 60 W bulb have on a 120-V line? (HINT: Use both formulas.) 240 Ω

29—32. Use the given rule to write your own formula. Then use it to solve the problems.

For the revolutions per minute (rev/min) of two gears *A* and *B*:

number of teeth in *A* x rev/min of *A* = number of teeth in *B* x rev/min of *B*



29. If gear *B* rotates at a rate of 50 rev/ min, how fast will *A* turn? 80 rev/min

30. If *A* rotates at 120 rev/min, how fast will *B* turn? 75 rev/min

Total wages equal pay per hour times the number of hours.

31. At $5.80/h, how much would you earn in 8 h? $46.40

32. If your wages were $52.50 for a 7h day, what is your hourly rate? $7.50/h

CAREER CORNER

MATH IN APPRENTICESHIPS

To become a carpenter, an electrician, a bricklayer, or other skilled worker, you need special training. The training period is called an *apprenticeship*. Some apprentices get all their training on the job. Others have to go to some classes.

Here are some problems an apprentice may have to solve with a formula.

Van Cleve/Van Cleve

1. *Electrician's apprentice:* Find the watt rating of a 10 Ω heater that is connected to a 120 V line. 1440 W

$$W = \frac{E \times E}{R}$$

R: ohms
E: volts
W: watts

Van Cleve/Van Cleve

2. *Bricklayer's apprentice:* Find the number of common bricks it takes to build a wall 4 m long, 3 m high and 50 cm wide. 3840 bricks

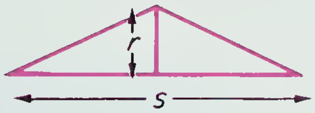
For a wall of common brick:

$$N = 640 \times \ell \times w \times h$$

N: number of bricks
ℓ : length (metres)
w: width (metres)
h: height (metres)

Grant Heilman

3. *Carpenter's apprentice:* A roof is being built over a span of 9 m. How long should lumber be cut for the rise if the pitch is to be $\frac{1}{6}$? 1.5 m



$$r = ps$$

r: rise (metres)
s: span (metres)
p: pitch (amount of slope)

Photri

4. *Pipe-fitter's apprentice:* How many millimetres of expansion must be allowed for when laying 100 m of brass pipe for water that will vary in temperature from 80°C to 5°C? 142.5 mm

For a brass pipe:

$$E = 0.019\,(T - t)$$

E: expansion (mm)
L: length (m)
T: high temperature
t: low temperature

Test for Chapter 3

Tell whether each sentence is true, false, or open. (3.1)

1. $7 + n = 18$ Open **2.** $3 \times 5 = 11 + 4$ True

3. $4 + 18 = 2 \times 12$ False **4.** $20 = 4x + 5$ Open

Find each answer when n is replaced with 3. (3.2)

5. $2n$ 6 **6.** $5(n + 2)$ 25 **7.** $n(6 - 4)$ 6

Find each answer. (3.3)

8. $16 - 9 + 5$ 12 **9.** $35 \div 5 + 2$ 9 **10.** $3(6) - 15 \div 3$ 13

Use the given formula to solve. (3.4)

11. What distance will a car go in 3 h at 90 km/h? ($d = rt$) 270 km

12. Find the perimeter of a rectangle 8 m wide and 14 m long. ($P = 2\ell + 2w$) 44m

Solve and check.

13. $x + 13 = 29$ 16 **14.** $8 = y - 27$ 35 (3.5)

15. $4n = 68$ 17 **16.** $12 = \frac{w}{3}$ 36 (3.6)

17. $4a - 3 = 17$ 5 **18.** $3 = \frac{b}{4} - 9$ 48 (3.7)

19. $3n = 7$ $\frac{7}{3}$ **20.** $10 = 5x + 6$ $\frac{4}{5}$ (3.8)

21. $\frac{3}{5}t = 18$ 30 **22.** $7 = \frac{2}{3}c - 3$ 15 (3.9)

RATE YOURSELF

Number Correct	
23–24	Superior
21–22	Excellent
18–20	Good
13–17	Fair
0–12	Poor

Use the given formula. (3.10)

23. A boat went 26 h on 39 L of gas. How many hours per litre did the boat get? 0.67 h or 40 min

$t = Lh$

t: time

L: number of litres

h: hours per litre

24. Allen ordered 5 shirts from a catalog. Shipping and handling were \$2. If the total cost was \$27, what was the price of each shirt? \$5

$C = pn + a$

C: total cost

p: price per item

n: number of items

a: additional charge

CONSUMER CORNER

MONEY MATTERS

Many problems involving money are solved with an idea or a rule that can be written as a formula. Even when we don't use a formula, we still use the rule.

At the store

How would you find the cost of 3 stereo LP's at \$5 apiece? You would multiply 3 × 5 and get \$15. You are using this rule:

To find the cost, multiply the number of items times the price per item.

This rule can be written as a formula: $c = np$

c: cost
n: number of items
p: price per item

Since there is usually a sales tax, the formula gets a little more complicated. Write this rule as a formula:

To find the cost, multiply the number of items times the price per item, and then add the tax.

$c = np + t$

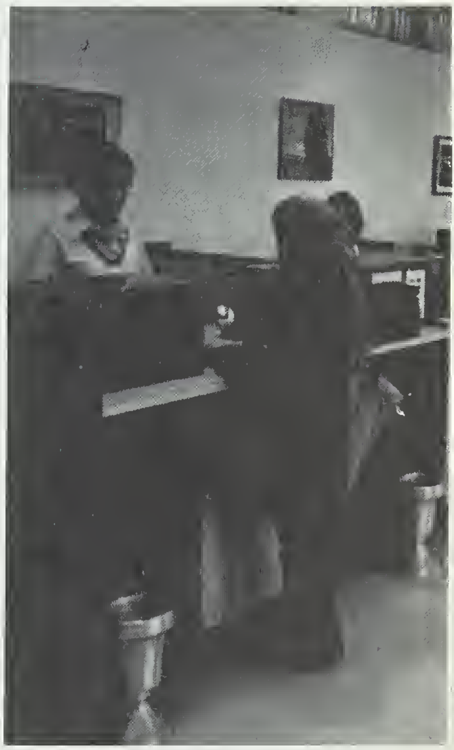


Photo Research International

At the bank

If you deposit an amount of money (called the principal) into a savings account, how can you find how much you will have at a later date? Write this rule for simple interest as a formula:

To find the amount, multiply the principal times the rate of interest times the number of years, and then add this to the principal.

$A = Prt + P$

In the home

Suppose you want to figure the cost of carpeting a room. Room sizes are usually given in metres. Carpet prices are usually given per square metre. You need two rules. Write both rules as formulas:

Area in square metres is length (metres) times width (metres).

Cost is number of square metres times the price per square metre.

$A = lw$, $c = np$

Try these problems using either a rule or a formula.

1. What is the cost of 6 pairs of stockings at \$1.98 a pair with a sales tax of \$.58? \$12.46
2. If you deposit \$250 in a savings account that pays 8%(0.08) simple interest a year, how much will you have in three years? \$310.00
3. Find the cost of carpeting a room 6 m long and 4.5 m wide if the carpeting costs \$7.95/m,^{2} \$214.65

THE DATE'S DAY

Here is a formula to end all formulas. It will give the *day* of the week on which any given date falls.

$$W = d + 2m + [(3m + 3) \div 5] + y + \left[\frac{y}{4}\right] - \left[\frac{y}{100}\right] + \left[\frac{y}{400}\right] + 2$$

d: day of the month (From 1 to 31)
m: month (March is 3, April is 4, and so on, up to December is 12. January is 13 and February is 14.)
y: year (For dates in January or February, use the previous year.)
Examples: For February 29, 1980, d is 29, m is 14, and y is 1979.
For July 4, 1776, d is 4, m is 7, and y is 1776.

The brackets [ ] or $\Big[\ \Big]$ mean do the division inside them, discard any remainder and use only the whole-number part of the quotient. Example: for March the part $+\ [(3m + 3) \div 5]\ +$ looks like this:

$$+\ [(3(3) + 3) \div 5]\ +$$
$$+\ [(9 + 3) \div 5]\ +$$
$$+\ [12 \div 5]\ +$$
$$+\ [2\tfrac{2}{5}]\ +$$
$$+\ 2\ +$$

When you work out each part and then add and subtract from left to right, you will know what W is. Divide W by 7. The remainder will tell you the day of the week. 0 is Saturday, 1 is Sunday, 2 is Monday, and so on, up to 6 is Friday.

1–3. *Answers will vary.*

1. See if the formula works for today's date.
2. On which day of the week were you born?
3. On which day of the week will Christmas fall this year?
4. On which day of the week will New Year's Eve fall in 1999?
Friday

Reininger/De Wys, Inc.

Chapter 3 Review

Tell whether each sentence is true, false, or open. (3.1)

1. $4 \times 7 = 16 + 14$ False **2.** $18 = 4b$ Open

3. $2w - 7 = 15$ Open **4.** $6 + 12 = 6 \times 3$ True

Find each answer. (3.2)

5. $(2 + 9) - 5$ 6 **6.** $8 - (4 + 3)$ 1

7. $(4 \times 3) + 5$ 17 **8.** $8 + (20 \div 4)$ 13

Find each answer when y is replaced with 4.

9. $3y$ 12 **10.** $6y$ 24

11. $2(y + 5)$ 18 **12.** $4(y - 1)$ 12

Find each answer. (3.3)

13. $14 - 9 + 5$ 10 **14.** $20 + 2 - 8$ 14

15. $2(8) - 3(4)$ 4 **16.** $8 \div 4 + 2(3)$ 8

Find each answer when x is replaced with 7.

17. $3x + 1$ 22 **18.** $2x - 9$ 5

19. $4(x - 5)$ 8 **20.** $3(x + 3)$ 30

Use the given formula. (3.4)

$P = \frac{a}{2} + 110$

P: blood pressure

a: age

21. Find the blood pressure of a person who is 30 years old. 125

$W = pt$

W: total wages

p: hourly pay rate

t: hours worked

22. Suppose you worked 48 h a week at \$5/h. What would your week's wages be? \$240.

Solve and check. (3.5)

23. $a + 18 = 25$ 7 **24.** $b - 13 = 22$ 35

25. $36 = x - 10$ 46 **26.** $y + 9 = 46$ 37

27. $4.5 = p + 3.2$ 1.3 **28.** $6.5 = c - 2.6$ 9.1

Solve and check. (3.6)

29. $6n = 78$ 13 **30.** $36 = 9p$ 4

31. $\frac{x}{5} = 12$ 60 **32.** $17 = \frac{w}{8}$ 136

33. $105 = 7y$ 15 **34.** $11r = 132$ 12

35. If a plane flies a distance of 2790 km in 3 h, what is its rate of speed? ($d = rt$) 930 km/h

Solve and check. (3.7)

36. $2a - 3 = 7$ 5 **37.** $25 = 3b + 4$ 7

38. $\frac{x}{6} - 5 = 3$ 48 **39.** $14 = \frac{y}{2} + 9$ 10

40. $40 = 4w + 4$ 9 **41.** $3t + 2 = 35$ 11

42. If 22 m of fencing are used to enclose a lot 4 m wide, how long will the fence be? ($P = 2\ell + 2w$) 7 m

Solve and check. (3.8)

43. $3x = 11$ $\frac{11}{3}$ **44.** $13 = 2y$ $\frac{13}{2}$

45. $10 = 4n + 7$ $\frac{3}{4}$ **46.** $3m - 2 = 3$ $\frac{5}{3}$

47. $2z - 4 = 5$ $\frac{9}{2}$ **48.** $11 = 5w - 8$ $\frac{19}{5}$

49. $15 = \frac{3}{7}x$ 35 **50.** $\frac{2}{9}y = 4$ 18 (3.9)

51. $\frac{3}{2}c + 1 = 10$ 6 **52.** $\frac{2}{9}n + 2 = 4$ 9

53. $12 = \frac{3}{4}a - 3$ 20 **54.** $20 = \frac{7}{3}p - 1$ 9

Use the given formula. (3.10)

$a = \frac{h}{b}$

a: average
h: hits
b: at bats

55. Sarah got 9 hits in 25 at bats. What is her average? 0.360

56. If your average was 0.250 for 36 at bats, how many hits did you get? 9 hits

$F = 1.15C + 1.25$

57. Find the foreign value (F) for \$200 Canadian (C). \$231.25

58. What is the Canadian equivalent for \$403.75? \$350

Preview Test for Chapter 4

Give the ratio of circles to triangles. (4.1)

1. ○○ △△△△△ $\frac{2}{5}$ 2. ○△△○○○△ $\frac{4}{3}$

Give each ratio in lowest terms. (4.2)

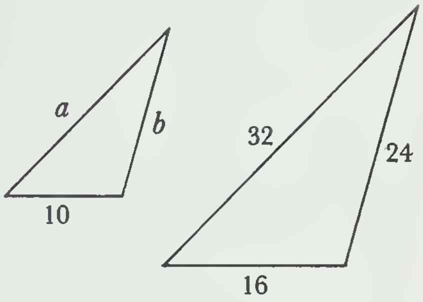
3. $\frac{4}{6}$ $\frac{2}{3}$ 4. $\frac{8}{20}$ $\frac{2}{5}$ 5. $\frac{18}{42}$ $\frac{3}{7}$

Solve each proportion. (4.3)

6. $\frac{2}{3} = \frac{8}{x}$ 12 7. $\frac{8}{a} = \frac{10}{35}$ 28 8. $\frac{18}{10} = \frac{45}{n}$ 25

Solve with a proportion. (4.4)

9. If a car can go 144 km on 20 L of gas, how far can it go on 30 L? 216 km



Use the similar figures at the right. (4.5)

10. Find the values of a and b. $a = 20$, $b = 15$

Find the better buy. (4.6)

11. 125 g for 39¢ 6625 12. 325 g for 71¢ 30 225
200 g for 53¢ 7800 ← 450 g for 93¢ 31 950 ←

Ex. 11–14: Answers given in order; arrow indicates better buy.

Find the best buy—find how much of each for $1.00. (4.7)

13. 150 mL for 40¢, 175 mL for 50¢, 300 mL for 75¢
375, 350, 400 ←

Find the best buy by finding the cost of one gram of each.

14. 75 g for 54¢, 100 g for 82¢, 125 g for 95¢ 0.72 ← ,0.82,0.76

Change to a percent. 15. $\frac{3}{12}$ 25% 16. 12 out of 15 80% (4.8)

Find each number. 17. 40% of 60 24 18. 36% of 75 27 (4.9)

Solve each problem with a proportion. (4.10)

19. If 38 of 50 people use Brand X, what percent use Brand X? 76%

20. How much would you save buying a $40 watch at 15% off? $6

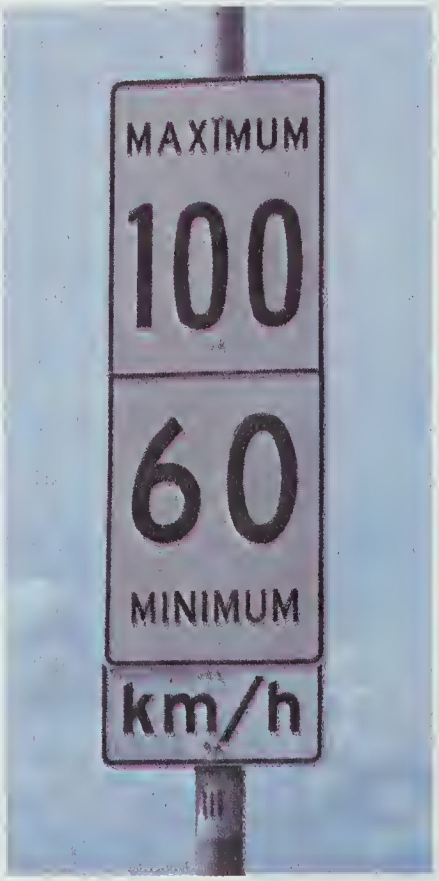
Find by multiplying. 21. 32% of 70 22.4 22. 17% of 145 24.65 (4.11)

Estimate using $\frac{1}{10}$. 23. 19% of 51 10 24. 31% of 4.99 1.50 (4.12)
Ex. 23–24: Estimates

4
Ratio and Proportion

PICTURE PREVIEW: An understanding of ratio and proportion is helpful for working with many problems of ordinary life. We all need to know how to get the best buy for our money. We need to be able to understand facts and figures given as percents. The pictures here give some idea of the many uses for the ideas in this chapter.

Camerique



Robert Brown

Simpson/De Wys, Inc.

2/4
GIFTS
POSTERS
Jungle Puzzle

Camerique

4.1 Ratios

Camerique

Performance Objective: The student can write ratios to show comparisons from either diagrams or verbal statements.

If you were asked how the team is doing, you might say, "They won 6 out of 10 games."

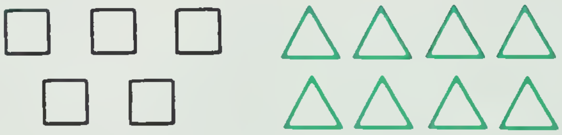
When you say "6 out of 10," you are using a *ratio*.

A ratio compares two numbers by division.

The ratio "6 out of 10" can be written in 3 ways:

$\frac{6}{10}$ 6 to 10 6:10

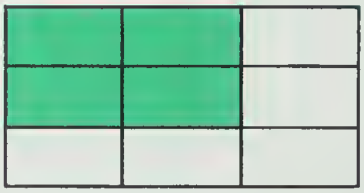
Example 1:



The ratio of squares to triangles is $\frac{5}{8}$ or 5 to 8 or 5:8.

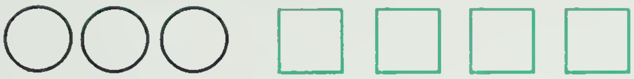
The ratio of triangles to squares is $\frac{8}{5}$ or 8 to 5 or 8:5.

Example 2:



The ratio of green parts to all parts is $\frac{4}{9}$.
The ratio of white parts to green parts is $\frac{5}{4}$.
The ratio of all parts to white parts is $\frac{9}{5}$.

Exercises



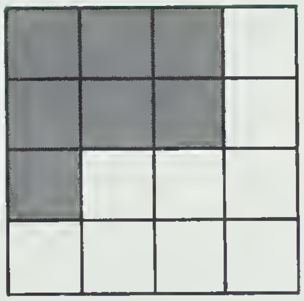
1. For 3 circles there are 4 squares.

2. The ratio of circles to squares is 3 to 4.

3. For 4 squares there are 3 circles.

4. The ratio of squares to circles is 4 to 3.

5–14. Give each ratio as a fraction.



5. shaded squares to all squares $\frac{7}{16}$

6. unshaded squares to all squares $\frac{9}{16}$

7. all squares to unshaded squares $\frac{16}{9}$

8. shaded squares to unshaded squares $\frac{7}{9}$

9. green parts to all parts $\frac{5}{12}$

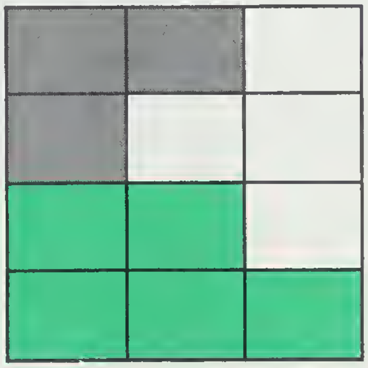
10. gray parts to all parts $\frac{3}{12}$

11. white parts to gray parts $\frac{4}{3}$

12. green parts to white parts $\frac{5}{4}$

13. green parts to gray parts $\frac{5}{3}$

14. all parts to white parts $\frac{12}{4}$

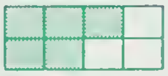


15–18. Make a drawing to show each ratio.

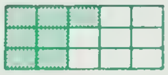
Ex. 15–18: Typical answers

15. The ratio of circles to squares is $\frac{6}{5}$. ○○○○○○ □□□□□

16. The ratio of shaded parts to all parts is 5 to 8.



17. 7 out of 15 parts are shaded.



18. The ratio of triangles to circles is 4:7. △△△△ ○○○○○○○

19–28. Give each ratio as a fraction.

19. A hall is 9 m long and 5 m wide. What is the ratio of length to width? $\frac{9}{5}$

20. A woman has a mass of 49 kg. Her husband has a mass of 75 kg. What is the ratio of her mass to his? $\frac{49}{75}$

21. What is the ratio of territories to provinces? $\frac{2}{10}$ or $\frac{1}{5}$

Forsyth/Monkmeyer Press Photo Service

100 L/min

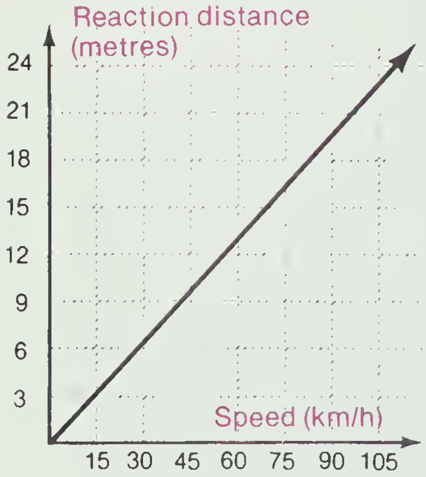
Forsyth/Monkmeyer Press Photo Service

8 L/min

22. What is the ratio of air intake during rest to air intake during vigorous activity? $\frac{8}{100}$
See the pictures.

23. A bricklayer mixes mortar with 3 parts cement, 10 parts sand, and 1 part lime. What is the ratio of sand to cement? $\frac{10}{3}$

24. A chemical technician analyzed a sample and found 6 parts carbon, 2 parts sulfur, and 10 parts hydrogen. What is the ratio of carbon to hydrogen? $\frac{6}{10}$



The distance a car travels between the time a driver realizes he must stop until he actually hits the brakes is called the *reaction distance*. At a speed of 60 km/h, for example, the reaction distance is about 13 m. *See the graph.* Write each ratio.

25. reaction distance at 30 km/h to that at 60 km/h $\frac{7}{13}$

26. reaction distance at 90 km/h to that at 60 km/h $\frac{20}{13}$

27. reaction distance at 105 km/h to that at 75 km/h $\frac{23}{16}$

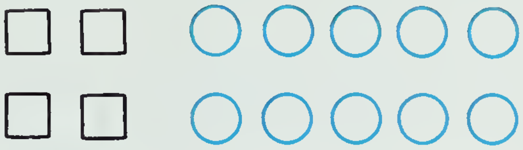
28. reaction distance at 30 km/h to that at 75 km/h $\frac{7}{16}$

Performance Objective:
The student can demonstrate equivalent ratios using diagrams or computation.

Equivalent Ratios

4.2

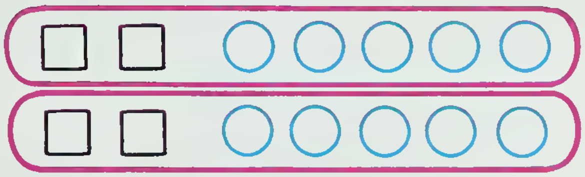
The ratio $\frac{4}{10}$ can be shown with a diagram.



For **4** squares, there are **10** circles.

These give the *same* comparison.

But each row can be grouped separately.



For every **2** squares, there are **5** circles.

This shows the ratio $\frac{2}{5}$.

So it must be true that $\frac{4}{10} = \frac{2}{5}$. $\frac{4}{10}$ and $\frac{2}{5}$ are *equivalent ratios*.

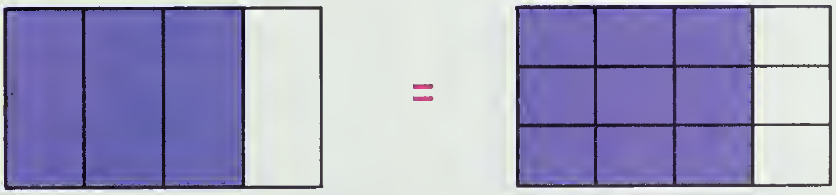
Notice that $\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$.

Equivalent ratios are ratios for the same comparison.
Equivalent ratios are named by equivalent fractions.

Example 1: Find an equivalent ratio for $\frac{16}{24}$ in **lowest terms.**

$$\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$$

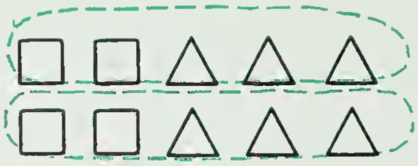
Example 2: Show $\frac{3}{4} = \frac{9}{12}$ with a diagram.

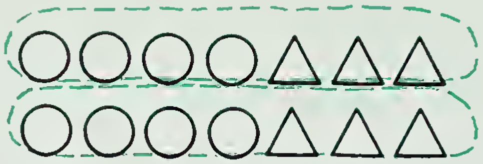


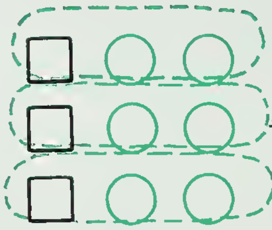
The two diagrams are the same size and have the *same* amount shaded. The first has 3 out of 4 parts shaded, and the second has 9 out of 12 parts shaded. So $\frac{3}{4} = \frac{9}{12}$.

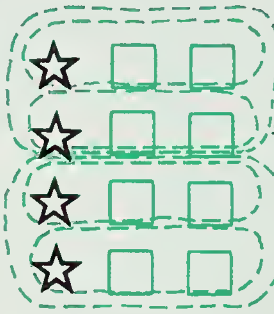
Exercises

1–4. Copy each diagram and group to show that the given ratios are equivalent. *Ex. 1–4: Answers given in dashed lines*

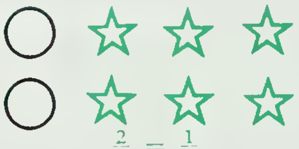
1.

$\frac{4}{6} = \frac{2}{3}$

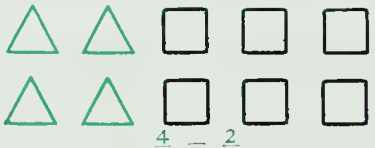
2.

$\frac{8}{6} = \frac{4}{3}$

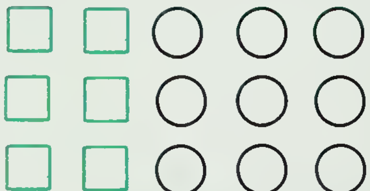
3.

$\frac{3}{6} = \frac{1}{2}$

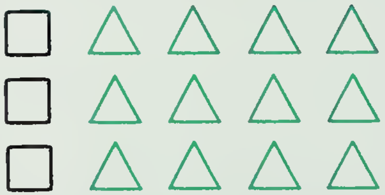
4.

$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$

5–8. Give the equivalent ratios shown by each diagram.

5.

$\frac{2}{6} = \frac{1}{3}$

6.

$\frac{4}{6} = \frac{2}{3}$

7.

$\frac{6}{9} = \frac{2}{3}$

8.

$\frac{3}{12} = \frac{1}{4}$

9–16. Show by multiplying or dividing that the ratios in each pair are equivalent.

Example: $\frac{4}{6}, \frac{32}{48}$ $\quad \frac{4}{6} = \frac{4 \times 8}{6 \times 8} = \frac{32}{48}$

9. $\frac{3}{7}, \frac{9}{21}$ $\quad \frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$

10. $\frac{4}{5}, \frac{8}{10}$ $\quad \frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

11. $\frac{20}{12}, \frac{5}{3}$ $\quad \frac{20}{12} = \frac{20 \div 4}{12 \div 4} = \frac{5}{3}$

12. $\frac{12}{18}, \frac{2}{3}$ $\quad \frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$

13. $\frac{8}{3}, \frac{32}{12}$ $\frac{8}{3} = \frac{8 \times 4}{3 \times 4} = \frac{32}{12}$

14. $\frac{5}{3}, \frac{25}{15}$ $\frac{5}{3} = \frac{5 \times 5}{3 \times 5} = \frac{25}{15}$

15. $\frac{20}{24}, \frac{5}{6}$ $\frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}$

16. $\frac{7}{12}, \frac{21}{36}$ $\frac{7}{12} = \frac{7 \times 3}{12 \times 3} = \frac{21}{36}$

17–22. Give *any* two ratios equivalent to each ratio below.

Ex. 17–22: Typical answers

Example: $\frac{4}{6}$ $\quad \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}, \quad \frac{4}{6} = \frac{4 \times 3}{6 \times 3} = \frac{12}{18}$

17. $\frac{2}{6}$ $\frac{1}{3}, \frac{4}{12}$

18. $\frac{8}{6}$ $\frac{4}{3}, \frac{16}{12}$

19. $\frac{2}{5}$ $\frac{4}{10}, \frac{6}{15}$

20. $\frac{6}{7}$ $\frac{12}{14}, \frac{18}{21}$

21. $\frac{4}{3}$ $\frac{8}{6}, \frac{12}{9}$

22. $\frac{3}{8}$ $\frac{6}{16}, \frac{9}{24}$

23–34. Find an equivalent ratio in lowest terms.

23. $\frac{8}{10}$ $\frac{4}{5}$

24. $\frac{4}{12}$ $\frac{1}{3}$

25. $\frac{20}{36}$ $\frac{5}{9}$

26. $\frac{9}{18}$ $\frac{1}{2}$

27. $\frac{39}{21}$ $\frac{13}{7}$

28. $\frac{28}{18}$ $\frac{14}{9}$

29. $\frac{20}{28}$ $\frac{5}{7}$

30. $\frac{20}{4}$ $\frac{5}{1}$

31. $\frac{12}{42}$ $\frac{2}{7}$

32. $\frac{72}{27}$ $\frac{8}{3}$

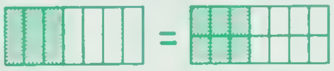
33. $\frac{35}{14}$ $\frac{5}{2}$

34. $\frac{54}{27}$ $\frac{2}{1}$

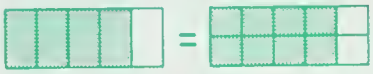
35–38. Draw a diagram to show that the ratios in each pair are equivalent.

Ex. 35–38: Typical answers

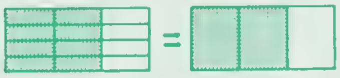
35. $\frac{3}{7}, \frac{6}{14}$



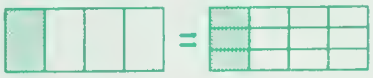
36. $\frac{4}{5}, \frac{8}{10}$



37. $\frac{8}{12}, \frac{2}{3}$



38. $\frac{1}{4}, \frac{3}{12}$



39–44. Write each ratio in lowest terms.

Example: 4 h to 15 min

$$\frac{4\text{ h}}{15\text{ min}} = \frac{240\text{ min}}{15\text{ min}} = \frac{240}{15} = \frac{240 \div 15}{15 \div 15} = \frac{16}{1}$$

39. 10 m to 4 cm $\frac{250}{1}$

40. 3 h to 5 min $\frac{36}{1}$

41. 2 h to 20 min $\frac{6}{1}$

42. 75 cm to 2 m $\frac{3}{8}$

43. 450 g to 2 kg $\frac{9}{40}$

44. 150 L to 0.4 mL $\frac{80}{3}$

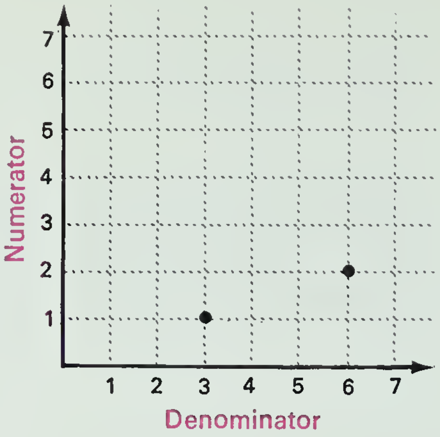
GRAPHING EQUIVALENT RATIOS

A ratio can be graphed by plotting the ordered pair (denominator, numerator).

Example: Graph $\frac{1}{3}$ and $\frac{2}{6}$.

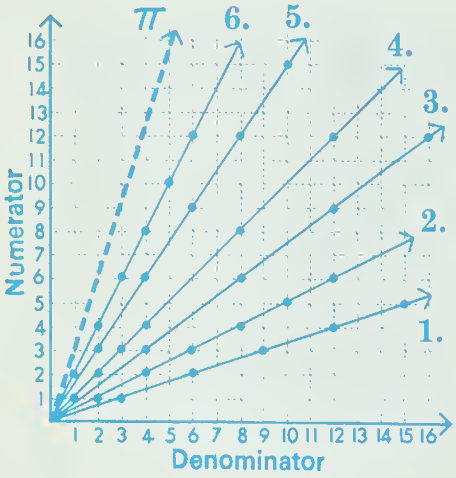
The ordered pair for $\frac{1}{3}$ is (3, 1).

The ordered pair for $\frac{2}{6}$ is (6, 2).



Graph each set of equivalent ratios on the same grid. Connect the points of each set.

1–6. See graph at left.



1. $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}\}$

2. $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}\}$

3. $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}\}$

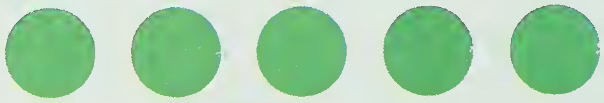
4. $\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{8}{8}, \frac{12}{12}\}$

5. $\{\frac{3}{2}, \frac{6}{4}, \frac{9}{6}, \frac{12}{8}, \frac{15}{10}\}$

6. $\{\frac{2}{1}, \frac{4}{2}, \frac{6}{3}, \frac{8}{4}, \frac{10}{5}, \frac{12}{6}\}$

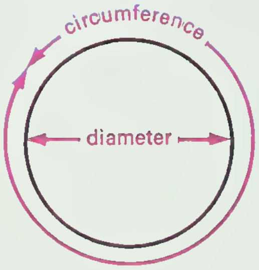
7. What does the graph of a set of equivalent ratios look like? Straight line

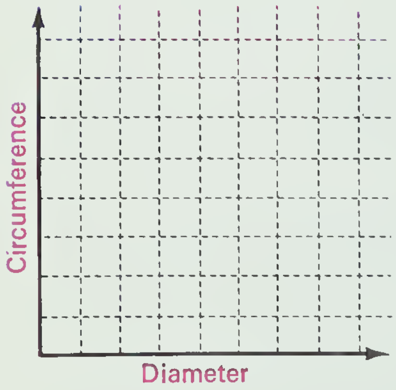
8. If the graph were larger, how many more equivalent ratios could you find for each set? Infinitely many



There is a curious relationship between the diameter and the circumference of circular objects.

As an experiment, use a tape measure to find the diameter and the circumference of 6 circular objects, such as tin cans or roles of tape.





Find and graph each ratio $\frac{\text{circumference}}{\text{diameter}}$ using the ordered pair (circumference, diameter).

1. What pattern is formed on the graph? Straight line

2. If any ratio $\frac{c}{d}$ is divided out, what number results? π, about 3.14

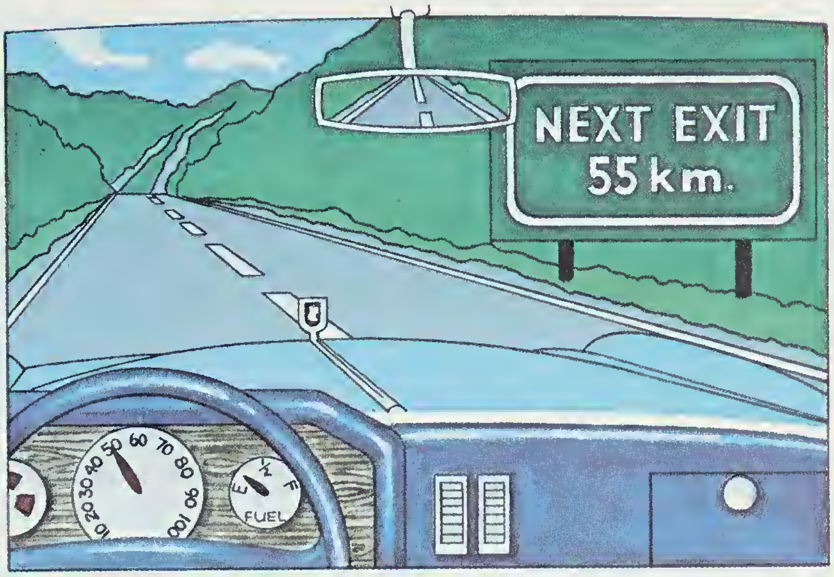
3. How do you suppose you could find the circumference of a circle if you know the diameter? $c = \pi \times d$

Performance Objective: The student can use cross-products to solve proportions.

Proportions

4.3

Louise has driven 300 km since last filling her car's 75 L gas tank. She has used 50 L. Can she make the next exit 100 km away?



300 km on
50 L

? km on
25 L

Louise used equivalent ratios to make a quick calculation.

$$\frac{300}{50}\begin{array}{c}\longleftarrow\text{distance}\longrightarrow\\ \longleftarrow\text{litres}\longrightarrow\end{array}\frac{K}{25}$$

Since $\frac{300}{50} = \frac{300 \div 2}{50 \div 2} = \frac{150}{25}$, K must be 150.

She can go another 150 km, so she can reach the next exit.

The sentence $\frac{300}{50} = \frac{K}{25}$ is called a *proportion.*

A **proportion** is a sentence that states two ratios are equivalent.

To help in solving proportions, there is a useful fact about equivalent ratios.

$$12 \nwarrow \frac{2}{3} = \frac{4}{6} \nearrow 12$$

The cross-products are both 12.

$$30 \nwarrow \frac{3}{5} = \frac{6}{10} \nearrow 30$$

The cross-products are both 30.

Whenever two ratios are equivalent, the **cross-products** are equal.

Example 1: Solve $\frac{300}{50} = \frac{K}{25}$

$$\frac{300}{50} = \frac{K}{25}$$

$$300 \times 25 = 50 \times K$$ Take the cross-products.

$$7500 = 50K$$

$$\frac{7500}{50} = \frac{50K}{50}$$ Divide each side by 50.

$$150 = K$$

Example 2: Solve $\frac{4}{5} = \frac{6}{x}$

$$\frac{4}{5} = \frac{6}{x}$$

$$4 \times x = 5 \times 6$$ Take the cross-products.

$$4x = 30$$

$$\frac{4x}{4} = \frac{30}{4}$$ Divide each side by 4.

$$x = 7\tfrac{1}{2} \text{ or } 7.5$$

Exercises

1. If two ratios are equivalent, the ______ are equal. Cross-products

2. A sentence stating that two ratios are equivalent is called a ______. Proportion

3. $\frac{180}{24} = \frac{K}{8}$ might mean 180 km on 24 L is the same as K kilometres on 8 L.

4. $\frac{2}{3} = \frac{4}{6}$ means 2 out of __3__ is the same as 4 out of __6__.

5–22. Use cross-products to solve each proportion.

5. $\frac{2}{5} = \frac{N}{20}$ 8

6. $\frac{3}{7} = \frac{x}{21}$ 9

7. $\frac{6}{9} = \frac{26}{y}$ 39

8. $\frac{4}{7} = \frac{20}{a}$ 35

9. $\frac{12}{15} = \frac{16}{b}$ 20

10. $\frac{4}{5} = \frac{12}{c}$ 15

11. $\frac{21}{r} = \frac{6}{8}$ 28

12. $\frac{20}{a} = \frac{8}{12}$ 30

13. $\frac{m}{16} = \frac{27}{72}$ 6

14. $\frac{n}{51} = \frac{25}{17}$ 75

15. $\frac{x}{15} = \frac{3}{2}$ $22\frac{1}{2}$

16. $\frac{2}{5} = \frac{7}{M}$ $17\frac{1}{2}$

17. $\frac{4}{9} = \frac{w}{12}$ $5\frac{1}{3}$

18. $\frac{45}{n} = \frac{10}{8}$ 36

19. $\frac{3}{x} = \frac{5}{100}$ 60

20. $\frac{c}{10} = \frac{56}{16}$ 35

21. $\frac{75}{25} = \frac{45}{a}$ 15

22. $\frac{20}{85} = \frac{32}{w}$ 136

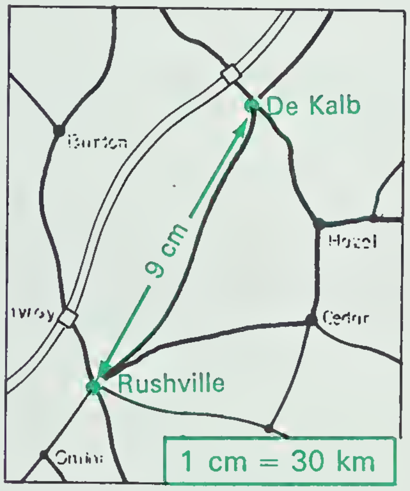
23–28. Use proportions to solve.

23. A car can go 352 km on 40 L of gas. How far can it go on 25 L? 220 km

$$\frac{352}{40} \quad \begin{matrix} \longleftarrow \text{distance} \longrightarrow \\ \longleftarrow \text{litres} \longrightarrow \end{matrix} \quad \frac{d}{25}$$

24. A secretary typed 6 pages in 32 min. How long will 15 pages take? 80 min

$$\frac{6}{32} \quad \begin{matrix} \longleftarrow \text{pages} \longrightarrow \\ \longleftarrow \text{time} \longrightarrow \end{matrix} \quad \frac{15}{t}$$



25. How many kilometres is it from De Kalb to Rushville? 270 km

$$\frac{30}{1} \quad \begin{matrix} \leftarrow \text{real distance (km)} \rightarrow \\ \leftarrow \text{map distance (cm)} \rightarrow \end{matrix} \quad \frac{k}{9}$$

26. If two towns are actually 360 km apart, what is the map distance? 12 cm

$$\frac{30}{1} \quad \begin{matrix} \leftarrow \text{real distance (km)} \rightarrow \\ \leftarrow \text{map distance (cm)} \rightarrow \end{matrix} \quad \frac{360}{m}$$

27. A printer needs a photograph reduced in size for a book page. If the photograph is to be 9 cm tall, how wide will it be? 6 cm

$$\frac{w}{9} \quad \begin{matrix} \longleftarrow \text{width} \longrightarrow \\ \longleftarrow \text{height} \longrightarrow \end{matrix} \quad \frac{10}{15}$$

28. A water tank holds 6800 L when it is filled to the 4 m mark. How many litres does it contain when the water is 2 m deep? 3400 L

$$\frac{6800}{4} \quad \begin{matrix} \longleftarrow \text{litres} \longrightarrow \\ \longleftarrow \text{depth} \longrightarrow \end{matrix} \quad \frac{L}{2}$$

10 cm wide
15 cm tall

A. Devaney, Inc.

4.4 Solving Problems with Proportions

Performance Objective: The student can solve problems by writing appropriate proportions and solving the proportions.

Regulation pool tables are made in different sizes. But the length and width have the same ratio. A pool table 220 cm long will be 110 cm wide.

Camerique

Example 1: If a pool table is 190 cm long, how wide is it?

Two different proportions can be used to solve this problem. You can write ratios to compare:

first table to second table

220 ← lengths → 190
110 ← widths → w

$$\frac{220}{110} = \frac{190}{w}$$

$$220 \times w = 190 \times 110$$

$$220w = 20\ 900$$

$$\frac{220w}{220} = \frac{20\ 900}{220}$$

$$w = 95$$

or

lengths to widths

220 ← first table → 110
190 ← second table → w

$$\frac{220}{190} = \frac{110}{w}$$

$$220 \times w = 190 \times 110$$

$$220w = 20\ 900$$

$$\frac{220w}{220} = \frac{20\ 900}{220}$$

$$w = 95$$

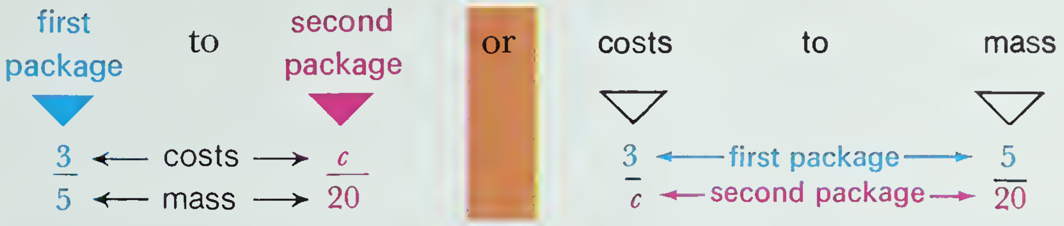
Cross products

Divide by 220

Each proportion gives the same answer. You can choose either proportion. But when you write a proportion, comparisons must be made in the *same order* for both ratios.

Example 2: It costs \$3 to ship a 5 kg package. At the same rate, what will a 20 kg package cost?

There is more than one correct proportion. You could compare:



$$\frac{3}{c} = \frac{5}{20}$$

Take the cross-products. $60 = 5c$

Divide each side by 5. $12 = c$

The 20 kg package will cost \$12.

Exercises

1–10. Copy and complete the table.

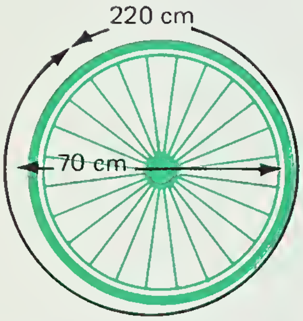
First information	Second information	First ratio	Second ratio	True proportion
Example: 22 km, 2 L	77 km, 7 L	$\frac{22}{2}$	$\frac{77}{7}$	$\frac{22}{2} = \frac{77}{7}$
1. 21 km, 3 L	119 km, 17 L	$\frac{21}{3}$	$\frac{119}{17}$	$\frac{21}{3} = \frac{119}{17}$
2. 21 km, 3 L	119 km, 17 L	$\frac{21}{119}$	$\frac{3}{17}$	$\frac{21}{119} = \frac{3}{17}$
3. 6 m long, 4 m wide	15 m long, 10 m wide	$\frac{6}{4}$	$\frac{15}{10}$	$\frac{6}{4} = \frac{15}{10}$
4. 300 g, 14 cm	450 g, 21 cm	$\frac{14}{21}$	$\frac{300}{450}$	$\frac{14}{21} = \frac{300}{450}$
5. 8 baskets, 36 shots	6 baskets, 27 shots	$\frac{6}{27}$	$\frac{8}{36}$	$\frac{6}{27} = \frac{8}{36}$
6. 4 h, 20 min	6 h, 30 min	$\frac{20}{30}$	$\frac{4}{6}$	$\frac{20}{30} = \frac{4}{6}$
7. 12 mg, 72¢	7 mg, 42¢	$\frac{7}{42}$	$\frac{12}{72}$	$\frac{7}{42} = \frac{12}{72}$
8. 80 km, 50 km	30 km, x km	$\frac{80}{30}$	$\frac{50}{x}$	$\frac{80}{30} = \frac{50}{x}$
9. 20 ups, 23 downs	25 ups, y downs	$\frac{20}{25}$	$\frac{23}{y}$	$\frac{20}{25} = \frac{23}{y}$
10. 10.5 zips, 16.5 zaps	7 zips, 11 zaps	$\frac{11}{16.5}$	$\frac{7}{10.5}$	$\frac{11}{16.5} = \frac{7}{10.5}$

11—32. Solve each problem with a proportion.

11. If it costs $2 to ship a 10 kg package, what does it cost for a 25 kg package? $5

12. A car went 64 km on 11 L of gas. How much gas would it take to go 320 km? 55 L

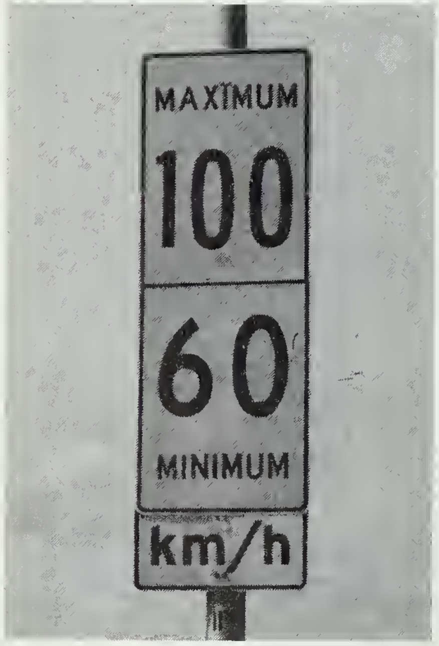
13. A 14 kg mass stretches a spring 6 cm. How far will a 35 kg mass stretch the spring? 15 cm



14. A recipe for 6 calls for 2 L of water. If a cook is preparing for 15 people, how much water should be used? 5 L

15. A 70 cm bicycle wheel has a circumference of 220 cm. What is the circumference of a 49 cm wheel? 154 cm

16. 4 m of brass pipe has a mass of 12 kg. What is the mass of 10 m of the same pipe? 30 kg



Robert Brown

At 50 km/h a car is traveling 14 m/s.

17. How many metres per second does a car travel at 75 km/h? 21 m/s

18. How many kilometrtes per hour is 7 m/s? 25 km/h

At the left is a speed-limit sign with both maximum and minimum speeds. Use a proportion to solve each problem.

19. If the maximum is 90, what would the minimum be? 54 km/h

20. If the minimum speed is 40 km/h, what would the maximum be? 66.7 km/h

21. A printer wants to reduce a photograph 10 cm wide by 8 cm tall to a size 4 cm wide. How tall will it be? 3.2 cm

22. Nora made $17 for four hours work. How much could she make if she worked 10 h. $42.50

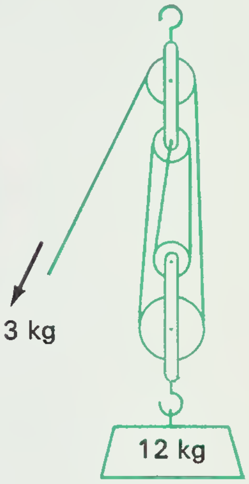
23. A snowmobile needs a fuel mixture of 2 L of oil for every 20 L of gas. How much oil for 12 L of gas? 1.2 L

24. If a 3 kg mass stretches a spring 5 cm, what mass will stretch it 7 cm? 4.2 kg

25. Ken paid $3.50 for 14 L of gas. At that rate, what would it cost him to fill his 70 L gas tank? $17.50

26. Mrs. Rackow won an election by a 5 to 2 margin. Her opponent got 2800 votes. How many votes did she get? 7000 votes

A certain pulley will lift 12 kg when the rope is pulled with a force of 3 kg.



27. If you pulled the rope with a force of 20 kg, what mass could you lift? 80 kg

28. How hard does a garage mechanic have to pull to lift a 180-kg auto engine with the pulley? 45 kg

Boat speeds are usually given in knots instead of km/h.
20 knots is the same as 37 km/h.

29. A boat's top speed is 35 knots. How many km/h is that? 64.75 km/h

30. What is the speed in knots of a boat going 74 km/h? 40 knots

De Wys, Inc.

31. If property taxes are assessed at $2.12 per $100, what is the tax on a $65,800 house? $1,394,96

32. An engine is turning at the rate of 8400 revolutions every seven minutes. How many revolutions in ten minutes? 12 000 rev

Wanda Witch worked 12 wonderful whammies with 20 wand waves. How many waves for 15 fabulous feats? 25

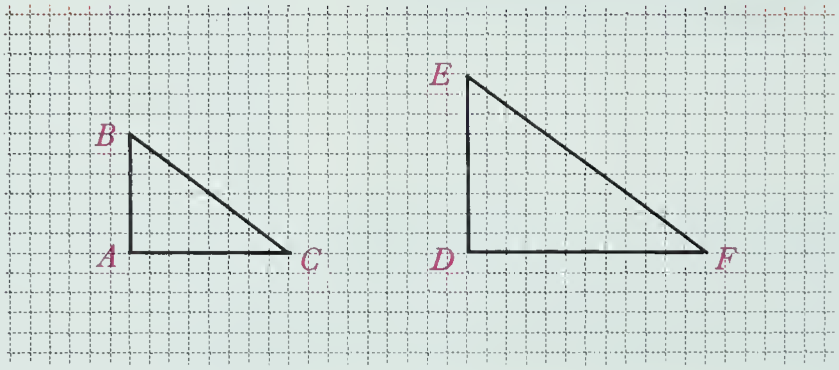
4.5 Similarity

Performance Objective: The student can use proportions to find the missing measures of similar figures.

Mosallem/De Wys, Inc.

When you get a picture enlarged, you expect it to look like the original. You wouldn't want the picture made 3 times as tall but only 2 times as wide. Everything would look tall and skinny. You expect everything to keep the same shape.

Similar figures have the same shape but may have different sizes.



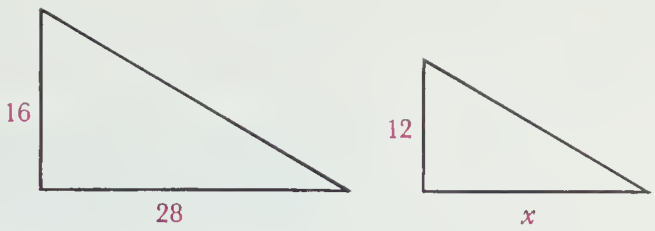
Triangle ABC and triangle DEF are similar. Take the ratios of corresponding sides.

$$\frac{AB}{DE} = \frac{6}{9} = \frac{2}{3} \qquad \frac{AC}{DF} = \frac{8}{12} = \frac{2}{3} \qquad \frac{BC}{EF} = \frac{10}{15} = \frac{2}{3}$$

In lowest terms, all the ratios are equal!

In similar figures, the ratios of corresponding lengths are equal.

Example 1: The triangles are similar. Find x.



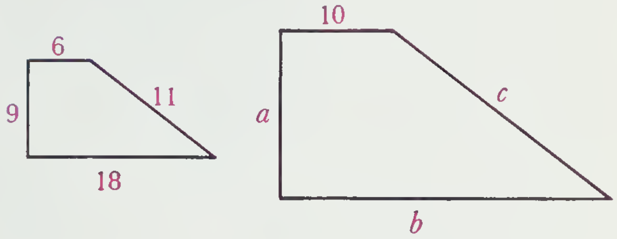
Since the ratios of corresponding lengths are equal, a proportion can be set up.

$$\frac{16}{12} = \frac{28}{x}$$

$16x = 336$ Take cross-products.

$x = 21$ Divide by 16.

Example 2: The figures are similar. Find the missing values.



Set up a proportion for each missing value.

$$\frac{6}{10} = \frac{9}{a} \qquad \frac{6}{10} = \frac{18}{b} \qquad \frac{6}{10} = \frac{11}{c}$$

$$6a = 90 \qquad 6b = 180 \qquad 6c = 110$$

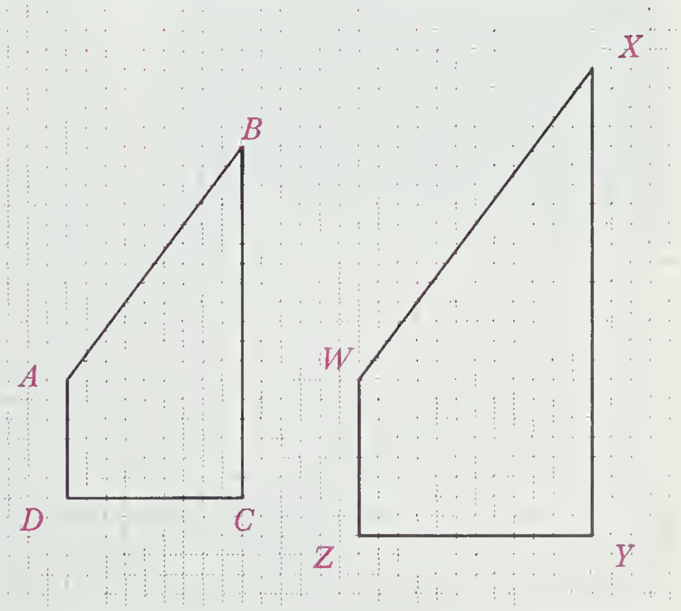
$$a = 15 \qquad b = 30 \qquad c = 18\tfrac{1}{3}$$

Exercises

1. Copy and complete the chart.

To measure slanted sides, mark off the units of the grid on the edge of a sheet of paper, and use that as a ruler.

Ratios of corresponding sides			Ratios in lowest terms	
$\frac{AD}{WZ}$	=	$\frac{6}{8}$	=	$\frac{3}{4}$
$\frac{CD}{YZ}$	=	$\frac{9}{12}$	=	$\frac{3}{4}$
$\frac{BC}{XY}$	=	$\frac{18}{24}$	=	$\frac{3}{4}$
$\frac{AB}{WX}$	=	$\frac{15}{20}$	=	$\frac{3}{4}$



2. Copy and complete the chart. Use the pictures on page 158. Take measurements in millimetres.

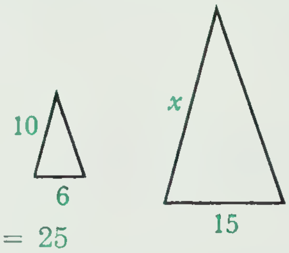
Measurement	Small picture	Large picture	Ratio	
			$\frac{\text{small}}{\text{large}}$	in lowest terms
distance between eyes	3	6	$\frac{3}{6}$	$= \frac{1}{2}$
distance between ears	10	20	$\frac{10}{20}$	$= \frac{1}{2}$
width of mouth	4	8	$\frac{4}{8}$	$= \frac{1}{2}$

All the ratios in the last column should be the same. Is it possible that the ratios may not be exactly equal? Why?

The ratios may not be equal since measurements are not exact.

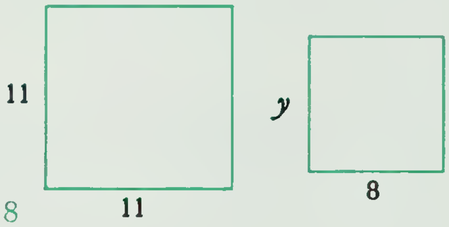
3–12. Find the missing values for each pair of similar figures.

3.



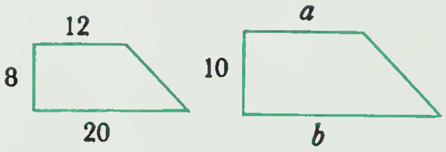
$x = 25$

4.



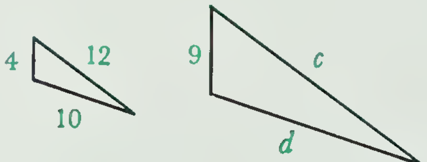
$y = 8$

5.



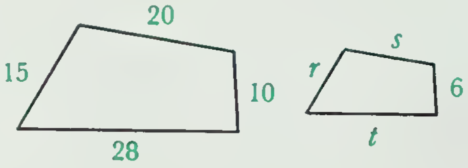
$a = 15, b = 25$

6.



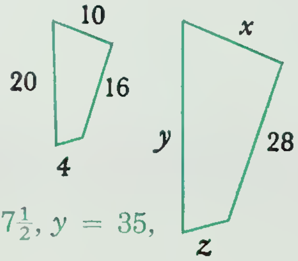
$c = 27, d = 22\frac{1}{2}$

7.


20
15
10
28
s
r
6
t

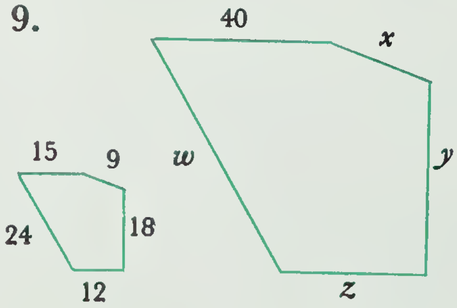
$r = 9, s = 12, t = 16\frac{4}{5}$

8.


10
20
16
4
x
y
28
z

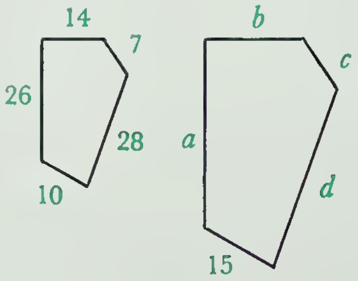
$x = 17\frac{1}{2}, y = 35,$
$z = 7$

9.


40
x
15
9
w
y
24
18
12
z

$w = 64, x = 24, y = 48, z = 32$

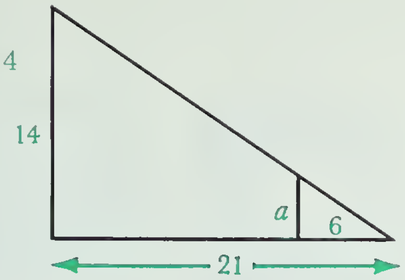
10.


14
7
b
c
26
28
a
10
d
15

$a = 39, b = 21, c = 10\frac{1}{2}, d = 42$

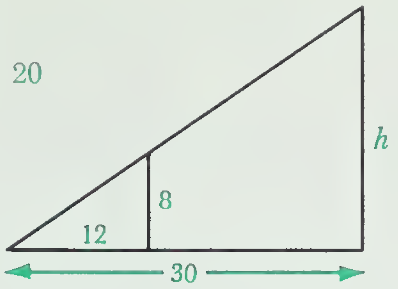
11.

$a = 4$



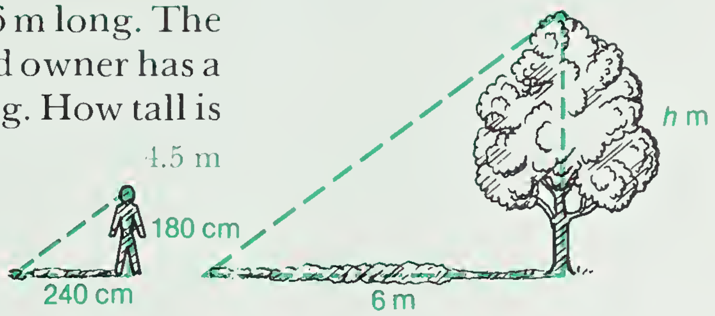
12.

$h = 20$

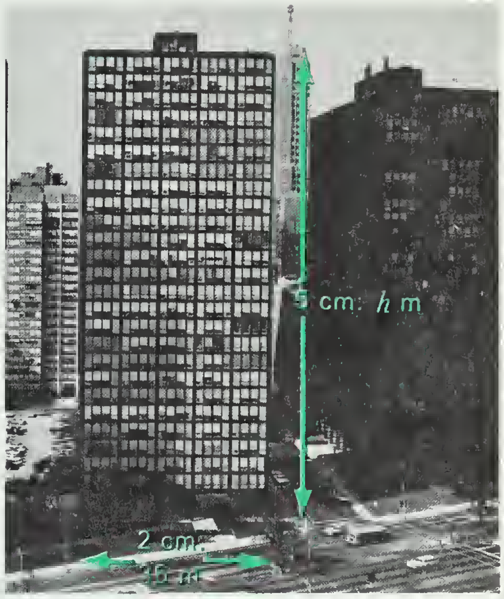


13–16. Use a proportion to solve each problem.

13. A tree's shadow is 6 m long. The 180 cm tall orchard owner has a shadow 240 cm long. How tall is the tree? 4.5 m

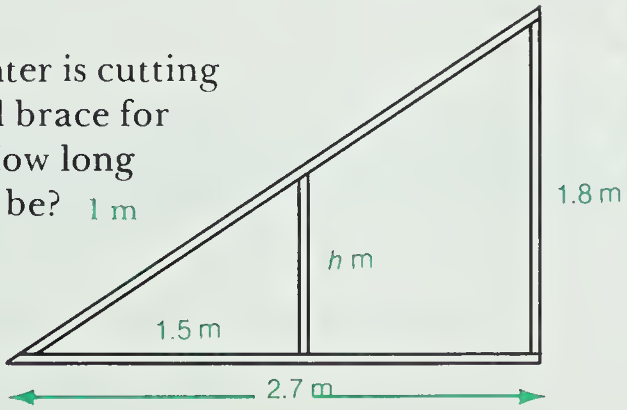


14. A photograph of a building measures 2 cm wide and 5 cm tall. The actual building measures 36 m wide along the ground. Find the height h for the actual building. 90 m



Futagawa/Retoria

15. A carpenter is cutting a vertical brace for a roof. How long should it be? 1 m

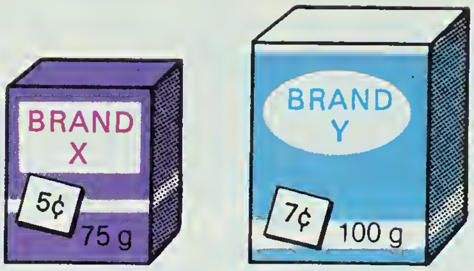


16. You can just see the top of a building over a 7 m light pole. You are 2 m from the pole and 14 m from the building. How tall is the building? (HINT: Draw a figure like the one in Exercise 12.) 49 m

4.6 Proportions and Comparing

Performance Objective: The student can use cross-products to find which of two choices gives the larger amount for the money.

Which is the better buy?
Which gives more for the money?



One way to find out is to see how much of each can be bought for the *same* money.

75 g for 5¢ 100 g for 7¢

Write the ratios $\frac{\text{amount}}{\text{cost}}$. $\frac{75}{5}$ $\frac{100}{7}$

We want to find the *larger* ratio.

35 is a common denominator. $\frac{75 \times 7}{5 \times 7}$ $\frac{100 \times 5}{7 \times 5}$

Which ratio is larger? $\frac{525}{35}$ $\frac{500}{35}$

525 g for 35¢ 500 g for 35¢

Brand X is the better buy,
larger amount for the same money

But there is an easier way!

Write the ratios next to each other. $\frac{75}{5}$ $\frac{100}{7}$

Now take the cross-products. $525 \leftarrow \frac{75}{5} \quad \frac{100}{7} \rightarrow 500$

525 is a larger number than 500.
525 came from the numerator 75.
So 75 g for 5¢ is the better buy.

The ratio whose numerator (amount) gives the larger cross-product is the better buy.

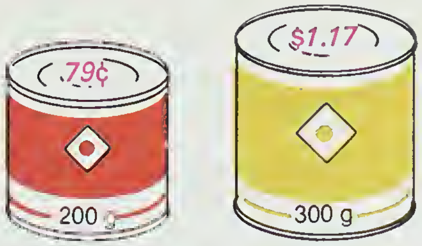
Example 1: Which ratio is larger, $\frac{4}{7}$ or $\frac{5}{8}$?

$\frac{4}{7}$ $\frac{5}{8}$ Write the ratios.

4×8 7×5 Take the cross-products.

32 35 $\frac{5}{8}$ is larger since its numerator gives the larger cross-product.

Example 2: Find the better buy.



Write the ratios $\frac{\text{amount}}{\text{cost}}$. $\frac{200}{79}$ $\frac{300}{117}$

Note: Always use the same units.

Take the cross-products. 200×117 79×300

$23\ 400$ $23\ 700$

The 300 g size gives more for the money. better buy

Exercises

1. The better buy is the one with the larger ratio of ______ to cost. Amount

2. For two ratios, the ratio whose ______ gives the larger cross-product is the larger ratio. Numerator

3–8. Rewrite each pair of ratios so that they have a common denominator. Then tell which original ratio is larger.

3. $\frac{2}{3}, \frac{3}{5}$ $\frac{10}{15}, \frac{9}{15}; \frac{2}{3}$

4. $\frac{5}{8}, \frac{3}{5}$ $\frac{25}{40}, \frac{24}{40}; \frac{5}{8}$

5. $\frac{3}{13}, \frac{2}{9}$ $\frac{27}{117}, \frac{26}{117}; \frac{3}{13}$

6. $\frac{4}{11}, \frac{6}{17}$ $\frac{68}{187}, \frac{66}{187}; \frac{4}{11}$

7. $\frac{13}{15}, \frac{20}{23}$ $\frac{299}{345}, \frac{300}{345}; \frac{20}{23}$

8. $\frac{12}{17}, \frac{16}{21}$ $\frac{252}{357}, \frac{272}{357}; \frac{16}{21}$

9–14. Take the cross-product of each pair of ratios. Then tell which ratio is larger.

9. $\frac{5}{7}, \frac{6}{8}$ 40, 42; $\frac{6}{8}$

10. $\frac{5}{8}, \frac{3}{5}$ 25, 24; $\frac{5}{8}$

11. $\frac{3}{13}, \frac{2}{9}$ 27, 26; $\frac{3}{13}$

12. $\frac{4}{11}, \frac{6}{17}$ 68, 66; $\frac{4}{11}$

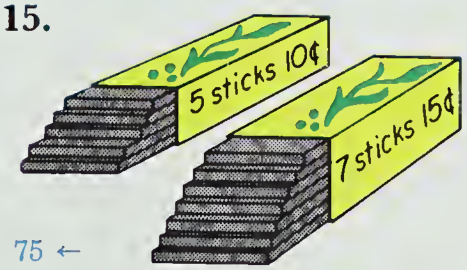
13. $\frac{13}{15}, \frac{20}{23}$ 299, 300; $\frac{20}{23}$

14. $\frac{12}{17}, \frac{16}{21}$ 252, 272; $\frac{16}{21}$

15–30. Find the better buys.

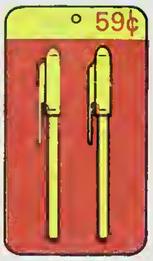
Ex. 15–30: Cross-products are given in order; arrow indicates better buy.

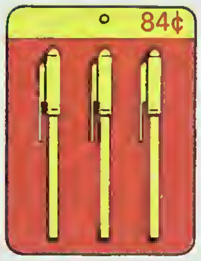
15.



75 ←
70

16.





168
177 ←

17. 200 g for 59¢ 15 800
300 g for 79¢ 17 700 ←

18. 150 mL for 43¢ 11 700
325 mL for 78¢ 13 975 ←

19. 4 kg for 39¢ 268
7 kg for 67¢ 273 ←

20. 3 kg for 75¢ 288
4 kg for 96¢ 300 ←

21. 200 g for 32¢ 15 600
500 g for 78¢ 16 000 ←

22. 400 g for 44¢ 31 200 ←
700 g for 78¢ 30 800

23. 150 g for 94¢ 33 750
500 g for $2.25 47 000 ←

24. 250 g for 79¢ 29 750
450 g for $1.19 35 550 ←

25. 36 tablets for $.69 5940
100 tablets for $1.65 6900 ←

26. 100 tablets for $2.65 65 000
250 tablets for $6.50 66 250 ←

27. 2.5 kg for $1.21 497.5 ←
4 kg for $1.99 484

28. 200 mL for $1.19 41 800
500 mL for $2.09 59 500 ←

29. 1100 mL for $1.65 229 900
1500 mL for $2.09 247 500 ←

30. 5.5 kg for $.88 819.5 ←
9 kg for $1.49 792

Quick Quiz

Check these skills:

- *dividing whole numbers by whole numbers*
- *rounding quotients to the nearest tenth*

Example: Find $\frac{48}{13}$ to the nearest tenth.

$$\begin{array}{r} 3.69 \\ 13\overline{)48.00} \\ \underline{39} \\ 9\,0 \\ \underline{7\,8} \\ 1\,20 \\ \underline{1\,17} \\ 3 \end{array}$$

$\frac{48}{13} = 3.7$ to the nearest tenth.

1–8. Find each quotient to the nearest tenth.

1. $29 \div 7$ 4.1

2. $8\overline{)241}$ 30.1

3. $\frac{123}{6}$ 20.5

4. $\frac{294}{9}$ 32.7

5. $68 \div 18$ 3.8

6. $15\overline{)262}$ 17.5

7. $\frac{345}{23}$ 15.0

8. $\frac{864}{19}$ 45.5

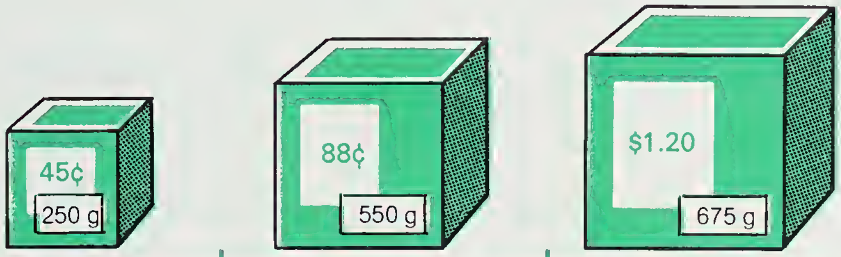
4.7

Common Values for Comparison

If you have 3 or more items to compare, you can use proportions to find how much of each you can buy for the same amount of money. An easy amount to use is $1.00, or 100 cents.

Performance Objective: The student can employ proportions to compare prices using either unit costs or the common value 100.

Example 1: Find the rate that gives the most for $1.00.



$\frac{\text{amount}}{\text{cost}}$	$\frac{250}{45} = \frac{a}{100}$	$\frac{550}{88} = \frac{a}{100}$	$\frac{675}{120} = \frac{a}{100}$
	$2500 = 45a$	$55\,000 = 88a$	$67\,500 = 120a$
	$555.55\ldots = a$	$625 = a$	$562.5 = a$
	555.6g for \$1.00	625 g for \$1.00	562.5 g for \$1.00

In each case, we want to know: "How many grams for 100 cents?"

The *most* for the *same* money—best buy!

The three dots in 555.55 . . . mean more numbers follow. You could get as many numbers as you want if you keep including zeros and dividing. One place after the decimal point is usually enough for comparisons. So 555.55 . . . is rounded to the nearest tenth.

You can also find the best buy by finding the cost of one gram of one kilogram. This is called the **unit cost.** You are looking for the *lowest* cost per gram.

Example 2: Find the one with the lowest cost per gram.

	200 g for 48¢	400 g for 88¢	775 g for \$1.63
$\frac{\text{amount}}{\text{cost}}$	$\frac{200}{48} = \frac{1}{c}$	$\frac{400}{88} = \frac{1}{c}$	$\frac{775}{163} = \frac{1}{c}$
	$200c = 48$	$400c = 88$	$775c = 163$
	$c = 0.24$	$c = 0.22$	$c = 0.210$
	1 g for 0.24¢	1 g for 0.22¢	1 g for 0.21¢

Answer is rounded

In each case, we want to know: "What does 1 g cost?"

The *same* for the *least* money—best buy!

Exercises

1. When you compare items by using the unit cost, the best buy gives the same amount for the Least money.

2. When you compare items by using the common value 100, the best buy gives the Largest amount for the same money.

3–14. Solve each proportion. If necessary, round to the nearest tenth.

3. $\frac{4}{5} = \frac{a}{100}$ 80 **4.** $\frac{7}{2} = \frac{x}{100}$ 350 **5.** $\frac{3}{8} = \frac{n}{100}$ 37.5 **6.** $\frac{9}{5} = \frac{x}{100}$ 180

7. $\frac{9}{16} = \frac{n}{100}$ 56.3 **8.** $\frac{11}{56} = \frac{a}{100}$ 19.6 **9.** $\frac{5}{20} = \frac{1}{c}$ 4 **10.** $\frac{3}{18} = \frac{1}{x}$ 6

11. $\frac{16}{24} = \frac{1}{w}$ 1.5 **12.** $\frac{14}{56} = \frac{1}{n}$ 4 **13.** $\frac{2}{85} = \frac{1}{m}$ 42.5 **14.** $\frac{12}{75} = \frac{1}{c}$ 6.3

15–20. Find the best buy by finding which rate gives the most for $1.00. *Ex. 15–26: Arrow indicates best buy.*

15. 50 g for 25¢ 200
75 g for 30¢ 250
150 g for 50¢ 300←

16. 75 g for 10¢ 750
175 g for 20¢ 875←
200 g for 25¢ 800

17. 400 mL for 25¢ 1600←
525 mL for 40¢ 1313
625 mL for 45¢ 1389

18. 175 g for 35¢ 500
300 g for 48¢ 625
400 g for 60¢ 667←

19. 425 g, 64¢ 664
600 g, 84¢ 714
950 g, $1.25 760←
1300 g, $1.85 703

20. 150 mL, 20¢ 750
400 mL, 45¢ 889
750 mL, 78¢ 962
1250 mL, $1.25 1000←

21–26. Find the best buy by finding the cost per gram for each rate.

21. 50 g for 14¢ 0.28
100 g for 24¢ 0.24
200 g for 40¢ 0.20←

22. 75 g for 33¢ 0.44←
125 g for 65¢ 0.52
175 g for 84¢ 0.48

23. 100 g for 24¢ 0.24
275 g for 58¢ 0.21
500 g for 99¢ 0.20←

24. 75 g for 36¢ 0.48
175 g for 77¢ 0.44←
200 g for 97¢ 0.49

25. 300 g, 60¢ 0.2
425 g, 79¢ 0.19
550 g, 90¢ 0.16←
775 g, $1.41 0.18

26. 175 g, 91¢ 0.52
400 g, $1.80 0.45←
600 g, $2.75 0.46
950 g, $4.48 0.47

MATH IN PUBLIC-SERVICE JOBS

CAREER CORNER

Many workers do not make a product, but they perform a service for the public. The problems a public-service worker may run into are often about money.

Some problems are given below. They can be solved with proportions, but other methods may work as well. Can you solve them?

Durango Mendoza

Supermarket cashier

What should be charged for 2 cans of soup marked 3 for 79¢? 53¢

Bookkeeper

A company's income for the first 3 months of the year was $45 380. Estimate the income for the year. $181 520

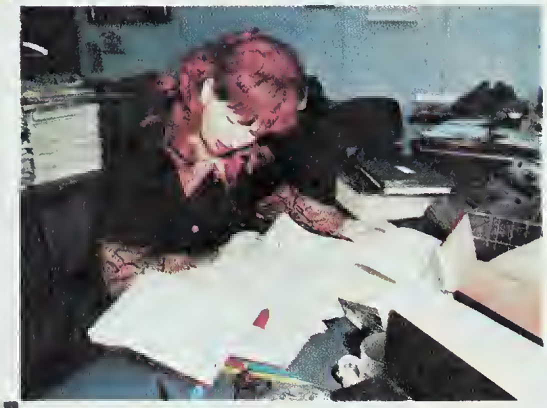


Photo Research International

Brent Jones

House painter

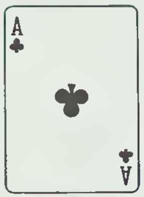
Primer paint can be made by mixing 2 parts linseed oil with 9 parts paint. How much linseed oil should be used with 6 L of paint? $1\frac{1}{3}$ L

Cabdriver

Some city taxi companies charge "rate and a half" for fares to the suburbs. What should be charged for a suburban fare when the taxi meter shows $5.40? $8.10

Milt and Joan Mann

PROPORTION RUMMY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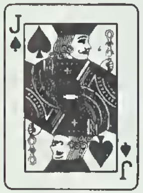
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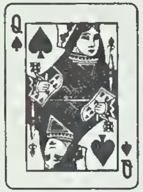
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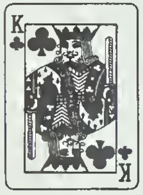
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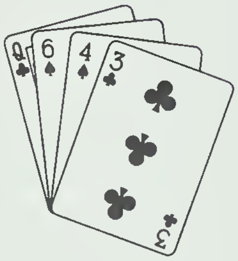
Equipment	An ordinary deck of 52 cards.
Setup	Deal four cards to each player. The rest becomes a "draw" pile.
A hand	You win a hand when you can lay down all four cards. You can do this when you can make a true proportion using the four values of your four cards.
Play	When your turn comes, if you can make a true proportion, you win the hand. If you cannot make a true proportion, you must discard one card *first*. Then draw a card either from the draw pile or from the discard pile (last discard only). If you can now make a true proportion, you win. If you cannot, your turn ends and it becomes the turn of the player on your left.
Scoring	The player who wins a hand scores *no* points. The others score the total of the values of the cards in their hand. Play until one person reaches 100 or 500. Low score wins the game.

Typical strategy: With this hand, there are several things you might do:



1. Discard the Queen, hoping to get an 8. $\frac{3}{4} = \frac{6}{8}$ or $\frac{3}{6} = \frac{4}{8}$
2. Discard the Queen, hoping to get a 2. $\frac{2}{3} = \frac{4}{6}$ or $\frac{2}{4} = \frac{3}{6}$
3. Discard the 6, hoping to get a 9. $\frac{3}{4} = \frac{9}{12}$ or $\frac{3}{9} = \frac{4}{12}$
4. Discard the 6, hoping to get an ace. $\frac{1}{4} = \frac{3}{12}$ or $\frac{1}{3} = \frac{4}{12}$
5. Discard the 4, hoping to get a 6. $\frac{3}{6} = \frac{6}{12}$
6. If you discarded the 3, with which two cards could you win? 2 or 8

SHOPPING IN THE NEWSPAPER

On the opposite page are some newspaper ads. Use any method from the last two sections to find which size from each pair of ads is the better buy. Ignore your own preferences or any special features.

1. Vitamins 365 tab
2. Shampoo 375 mL
3. Pizza 400 g
4. Fruit drinks 1300 mL
5. Filler paper 300
6. Antacid 300 mL
7. Mouthwash 650 mL
8. Dish detergent 1400 mL

¢

300 Page 5 Hole Filler Paper

69¢ Special Purchase!

Fits all standard binders.

Filler Paper

pkg. of 200 **59¢** reg. 99c

FRUIT DRINKS
ORCHARD PARK, GRAPE, PUNCH OR
Orange Drink
1800 mL BTL. **69¢**

Large 400 mL size!

BRECK
Shampoo
400 mL
For beautiful hair

Osco Reg. $1.87

99c

Pizza

REG. LOW PRICE 99¢

250 g BOX .. **79¢**

375 mL WHITE RAIN SHAMPOO

SAVE 36%

79¢

NORMA

Bullseye BONUS!

FROZEN 3 PACK CHEESE OR SAUSAGE
JOHN'S PIZZA
400 g CTN. **$1.09**
REG. LOW PRICE $1.29

Bullseye BONUS!

ORCHARD PARK 7 VARIETIES
FRUIT DRINKS
1300 mL CAN **39¢**
REG. LOW PRICE 48¢

Bullseye BONUS!

DISH DETERGENT 30¢ OFF LABEL
IVORY LIQUID
1400 mL BTL **$1.29**
REG. LOW PRICE 1.33

650 mL Scope Mouthwash

99¢

LIQUID DISH DETERGENT

Palmolive

$1.07

900 mL BTL.

Pepto-Bismol Antacid

200 mL
Relieves upset stomach

88c

750 mL
Listerine Mouthwash

$1.19

Multivitamins WITH IRON

BOTTLE OF 100 TABLETS

Compare at 3.39

99¢

OSCO
Multiple Vitamins
With Iron

Bottle of 365
Osco Reg. $3.19
$2

FAST ACTING ANTACID

MYLANTA ANTACID 300 mL **LIQUID**

1.29

89 CENTS OFF

2.18 VALUE

Dial
Very Dry
Anti-Perspirant
125 g
Osco Reg. 97c

66c

4.8 Percent as a Ratio

Performance Objective: The student can write ratios as percents and percents as ratios.

In the last section, you found the best buy by comparing to 100. When you compare a number to 100, you are using a percent.

Percent means "out of 100" or "for each 100."

William Means

Player	shots made	out of	total attempts
Alvin	6		10
Bernice	16		25
Cas	10		15

To find who has the best record, you can find a percent for each. Alvin made 6 out of 10. That is the same as what out of 100?

part out of whole $\frac{6}{10} = \frac{a}{100}$ what out of 100

$$600 = 10a$$

$$60 = a$$

If he continued to make baskets at the same rate, he would make 60 for every 100 attempts. His rate is 60%.

Example 1: Find the percents for Bernice and Cas.

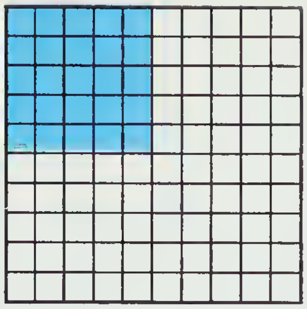
$\frac{16}{25} = \frac{b}{100}$		$\frac{10}{15} = \frac{c}{100}$
$1600 = 25b$	◀Take cross-products.▶	$1000 = 15c$
$64 = b$	◀ Divide. ▶	$66\frac{2}{3} = c$

Bernice made 64% of her attempts. She would make 64 out of each 100 attempts.

Cas's rate is $66\frac{2}{3}$%. This would mean 66 or 67 baskets made in each 100 tries.

When the records are given as percents, it is easy to see that Cas's rate of $66\frac{2}{3}$% is the best record.

Example 2: Show 25% on a diagram.



25 out of 100 parts are shaded.
($\frac{25}{100}$'s of the whole is shaded.)
Therefore, 25% is shaded.

Example 3: Write 56% as a ratio in lowest terms.

$$56\% = \frac{56}{100} = \frac{56 \div 4}{100 \div 4} = \frac{14}{25}$$

Example 4: Write the ratio $\frac{5}{8}$ as a percent.

$\frac{5}{8} = \frac{x}{100}$ ◀ Think: 5 out of 8 is what out of 100?

$500 = 8x$ Take the cross-products.

$62\frac{4}{8} = x$ Divide by 8.

So $\frac{5}{8} = 62\frac{1}{2}\%$ or 62.5%.

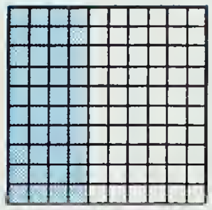
Exercises

1. If $60\% = \frac{n}{100}$, then $n =$ __60__.

2. 30% means __30__ out of 100 or 3 out of __10__.

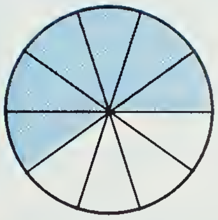
3–8. Copy and shade each figure to show the given percent.
Ex. 3–8: Typical answers are shown.

3.



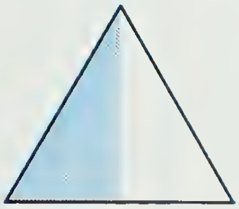
40%

4.



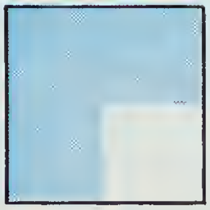
60%

5.



50%

6.

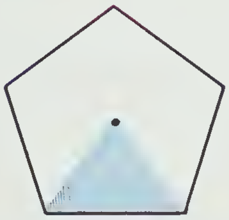


75%

7.

25%

8.



20%

9–16. Write each percent as a ratio in lowest terms.

9. 50% $\frac{1}{2}$ **10.** 75% $\frac{3}{4}$ **11.** 20% $\frac{1}{5}$ **12.** 15% $\frac{3}{20}$

13. 95% $\frac{19}{20}$ **14.** 12% $\frac{3}{25}$ **15.** 37% $\frac{37}{100}$ **16.** 49% $\frac{49}{100}$

17–28. Write each ratio as a percent.

17. $\frac{3}{10}$ 30% **18.** $\frac{1}{20}$ 5% **19.** 3:5 60% **20.** 1 out of 5 20%

21. $\frac{1}{3}$ $33\frac{1}{3}$% **22.** $\frac{4}{6}$ $66\frac{2}{3}$% **23.** 4 out of 25 16% **24.** 17:20 85%

25. $\frac{4}{9}$ $44\frac{4}{9}$% **26.** $\frac{7}{16}$ $43\frac{3}{4}$% **27.** 65:125 52% **28.** 66 out of 120 55%

29–32. Copy and complete each chart.

29. Compare the teams. Find a percent for each.

Team	Wins	Total games	%
Central H.S.	14	20	70%
East H.S.	18	27	$66\frac{2}{3}$%
West H.S.	19	25	76%

30. Find the percent commission each salesperson is getting.

	Commission	Total sales	%
Ann	$16	$80	20%
Mitch	$18	$96	$18\frac{3}{4}$%
Carla	$15	$60	25%

31. Which machine has the lowest proportion of defective parts? Find a percent for each.

Machine	Defective	Total parts	%
A	21	300	7%
B	18	225	8%
C	27	450	6%

32. Find the percent interest each bank pays.

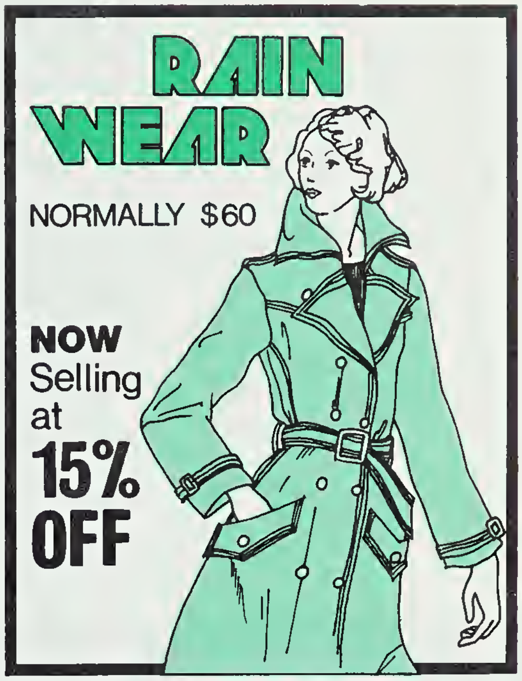
Bank	Interest for 1 year	Principal	%
Trust Co.	$18	$150	12%
Bank	$9.20	$80	$11\frac{1}{2}$%
Credit U.	$47	$400	$11\frac{3}{4}$%

Performance Objective: The student can find a percentage of a number.

Using Percent

4.9

By what amount has the price been reduced? What is 15% of 60?



Example 1: Find 15% of 60.

15% means 15 out of 100. We want to know what part out of 60.

What out of 60 = 15 out of 100?

actual amount off out of actual price → $\frac{a}{60} = \frac{15}{100}$ ← part off out of each 100

Cross-products $100a = 900$

Divide. $a = 9$

The price is reduced by $9.00.

Example 2: A $36 coat is reduced by 20%. What is the sale price?

First find the amount off.

$\frac{x}{36} = \frac{20}{100}$ Think: What out of 36 is the same as 20 out of 100?

$100x = 720$ Take the cross-products.

$x = 7.20$ Divide by 100.

Then subtract.

$36.00 - 7.20 = 28.80$ The sale price is $28.80.

Exercises

1. 25% of 24 is 6. That means 6 out of ____24____ is the same as ____25____ out of 100.

2. 50% of 14 is 7. That means ____7____ out of 14 is the same as ____50____ out of ____100____.

3–14. Find each number.

3. 25% of 12 3

4. 40% of 15 6

5. 75% of 36 27

6. 40% of 75 30

7. 76% of 90 68.4

8. 42% of 60 25.2

9. 65% of 24 15.6

10. 35% of 46 16.1

11. 62% of 150 93

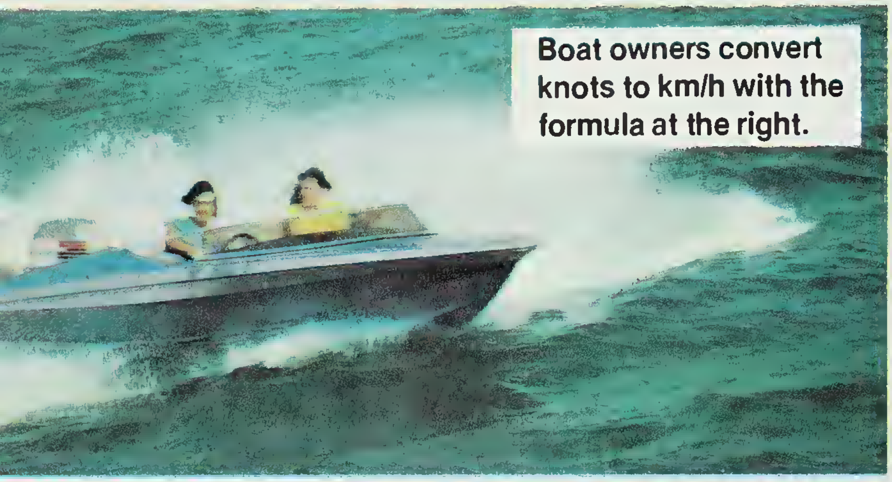
12. 26% of 250 65

13. 24% of 87 20.88

14. 64% of 34 21.76

15–26. Copy and complete each chart.

	Item	Regular price	Percent off	Amount off	Sale price
15.	Coat	$50	15%	$7.50	$42.50
16.	Shoes	$22	12%	$2.64	$19.36
17.	Ring	$75	10%	$7.50	$67.50
18.	Ties	$6.50	20%	$1.30	$5.20



Boat owners convert knots to km/h with the formula at the right.

A. Devaney, Inc.

	knots + 85% of knots = km/h		
	Knots	85% of knots	km/h
19.	20	17	37
20.	25	21.25	46.25
21.	30	25.5	55.5
22.	40	34	74

Find each salesperson's pay at the given commission rates.

	Salesperson	Sales	Rate	Pay
23.	Alvarez	$1275	20%	$255
24.	Berek	$980	25%	$245
25.	Wilson	$1450	22%	$319
26.	Young	$1033	18%	$185.94

27. A health insurance company will pay 80% of a claim of $65. How much will they pay? $52

28. The usual restaurant tip is 15%. How much for a $20 meal? $3

Solving Problems with Percents

4.10

Every percent problem has the same idea:

part out of whole equals percent amount out of 100

Type 1

What percent?

Performance Objective: The student can solve the three types of percent problems: percent, percentage, and base.

In section 4.8, you found the percent in problems like this: Find the percent: $\frac{14}{20}$ or 14 out of 20.

Think

14 out of 20 = what out of 100?

Solution

$$\frac{14}{20} = \frac{x}{100}$$
$$1400 = 20x$$
$$70 = x$$

Answer: 14 out of 20 is **70%**.

Or, in English, we also say: **70%** of 20 is 14.

Type 2

What part?

In section 4.9, you found the part in problems like this: Find 20% of 45.

Think

What part out of 45 = 20 out of 100?

Solution

$$\frac{x}{45} = \frac{20}{100}$$
$$100x = 900$$
$$x = 9$$

Answer: **9** out of 45 is 20%.

Or: 20% of 45 is **9**.

Type 3

What whole?

You can also solve for the whole in problems like this: 15% of what number is 12?

Think

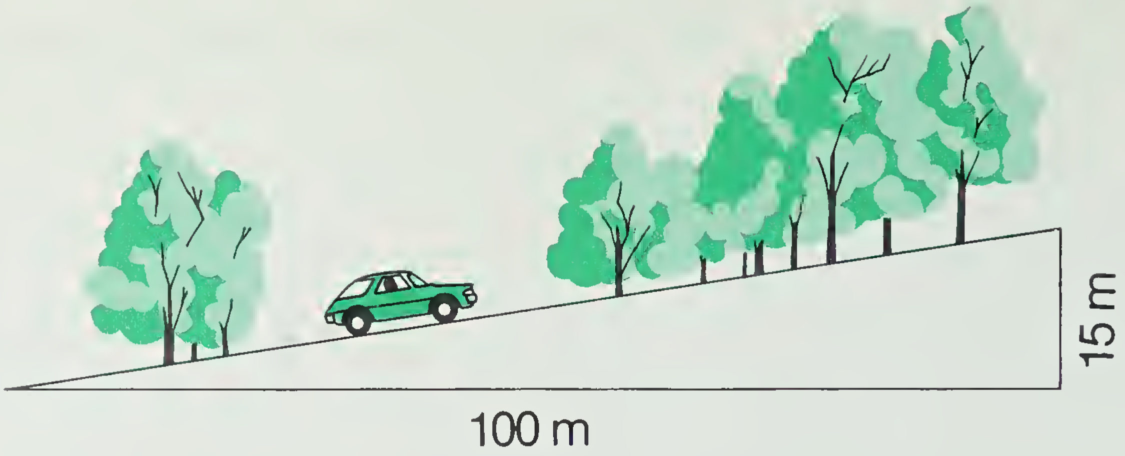
12 out of what whole = 15 out of 100?

Solution

$$\frac{12}{x} = \frac{15}{100}$$
$$1200 = 15x$$
$$80 = x$$

Answer: 12 out of **80** is 15%.

Or: 15% of **80** is 12.



A road that rises 15 m in a level distance of 100 m is said to have a *grade* or slope of 15%

Example 1: What is the percent grade of a road that rises 6 m in every 25 m?

Since you want a percent, this is a type-1 problem.

Think: 6 out of 25 = what out of 100?

$$\frac{6}{25} = \frac{x}{100}$$

Take cross-products. $600 = 25x$

Divide. $24 = x$

The road has a 24% grade.

Example 2: Beverly opened a 5% savings account. How much should she deposit to earn $12 interest in one year?

You can rephrase the problem: 5% of what is $12?
This is a type-3 problem.

12 out of what = 5 out of 100?

$$\frac{12}{x} = \frac{5}{100}$$

$$1200 = 5x$$

$$240 = x$$

She would have to deposit $240 to earn $12 interest.

Exercises

1–10. Copy and complete the charts by solving proportions.

	Part	Whole	%
1.	1	4	25%
3.	52	80	65%
5.	18	30	60%
7.	4	10	40%
9.	27	75	36%

	Part	Whole	%
2.	20	125	16%
4.	21	60	35%
6.	30	120	25%
8.	19	25	76%
10.	36	80	45%

11–16. Solve each type-1 problem.

11. You make 12 baskets out of 15 shots. Find your percent? 80%

12. On his English test, Jim got 36 of the 40 questions correct. What was his percent? 90%

13. A car just manages to climb 14 m in a distance of 40 m. What is the maximum percent grade it can climb? 35%

14. Gas was selling for 25¢/L. It went up 1¢. What percent increase is that? 4%

15. Suppose you bought a $250 item on the installment plan. If the total of your monthly payments comes to $45 more than that, what percent finance charges are you paying? 18%

16. Ann made $21 in commission on a $175 sale. What is her commission percent? 12%

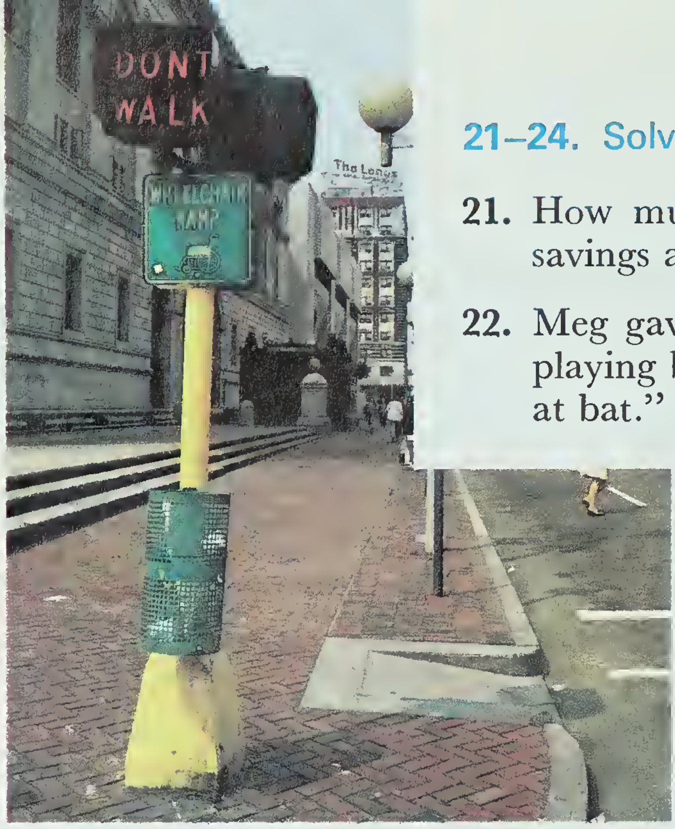
17–20. Solve each type-2 problem.

17. If 70% is passing, how many questions do you have to get right on a 30-question test? 21

18. A $65 bicycle is on sale at 20% off. Find the savings. $13

19. For winter protection a car's radiator should contain 40% antifreeze. If the radiator holds 15 L, how many litres of antifreeze should be used? 6 L

20. Suppose inflation is running at 8% a year. How much more will you have to pay for a $75 item if you wait a year to buy it? $6



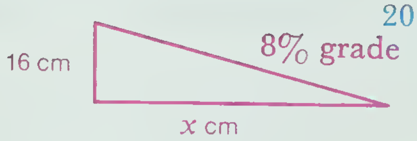
Artstreet

21–24. Solve each type-3 problem.

21. How much would you have to deposit in a 6% savings account to earn \$27 interest in a year? \$450

22. Meg gave her brother this puzzle: "I got 12 hits playing ball. That was 40% of my number of times at bat." How many times was she at bat? 30

23. A curb ramp is to be built for use by handicapped persons who must use wheelchairs. How much level distance is needed for a ramp up to a 16 cm curb if the grade is to be 8%? 200 cm



24. If the 84 people who preferred Brand X in a poll represented 60% of the total, how many people were polled? 140

25–28. Determine the type of each problem and solve.

25. If a province has an 8% sales tax, how much tax is charged on a \$250 purchase? Type 2, \$20

26. If the down payment on a \$700 motorcycle is 25%, how much money must you pay down? Type 2, \$175

27. Mr. Jones sold his car for \$2700. That was 45% of the original cost. What was the original cost? Type 3, \$6000

28. In the election for class president, Katie got 98 of the 175 votes cast. What percent of the votes did she get? Type 1, 56%

Quick Quiz

Check these skills:

- *multiplying by 100*
- *dividing by 100*

1–4. Multiply.

1. 24×100 2400
2. 3.6×100 360
3. 0.89×100 89
4. 0.07×100 7

5–8. Divide.

5. $\frac{75}{100}$ 0.75
6. $\frac{36}{100}$ 0.36
7. $\frac{12.6}{100}$ 0.126
8. $\frac{9}{100}$ 0.09

Percents in Decimal Form

4.11

A percent is a ratio, so it can be written as a fraction. Therefore it can also be written as a decimal.

Performance Objective: The student can write percents as decimals and use the decimal form to find percentages by multiplication.

Example 1: Change each percent to a decimal.

a. 75% $\qquad 75\% = \frac{75}{100} = 0.75$

b. $12\frac{1}{2}$ % $\qquad$ Since $12\frac{1}{2} = 12.5$,

$$12\tfrac{1}{2}\,\% = 12.5\% = \frac{12.5}{100} = 0.125$$

Example 2: Change each decimal to a percent.

a. 0.24 $\qquad 0.24 = \frac{24}{100} = 24\%$

b. 0.378 $\qquad 0.378 = \frac{0.378}{1} \times \frac{100}{100} = \frac{37.8}{100} = 37.8\%$

Writing percents as decimals makes it possible to solve percent problems by multiplication.

Example 3: Find 36% of 75.

Solution by proportion

$$\frac{x}{75} = \frac{36}{100}$$

$$100x = 2700$$

$$x = 27$$

Solution by multiplication

$$36\% = \frac{36}{100} = 0.36$$

Then $0.36 \times 75 = 27.00 = 27$

Example 4: Find 68% of 35 by multiplication.

$$0.68 \times 35 = 23.80 = 23.8$$

Exercises

1–12. Change each percent to a decimal.

1. 16% 0.16

2. 79% 0.79

3. 56% 0.56

4. 14% 0.14

5. 28% 0.28

6. 84% 0.84

7. 15.5% 0.155

8. 3% 0.03

9. 5% 0.05

10. 26.4% 0.264

11. $16\frac{1}{2}$% 0.165

12. $29\frac{1}{2}$% 0.295

13–24. Change each decimal to a percent.

13. 0.42 42%

14. 0.56 56%

15. 0.91 91%

16. 0.37 37%

17. 0.29 29%

18. 0.32 32%

19. 0.05 5%

20. 0.375 37.5%

21. 0.415 41.5%

22. 0.08 8%

23. 0.293 29.3%

24. 0.138 13.8%

25–40. Find each answer by multiplication.

25. 25% of 36 9

26. 60% of 35 21

27. 14% of 50 7

28. 15% of 40 6

29. 40% of 18 7.2

30. 30% of 24 7.2

31. 85% of 64 54.4

32. 18% of 40 7.2

33. 32% of 275 88

34. 48% of 325 156

35. 22% of 382 84.04

36. 28% of 270 75.6

37. 40% of 12.50 5

38. 60% of 14.80 8.88

39. 5% of 14.80 0.74

40. 8% of 16.50 1.32

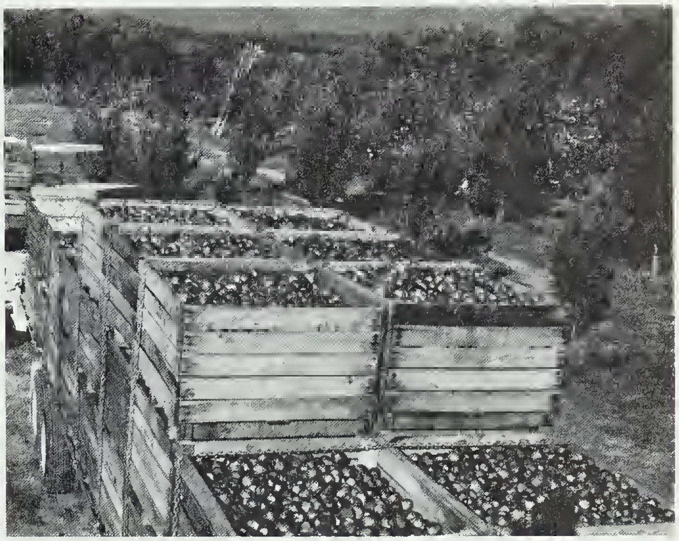
41–44. Solve each problem by multiplication.

41. How much would you save buying a $250 TV at 15% off? $37.50

42. You pay an 18% carrying charge if you buy a $70 coat on credit. How much is the carrying charge? $12.60

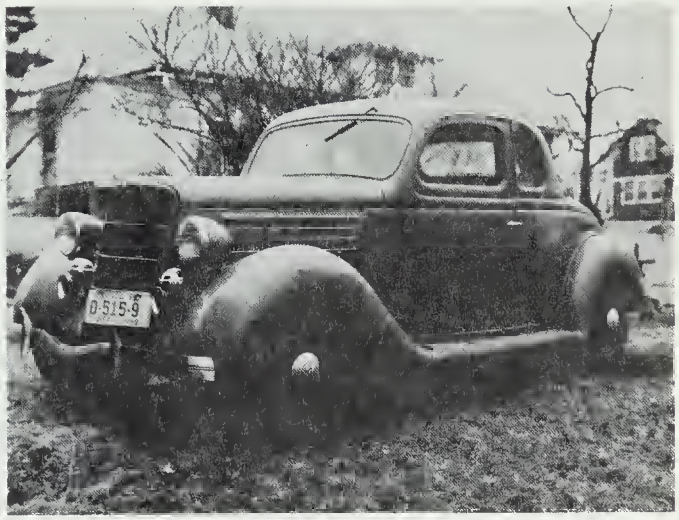
43. A town has 5300 registered voters. If the turnout for an election was 42%, how many people voted? 2226

44. A shipper expects 12% of a fresh fruit shipment to spoil before it is sold. How many cases can he expect to lose out of a 2100-case shipment? 252



Grant Heilman

45–48. Use multiplication for the first step of each problem.



Kuechmann/Bedford Photographic

45. Ted bought an old car for $160. He repaired it, intending to sell it for 80% more than he paid for it. What is his selling price? $288

46. Some new cars depreciate 25% the first year of ownership. If you buy a new car for $4400, what will it be worth one year later? $3300

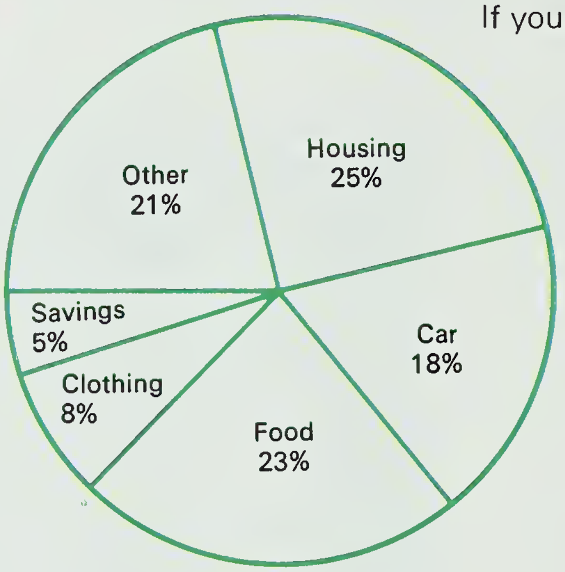
47. According to the tax rate schedule, Monica's income tax is $310 plus 19% of $750. What is her total tax? $452.50

48. Suppose you earn a salary of $350 a month plus 18% commission. Your sales for last month were $2575. What will your total earnings for the month be? $813.50

CONSUMER CORNER

BUDGETS

An average person's budget usually shows what part of his or her income will be spent on different things. Suppose you divided your income according to the graph below.



If your take-home pay was $800 a month,

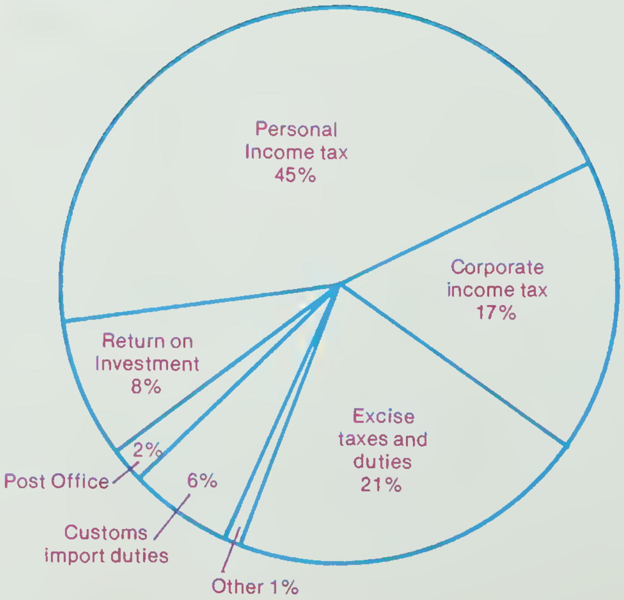
1. What would housing cost you? $200
2. How much would you spend on food? $184
3. How much for car payments, gas, parking, etc.? $144
4. What would you spend on clothing? $64
5. How much would you save? $40
6. What would you have left for medical and dental expenses, education, insurance, entertainment, etc.? $168

$ $ $ $ $ $ $ $ $ $ $ $ $

Governments and companies use budgets, too. Sometimes they show how their income will be spent. Other budgets show where their income is coming from. In 1976, Canada had a budget of nearly 34 billion dollars.

About how many billions were collected from each of the following?

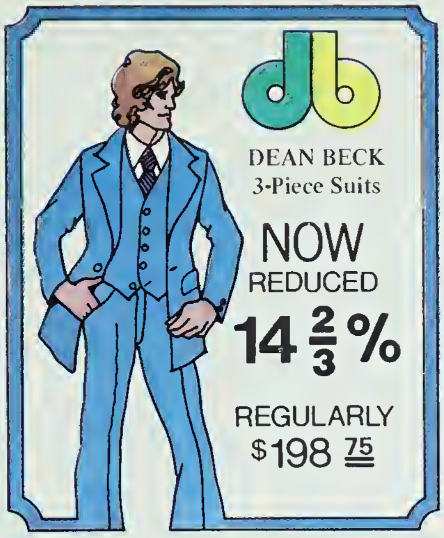
1. personal income taxes 15.3
2. corporate income taxes 5.8
3. customs 2
4. excise taxes 7
5. return on investment 3
6. post office 0.7



Source: Public Accounts of Canada, 1977

Estimating with Percents

4.12



How much is the price reduced?

Performance Objective: The student can estimate percentages using tenths, one-half of tenths, and multiples of tenths.

You could try to solve:

$$\frac{x}{198.75} = \frac{14\frac{2}{3}}{100}$$

You could try to multiply:

$$0.14\tfrac{2}{3} \times 198.75$$

But it is often good enough to estimate the reduction. First notice that 10% means $\frac{10}{100}$ or $\frac{1}{10}$. It is easy to find $\frac{1}{10}$ of a number—just move the decimal point one place to the left.

Example 1: Find each result exactly by using $\frac{1}{10}$.

a. Find 10% of 43.

$$10\% \text{ of } 43 = \tfrac{1}{10} \times 43 = 4.3$$

b. Find 30% of 70.

$$\begin{aligned} 30\% \text{ of } 70 &= 3 \times (10\% \text{ of } 70) \\ &= 3 \times (\tfrac{1}{10} \times 70) \\ &= 3 \times 7 \\ &= 21 \end{aligned}$$

This is true, since 30% = 3 × 10%.

c. Find 15% of 48.

$$\begin{aligned} 15\% \text{ of } 48 &= 10\% \text{ of } 48 + 5\% \text{ of } 48 && \text{Since } 15\% = 10\% + 5\% \\ &= \tfrac{1}{10} \times 48 + \tfrac{1}{2} \text{ of } (\tfrac{1}{10} \times 48) && \text{Since } 5\% = \tfrac{1}{2} \text{ of } 10\% \\ &= 4.8 + \tfrac{1}{2} \text{ of } 4.8 \\ &= 4.8 + 2.4 \\ &= 7.2 \end{aligned}$$

To estimate, you can choose easy numbers *close* to the given numbers and use the same method.

Example 2:

a. Estimate the price reduction on page 183.

$14\frac{2}{3}\%$ is close to 15%.

198.75 is about 200.

$$\begin{aligned} 15\% \text{ of } 200 &= 10\% \text{ of } 200 + 5\% \text{ of } 200 \\ &= \tfrac{1}{10} \times 200 + \tfrac{1}{2} \text{ of } (\tfrac{1}{10} \times 200) \\ &= 20 + \tfrac{1}{2} \text{ of } 20 \\ &= 20 + 10 \\ &= 30 \end{aligned}$$

The reduction is about \$30. The sale price is about \$170.

b. Estimate 26% of 79.

26% is about 25%.
79 is close to 80.

$$\begin{aligned} 25\% \text{ of } 80 &= 10\% \text{ of } 80 + 10\% \text{ of } 80 + 5\% \text{ of } 80 \\ &= \tfrac{1}{10} \times 80 + \tfrac{1}{10} \times 80 + \tfrac{1}{2} \text{ of } (\tfrac{1}{10} \times 80) \\ &= 8 + 8 + 4 \\ &= 20 \end{aligned}$$

Exercises

1–18. Find each result exactly by using $\frac{1}{10}$.

1. 10% of 90 9

2. 10% of 40 4

3. 10% of 48 4.8

4. 10% of 63 6.3

5. 20% of 60 12

6. 20% of 80 16

7. 20% of 46 9.2

8. 20% of 74 14.8

9. 15% of 80 12

10. 15% of 60 9

11. 30% of 160 48
12. 25% of 180 45
13. 25% of 280 70
14. 30% of 320 96
15. 5% of 120 6
16. 35% of 80 28
17. 35% of 60 21
18. 5% of 160 8

19–30. Estimate.

Ex. 19–38: Answers are estimates.

19. 11% of 59 6
20. 9% of 81 8
21. 19% of 52 10
22. 16% of 40 6
23. 21% of 148 30
24. 4% of 21 1
25. 24% of 282 70
26. 11% of 128 13
27. 6% of 78 4
28. 9½% of 240 24
29. 31% of 49.95 15
30. 21% of 99.50 20

31–34. Copy and complete the chart.

	Regular price	Discount %	Estimated reduction	Estimated sale price
31.	$19.99	11%	$2	$18
32.	$129.95	21%	$26	$104
33.	$299.50	9½%	$30	$270
34.	$24.95	19%	$5	$20

35–38. Solve each problem by estimating.

35. About how much will a $14.95 tennis racket cost at 15% off? About $13

36. Estimate the cost of a $49.50 item with a 5% sales tax. About $52

37. A 5½% savings account is opened with $759. About how much will be in the account 1 year from now? About $800

38. A farmer harvested 31 m^3 of wheat last year. This year's yield is expected to be 15% greater. About how many cubic metres can be expected this year? About 34 m^3

Test for Chapter 4

Give the ratio of circles to squares. (4.1)

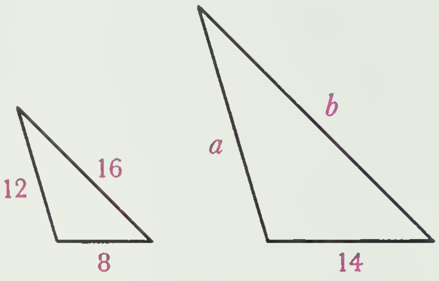
1. ○ ○ ○ □ □ □ □ $\frac{3}{4}$ 2. ○ ○ □ □ ○ ○ ○ $\frac{5}{2}$

Give each ratio in lowest terms. (4.2)

3. $\frac{4}{8}$ $\frac{1}{2}$ 4. $\frac{10}{15}$ $\frac{2}{3}$ 5. $\frac{15}{20}$ $\frac{3}{4}$

Solve each proportion. (4.3)

6. $\frac{3}{4} = \frac{x}{12}$ 9 7. $\frac{4}{14} = \frac{6}{a}$ 21 8. $\frac{n}{6} = \frac{24}{9}$ 16



Solve with a proportion. (4.4)

9. A 6 kg mass stretches a spring 10 cm. What mass will stretch it 25 cm? 15 kg

Use the similar figures at the left. (4.5)

10. Find the values of a and b. $a = 21, b = 28$

Use cross-products to find the better buy. (4.6)
Ex. 11–14: Answers are given in order; arrow indicates better buy.

11. 15 g for 59¢ 6075 ←
100 g for 81¢ 5900

12. 175 g for 39¢ 11 550
300 g for 66¢ 11 700 ←

Find the best buy—find how much of each for $1.00. (4.7)

13. 75 mL for 25¢, 175 mL for 50¢, 325 mL for 80¢ 300, 350, 406 ←

Find the best buy by finding the cost of one gram of each.

14. 75 g for 42¢, 100 g for 58¢, 175 g for 91¢ 0.56, 0.58, 0.52 ←

Change to a percent. 15. $\frac{2}{5}$ 40% 16. 8 out of 20 40% (4.8)

Find each number. 17. 20% of 85 17 18. 65% of 40 26 (4.9)

Solve each problem by using a proportion. (4.10)

19. A $40 coat is reduced 20%. How much money is that? $8

20. The finance charge on a $180 item bought on the installment plan is $27. What percent is that? 15%

Find by multiplying. 21. 32% of 95 30.4 22. 18% of 64 11.52 (4.11)

Estimate using $\frac{1}{10}$. 23. 31% of 49 15 24. 9% of 6.99 0.70 (4.12)
Ex. 23–24: Estimates

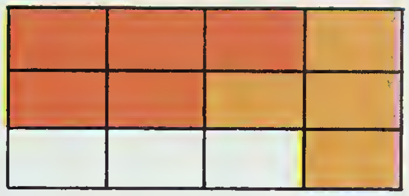
RATE YOURSELF

Number Correct	
23–24	Superior
21–22	Excellent
18–20	Good
13–17	Fair
0–12	Poor

Chapter 4 Review

Give each ratio as a fraction. (4.1)

1. circles to stars ○○☆☆☆☆☆ $\frac{2}{5}$



2. red parts to all parts $\frac{5}{12}$

3. orange parts to red parts $\frac{4}{5}$

Give each ratio in lowest terms. (4.2)

4. $\frac{4}{6}$ $\frac{2}{3}$

5. $\frac{9}{15}$ $\frac{3}{5}$

6. $\frac{12}{36}$ $\frac{1}{3}$

7. $\frac{15}{24}$ $\frac{5}{8}$

8. $\frac{21}{28}$ $\frac{3}{4}$

9. $\frac{30}{42}$ $\frac{5}{7}$

Solve each proportion. (4.3)

10. $\frac{2}{5} = \frac{4}{x}$ 10

11. $\frac{3}{8} = \frac{a}{24}$ 9

12. $\frac{10}{n} = \frac{6}{21}$ 35

13. $\frac{w}{45} = \frac{8}{18}$ 20

14. $\frac{20}{15} = \frac{m}{18}$ 24

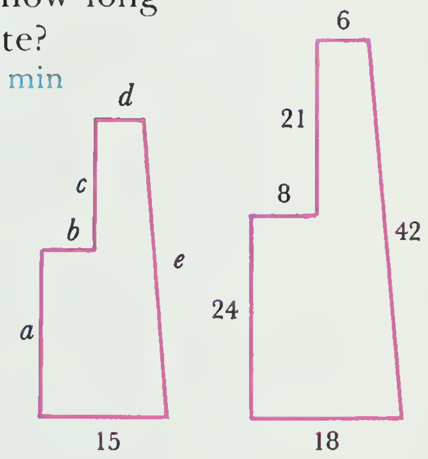
15. $\frac{15}{6} = \frac{85}{c}$ 34

Solve each problem with a proportion. (4.4)

16. A car traveled 150 km on 12 L of gas. How much gas will be used for 600 km? 48 L

17. If you walked a distance of 4 blocks in 7 min, how long would it take you to walk 10 blocks at the same rate? $17\frac{1}{2}$ min

Use the similar figures at the right. (4.5)



18. Find all the missing values.
$a = 20$, $b = 6\frac{2}{3}$, $c = 17\frac{1}{2}$, $d = 5$, $e = 35$

Find the better buy using cross-products. (4.6)

19. 125 g for 39¢ 7875 ←
200 g for 63¢ 7800

20. 100 g for 29¢ 8100
300 g for 81¢ 8700 ←

21. 150 mL for 78¢ 17 850
275 mL for $1.19 21 450 ←

22. 175 mL for 81¢ 18 725 ←
225 mL for $1.07 18 225

Find the best buy by finding which rate gives the most grams for $1.00 or 100 cents

23. 150 g for 40¢ 375
300 g for 72¢ 417
550 g for $1.25 440 ←

24. 125 for 64¢ 195 ←
175 g for 92¢ 190
375 g for $2.00 188

Find the best buy by finding which rate has the lowest cost per gram.

25. 175 g for 56¢ 0.32
300 g for 90¢ 0.3 ←
500 g for $1.56 0.31

26. 125 g for 29¢ 0.23
375 g for 77¢ 0.21
600 g for $1.17 0.195 ←

Write each ratio as a percent. (4.8)

27. $\frac{9}{36}$ 25% **28.** 14:25 56% **29.** 6 out of 40 15% **30.** $\frac{57}{75}$ 76%

Write each percent as a ratio in lowest terms.

31. 25% $\frac{1}{4}$ **32.** 60% $\frac{3}{5}$ **33.** 18% $\frac{9}{50}$ **34.** 84% $\frac{21}{25}$

Find each result by solving a proportion. (4.9)

35. 40% of 65 26 **36.** 75% of 24 18 **37.** 56% of 250 140
38. 24% of 200 48 **39.** 65% of 460 299 **40.** 15% of 700 105

Solve each problem by using a proportion. (4.10)

41. A team won 9 of the 15 games they played. What percent of their games did they win? 60%

42. If 70% is passing, how many correct answers would you have to give on a test that has 40 problems? 28

Find by multiplying. (4.11)

43. 27% of 50 13.5 **44.** 12% of 75 9 **45.** 37% of 62 22.94
46. 16% of 47 7.52 **47.** 23% of 126 28.98 **48.** 42% of 239 100.38

Estimate using $\frac{1}{10}$. *Ex. 49–54: Estimates* (4.12)

49. 39% of 75 30 **50.** 19% of 65 13 **51.** 21% of 320 64
52. 21% of 159 32 **53.** 9.8% of 19.88 2.00 **54.** 11% of 12.95 1.30

Tell whether each sentence is true, false, or open. (3.1)

1. $4 + n = 24$ Open

2. $6 \times 2 = 10 + 2$ True

3. $8 + 7 = 3 \times 4$ False

4. $18 = 3(6)$ True

Find each answer. (3.2)

5. $8 + (12 - 4)$ 16

6. $3(7 - 2)$ 15

7. $(4 \times 6) - 5$ 19

8. $9 + (15 \div 5)$ 12

Find each answer when n is replaced with 5.

9. $2n$ 10

10. $6n$ 30

11. $3(n - 3)$ 6

12. $n(7 + 4)$ 55

Find each answer. (3.3)

13. $12 - 5 + 2$ 9

14. $8(3) + 4$ 28

15. $5(4) - 12$ 8

16. $18 \div 3 + 3(2)$ 12

Find each answer when x is replaced with 8.

17. $2x - 5$ 11

18. $3x + 4$ 28

19. $3(x - 2)$ 18

20. $4(x + 2)$ 40

Use the given formula to solve. (3.4)

21. How many kilometres will a plane go in 4 h at a rate of 560 km/h? ($d = rt$) 2240 km

22. How many metres of fencing will be needed to enclose a lot 12 m long and 7 m wide? ($P = 2\ell + 2w$) 38 m

Solve and check. (3.5)

23. $x + 18 = 32$ 14

24. $y - 12 = 50$ 62

25. $57 = n - 23$ 80

26. $b + 14 = 28$ 14

27. $39 = a + 36$ 3

28. $9 = w - 42$ 51

Solve and check. (3.6)

29. $6x = 78$ 13

30. $84 = 12a$ 7

31. $17 = \frac{w}{9}$ 153

32. $\frac{n}{5} = 18$ 90

Solve and check. (3.7)

33. $17 = 3y + 2$ 5

34. $4m - 3 = 29$ 8

35. $12 = \frac{x}{5} - 8$ 100

36. $\frac{w}{7} + 4 = 10$ 42

37. $4c - 3 = 45$ 12

38. $31 = 3b + 4$ 9

Solve and check. (3.8)

39. $4x = 9$ $\frac{9}{4}$

40. $8y = 4$ $\frac{1}{2}$

41. $10 = 5d + 8$ $\frac{2}{5}$

42. $3m - 5 = 3$ $\frac{8}{3}$

Solve and check. (3.9)

43. $6 = \frac{2}{3}x$ 9

44. $\frac{4}{9}y = 12$ 27

45. $18 = \frac{5}{2}b + 3$ 6

46. $\frac{2}{7}z + 2 = 12$ 35

47. $\frac{9}{5}w - 3 = 15$ 10

48. $21 = \frac{3}{4}a - 3$ 32

Use the given formula to solve. (3.10)

$t = mn$

t: total mass
m: mass per unit
n: number of units

49. The mass of one can of peas is 250 g. What would be the mass of a case containing 24 cans? 6 kg

50. A supermarket employee is pricing tins. He has a 9 kg case containing 225 g tins. How many tins must he price? 40 tins

$P = \frac{a}{2} + 110$

P: blood pressure
a: age

51. Find the approximate normal blood pressure of a person who is 48 years old. 134

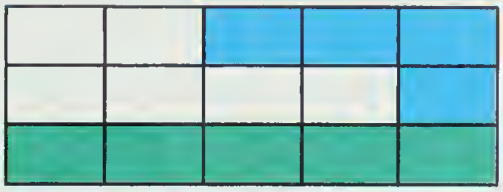
52. About how old is a person whose blood pressure is 145? 70

$F = 1.15C + 1.25$

53. Find the Canadian equivalent for \$633.75. \$550

54. Find the foreign equivalent for \$300. \$346.25

Give each ratio as a fraction. (4.1)



55. blue parts to all parts $\frac{4}{15}$

56. green parts to white parts $\frac{5}{6}$

57. green parts to blue parts $\frac{5}{4}$

58. all parts to white parts $\frac{15}{6}$

For each ratio give an equivalent one in lowest terms. (4.2)

59. $\frac{2}{8}$ $\frac{1}{4}$
60. $\frac{6}{10}$ $\frac{3}{5}$
61. $\frac{15}{18}$ $\frac{5}{6}$
62. $\frac{18}{24}$ $\frac{3}{4}$
63. $\frac{14}{35}$ $\frac{2}{5}$
64. $\frac{24}{40}$ $\frac{3}{5}$

Solve each proportion. (4.3)

65. $\frac{3}{7} = \frac{x}{35}$ 15
66. $\frac{6}{5} = \frac{24}{m}$ 20
67. $\frac{10}{w} = \frac{12}{18}$ 15
68. $\frac{18}{8} = \frac{c}{12}$ 27
69. $\frac{16}{28} = \frac{28}{x}$ 49
70. $\frac{n}{54} = \frac{4}{18}$ 12

Solve each problem by using a proportion. (4.4)

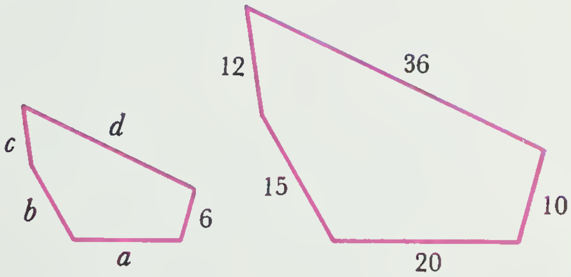
71. An 8 km mass stretches a spring 10 cm. How far will a 20 kg mass stretch the spring? 25 cm

72. If it costs $2 to ship a 15 kg package, what will a 42 kg package cost at the same rate? $5.60

73. If 4 rolls of wallpaper cover a wall 2 m long, how many rolls will be needed for a wall 3 m long? 6 rolls

Use the similar figures at the right. (4.5)

74. Find the missing values.
$a = 12$, $b = 9$, $c = 7.2$, $d = 21.6$



Find the better buy by using cross-products. (4.6)

75. 75 g for 44¢ 5400
125 g for 72¢ 5500 ←

76. 100 g for 53¢ 8900
75 g for 89¢ 9275 ←

77. 225 mL for 89¢ 30 600 ←
300 mL for $1.36 26 700

78. 150 mL for 81¢ 30 000
400 mL for $2.00 32 400 ←

Find the best buy by finding which rate gives the most for $1.00, or 100 cents. (4.7)

79. 50 g for 25¢ 200
150 g for 64¢ 234.4 ←
350 g for $1.52 230.3

80. 22 tablets for 55¢ 40
36 tablets for 75¢ 48
70 tablets for $1.25 56 ←

Find the best buy by finding which rate has the lowest cost per gram.

81. 100 g for 56¢ 0.56 ←
225 g for $1.35 0.6
300 g for $1.92 0.64

82. 150 g for 48¢ 0.32
350 g for 99¢ 0.28 ←
575 g for $1.65 0.29

Write each ratio as a percent. (4.8)

83. $\frac{14}{35}$ 40% **84.** 13:52 25% **85.** 8 to 40 20% **86.** $\frac{52}{80}$ 65%

Write each percent as a ratio in lowest terms.

87. 70% $\frac{7}{10}$ **88.** 45% $\frac{9}{20}$ **89.** 96% $\frac{24}{25}$ **90.** 22% $\frac{11}{50}$

Find each result by solving proportions. (4.9)

91. 25% of 48 12 **92.** 20% of 65 13 **93.** 16% of 75 12

94. 88% of 250 220 **95.** 45% of 400 180 **96.** 15% of 160 24

Solve each problem by using a proportion. (4.10)

97. Evan borrowed $700 from a bank at 14% interest. How much money must he pay in interest? $98

98. Peggy sold 32 of the 40 magazines she had to sell. What percent did she sell? 80%

Find by multiplying. (4.11)

99. 28% of 50 14 **100.** 40% of 65 26 **101.** 30% of 87 26.1

102. 39% of 80 31.2 **103.** 22% of 170 37.4 **104.** 77% of 250 192.5

Estimate using $\frac{1}{10}$. *Ex.* **105–110:** *Estimates* (4.12)

105. 29% of 150 45 **106.** 31% of 70 21 **107.** 9% of 60 6

108. 11% of 240 24 **109.** 14% of 39.95 6 **110.** 19% of 19.95 4

Preview Test for Chapter 5

Multiple choice.

1. The (metre, thumbwidth) is not a standard unit of measure. (5.1)

2. The (yard, cubit) is an ancient unit that is measured from the elbow to the end of the middle finger. (5.2)

Measure each segment to the nearest centimetre. (5.3)

3. ______________________ 4 cm

4. ________________ 3 cm

Complete the following: (5.4)

5. 25 mm is 2.5 cm

6. 5.4 km is 5400 m.

7. 4.2 t is 4200 kg

8. 4000 g is 4 kg. (5.5)

9. 15 kL is 15 000 L.

10. 3500 mL is 3.5 L (5.6)

Add or subtract. (5.7)

11. 8 h 28 min
+ 2 h 42 min
11 h 10 min

12. 8:05 p.m.
−3:23 p.m.
4:42

13. 1:46 p.m.
−9:30 a.m.
4:16

Write the time using the 24-hour clock.

14. 7:05 a.m. 07 05

15. 6:25 p.m. 18 25 (5.8)

Complete the following:

16. 25 min is 5/12 h

17. 4/5 h is 24 min (5.9)

18. What is the freezing point of water on the Kelvin scale? 273 K (5.10)

Use a protractor to draw an angle having each measure. (5.11)

19–20. *Drawings*

19. 53°

20. 107°

Measurement

5

smooth
peanut
butter
homogenized
1kg

Robert Brown

Photo Research International

Robert Brown

SALE
only $1.79 per metre

PICTURE PREVIEW: How did measurements come about? What are the most common measuring units? How are measurements used at home, at work, on the road, at the store, and so on? These are the questions this chapter will answer. And the pictures on these pages give an advance glimpse of uses of measurement.

5.1

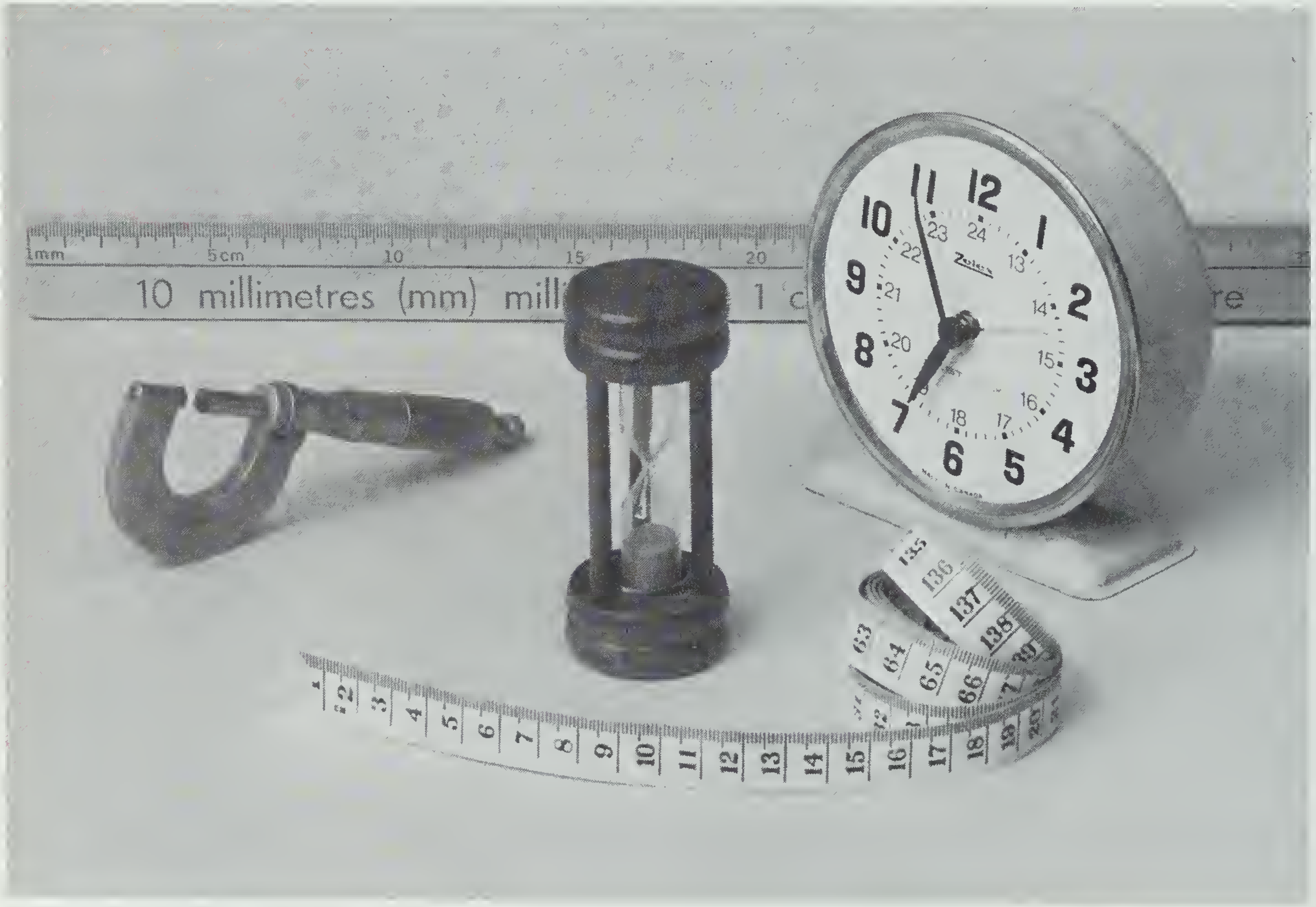
Picking a Unit of Measure

Performance Objective: The student can select appropriate units of measure or measuring instruments for given situations.

Measurements are used to describe such things as length, mass, volume, area, and time. Measurements like the ones below are seen often.

3.5 m
12.2 L
80 kg
78 cm
12 s
95 g

Many different measuring instruments are used to make measurements. Some of them are shown below.



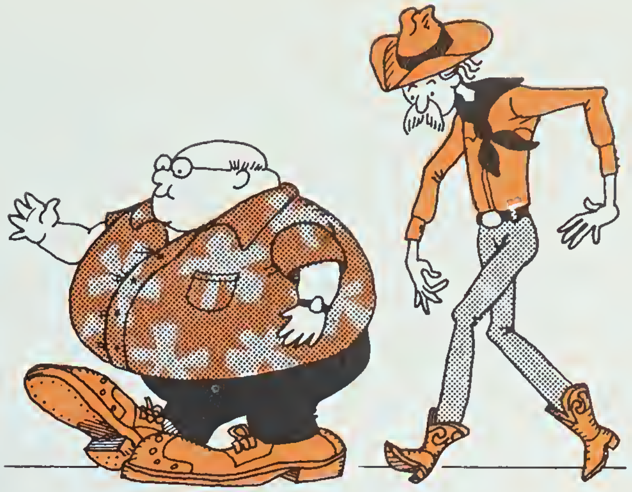
Robert Brown

Often more than one measuring instrument can be used to measure the same thing. A person's height can be measured with both a ruler and a tape measure.

Also, more than one **unit of measure** can be used to measure the same thing. Possible units of measure for your height include the centimetre and the metre.

Suppose the two people in the cartoon use their feet as units of measure. If they both measure the length of a room, their measurements will certainly not agree.

To avoid arguments, everyone needs to use the same units. These are called **standard units of measure**.



Exercises

Ex. 1–46: Typical answers

1–10. Name a unit of measure that could be used to find each of the following. More than one answer is usually possible.

1. your mass kilogram
2. the mass of a locomotive tonne
3. the time it takes to broil a steak minute
4. your age year
5. the mass of a spider gram
6. the height of the CN Tower metre
7. the height of King Kong centimetre
8. room temperature degree Celsius
9. the distance from Toronto to Ottawa kilometre
10. the amount of gasoline in a car's tank litre

Bettmann Archives

11–20. What measuring instrument would you use to find each of the following?

11. the length of a room Metrestick, tape measure
12. waist size Tape measure
13. water temperature Thermometer
14. the time it takes to run 100 m Stopwatch
15. your mass Scale
16. tire pressure Pressure gauge

17. the distance from one city to another Odometer

18. the amount of flour needed to bake a cake Measuring cup

19. the amount of oil in a car's crankcase Dipstick

20. spark-plug gap Gap gauge

21–30. Which do you think are standard units of measure?

21. metre Yes
22. truckload No
23. smidgin No
24. kilometre Yes
25. gram Yes
26. second Yes
26. pinch of salt No
28. scoop of ice cream No
29. litre Yes
30. pat of butter No

31–40. Look up the following standard units in a dictionary. Give a definition for each.

31. rod 5.03 m
32. light-year Distance light travels in 1 year
33. hand 10 cm
34. hectare 1 hm^2
35. furlong 201.2 m
36. nanometre 10^{-9} m
37. knot 1 nautical mile per hour
38. parsec 3.26 light years
39. nautical mile 1852 m
40. Mach number Ratio of speed of object to speed of sound in surrounding atmosphere

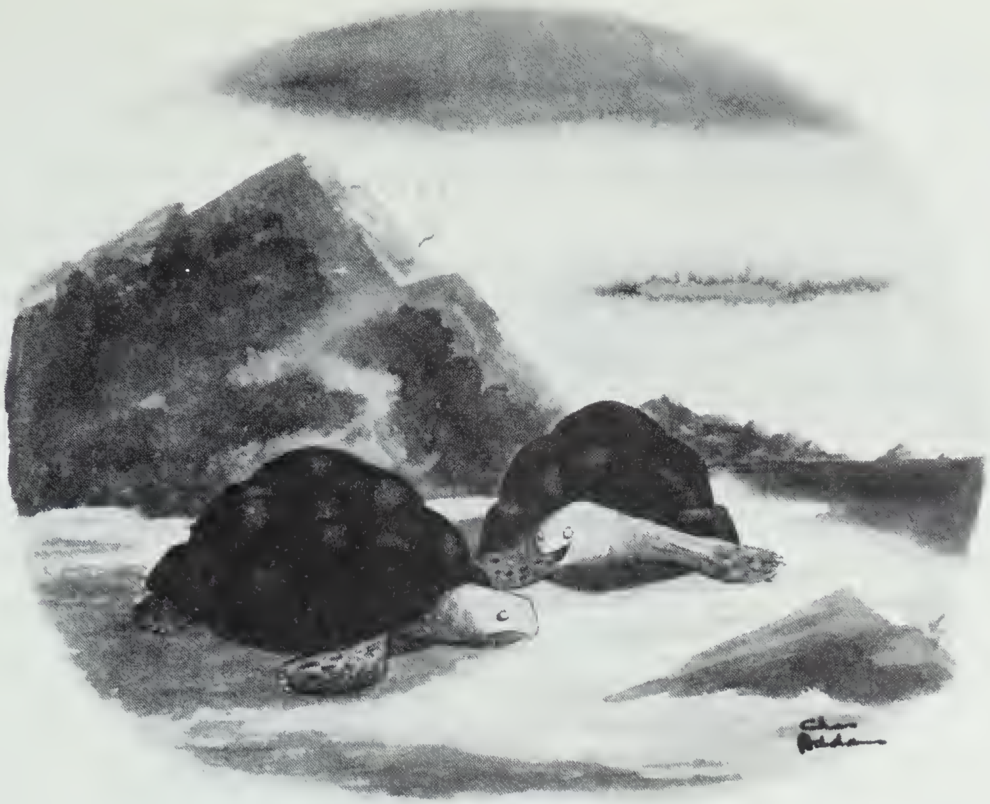
41–46. Look up the following in a dictionary or an encyclopedia and give a definition for each.

41. micrometer caliper Measures small linear distances
42. pluviometer Rain gauge
43. sphygmomanometer Measures blood pressure
44. theodolite Measures angles
45. tachometer Measures speed of rotation
46. ammeter Measures electric current

47–52. Tell which measuring instrument in Exercises **41–46** would probably be most useful for each person below.

47. nurse Sphygmomanometer
48. forest ranger Pluviometer
49. race-car driver Tachometer
50. electrician Ammeter
51. surveyor Theodolite
52. machinist Micrometer caliper

IT'S ABOUT TIME



Drawing by Chas. Addams; © 1957 The New Yorker Magazine, Inc.

"Nonsense, dear. Four hundred and thirty-eight isn't old!"

The cartoon shown above is actually misleading. According to the *Guinness Book of World Records*, the world's oldest tortoise lived to be 152 years old. Of course, there are claims of tortoises living much longer. But these claims often stretch the truth.

For humans, the claims of great age are even harder to believe. Recently a man who claimed to be 168 died in Russia. However, such claims are almost impossible to prove. The oldest person whose age has been proved beyond a doubt lived to be 113 years 124 days old.

The Guinness book lists these other age records:

oldest dog: 27 years

oldest cat: 36 years

oldest tree: 4900 years

Besides age records, there are many other records involving time. Who knows! Maybe you'll break one some day.

longest kiss: 96 hours 32 minutes

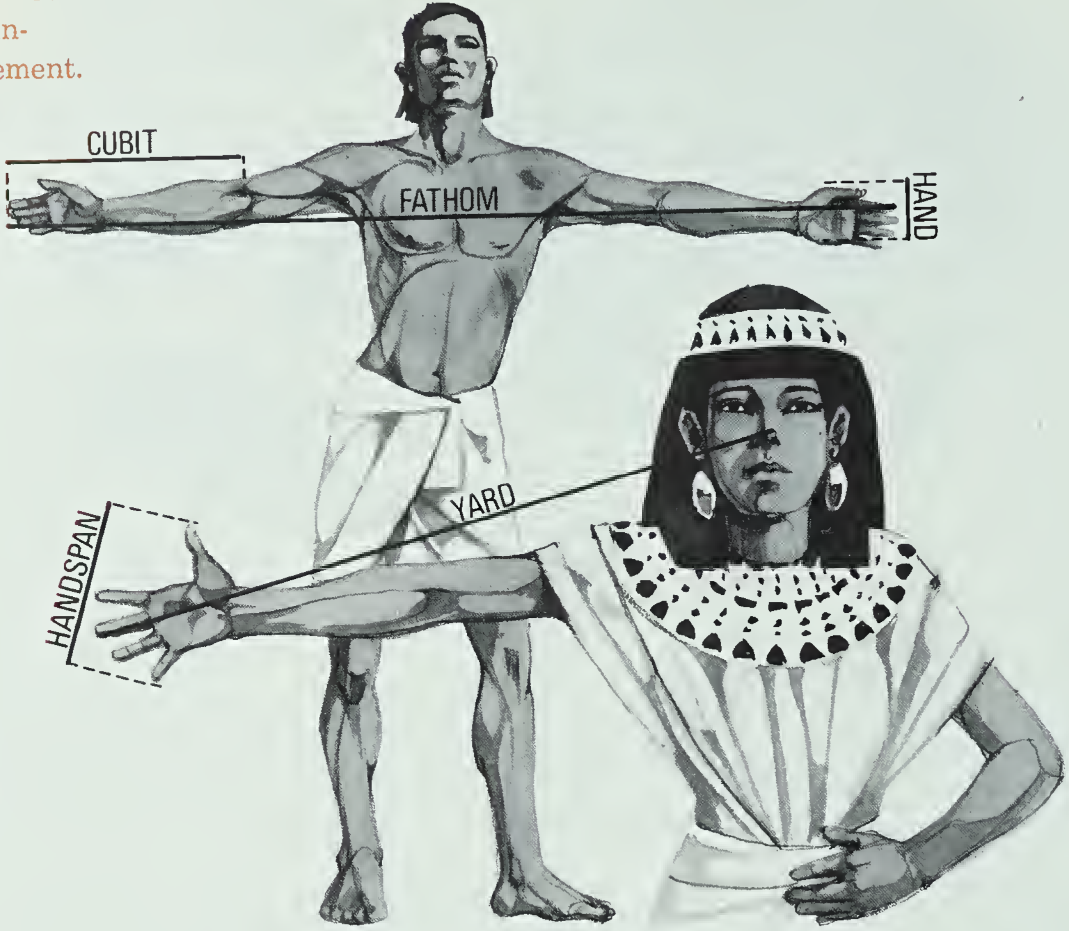
longest shower bath: 175 hours 7 minutes

longest phone call: 724 hours

5.2 Beginnings of Measurement

The pictures below show some ancient units of measure.

Performance Objective: The student can use ancient units of measurement.



yard: about 1 m

fathom: about 2 m

hand: about 10 cm

cubit: about 45 cm

handspan: about 20 cm

thumbwidth: about 2.5 cm

Such rough units as those above are useful for making estimates. But, of course, they cannot be used to make accurate measurements. So *standard units* like the centimetre and the metre were invented. People have agreed to keep these standard units the same from place to place.

Exercises

1–10. Refer to the ancient units on page 200.

1. Which unit is measured from the tip of the nose to the end of the outstretched arm? Yard

2. Which unit is the distance from the elbow to the tip of the middle finger? Cubit

3. Which unit is the distance between outstretched arms? Fathom

4. 1 fathom is about 2 metres.

5. 1 m is about 10 hands.

6. 1 handspan is about 20 centimetres.

7. 1 cubit is about 45 centimetres.

8. Which unit is about 2.5 cm? Thumbwidth

9. A boater says a lake is 3 fathoms deep. How many metres deep is this? 6 m

10. A horse trainer says a horse is 18 hands high. How many centimetres is this? 180 cm

11–14. Which word in the list best completes each sentence?

metres
handspans
cubits
thumbwidths
hands
fanthoms

11. The length of a caterpillar is about 2 Thumbwidths.

12. The depth of a pool for high diving is about 2 Fathoms.

13. The length of a metrestick is about 2 Cubits.

14. The height of a door is about 2 metres.

15–26. Answers will vary.

15–18. Find each measurement, using the units suggested.

15. width of this book in thumbwidths (HINT: Using both thumbs is quicker.)

16. length of your desk in handspans

17. width of your desk in hands

18. length of your foot in hands

19—24. Find the length, in centimetres, of each unit.

19. your cubit

20. your yard

21. your foot

22. your hand

23. your handspan

24. your thumbwidth

25. Make a list of the measurements everybody in your class found for the handspan in Exercise 20. Using this list, find

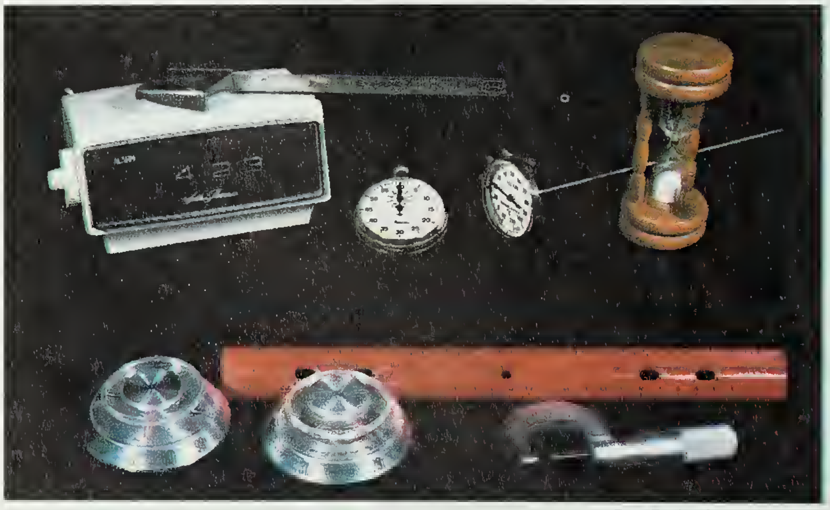
a. the range (difference between the largest and smallest)

b. the mode (most frequent)

c. the median (the midway measurement)

d. the mean (the average)

26. Repeat Exercise 25 for the cubit from Exercise 21.



Dr. E. R. Degginger

Photo Puzzle

How many measuring instruments in the photograph can you name?

Clockwise from upper left: clock, tape measure, stopwatch, cooking thermometer, hourglass, ruler, micrometer caliper, barometer, thermometer

Performance Objective: The student can measure with a ruler, draw segments of a given length and make estimations.

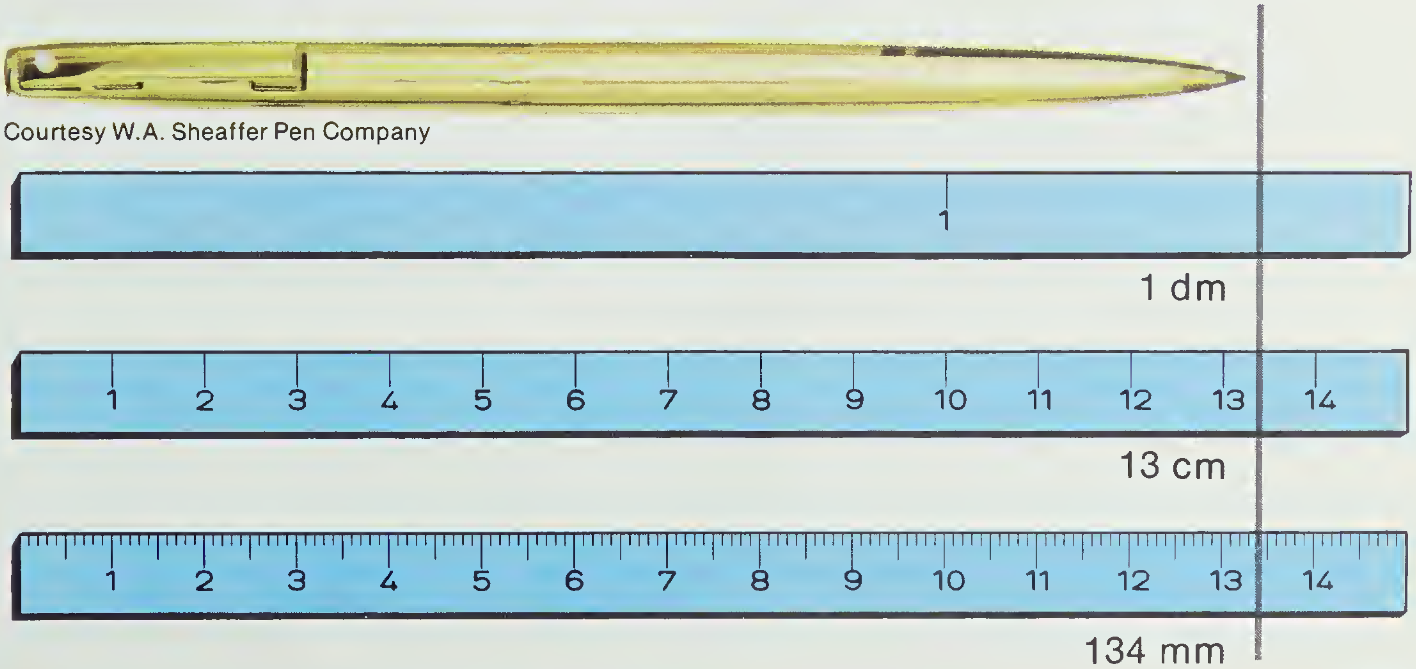
Measuring Length

5.3

We measure length in the metric system in metres. The metre is the base unit of length. Look at the table below. Each unit is 10 times larger than the next smaller unit.

PREFIX	UNIT	RELATION TO METRE	SYMBOL
kilo	**kilometre**	1000 m	km
hecto	hectometre	100 m	hm
deca	decametre	**10 m**	dam
	metre	1 m	m
deci	decimetre	0.1 m	dm
centi	**centimetre**	0.01 m	cm
milli	**millimetre**	0.001 m	mm

The pen below is measured with three different rulers. As the marked-off units get smaller, the measurements become more exact.



Courtesy W.A. Sheaffer Pen Company

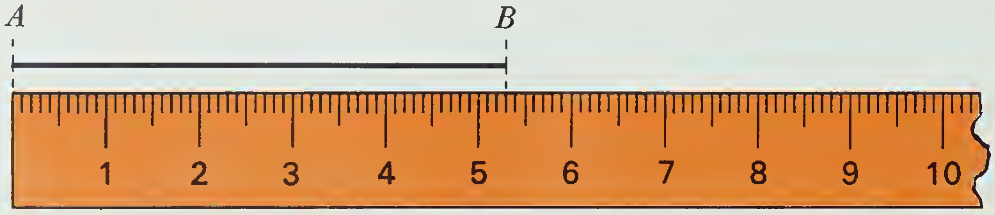
Courtesy United Airlines

Most common rulers are marked in centimetres and millimetres. Measuring to the nearest millimetre is close enough for most purposes. However, a machine operator might measure to the nearest micrometre. Of course, a special measuring instrument would be needed.

Because a ruler is marked in millimetres, this does not mean every measurement has to be made to the nearest millimetre.

The ruler shown in the following example is marked in millimetres and centimetres. The smaller units are millimetres; the larger units are centimetres.

Example:



Length *AB* is: 5 cm to the nearest centimetre.
53 mm to the nearest millimetre.

A measurement like 53 mm can also be expressed as 5.3 cm.

Exercises

1. The prefix *milli-* means (0.1, 0.01, 0.001).

2. The prefix *kilo-* means (10, 100, 1000).

3. Which is longer, a millimetre or a centimetre? Centimetre

4. Which is shorter, a metre or a kilometre? Metre

5. Which unit is about the width of your little finger? Centimetre

6. Which unit would be about the height of a doorknob? Metre

7—10. Measure each line segment in centimetres and then in millimetres.

7 (a) 4.2 cm (b) 42 mm

8 (a) 2.5 cm (b) 25 mm

9 (a) 5.2 cm (b) 52 mm

10 (a) 1.4 cm (b) 14 mm

11—14. Draw a segment of the given length.

11-14. *Drawings*

11. 2.3 cm

12. 5.9 cm

13. 113 mm

14. 76 mm

15—19. Estimate in length each of the following and then measure.

15–19. *Answers will vary*

	ESTIMATED LENGTH	MEASURED LENGTH
15. Length of your index finger.	____________	____________
16. Distance around your waist.	____________	____________
17. Length of your arm.	____________	____________
18. Distance around your wrist.	____________	____________
19. Length of your thumb.	____________	____________

20–23. On a TV game show, four people made the following statements. Was each telling the truth?

20. "I caught a fish that was 3 km long." No

21. "I live in a house that is 300 mm wide." No

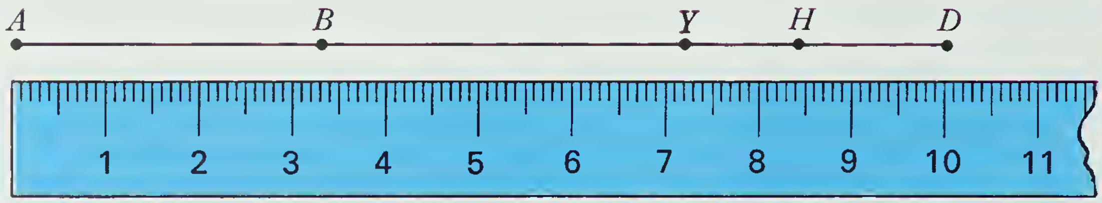
22. "My waist measurement is 75 cm." Yes

23. "My foot is 10 m long." No

24–27. Name the metric unit which would be best to find each measurement.

24. The thickness of an eyeglass lens.	millimetre
25. The length of the St. Lawrence River.	kilometre
26. The length of a car.	metre
27. The height of a person.	centimetre

28–31. Use the ruler below. Find the length of each segment to the nearest centimetre.



28. $\overline{AB}$ 3 cm

29. $\overline{AD}$ 10 cm

30. $\overline{AY}$ 7 cm

31. $\overline{AH}$ 8 cm

32–35. Use the ruler above. Find the length of each segment to the nearest millimetre.

32. $\overline{AH}$ 84 mm

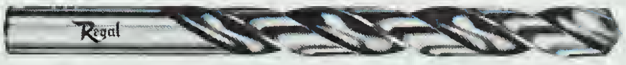
33. $\overline{AY}$ 72 mm

34. $\overline{AD}$ 100 mm

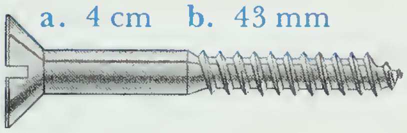
35. $\overline{AB}$ 33 mm

36–39. Find the length of each object below **(a)** to the nearest centimetre and **(b)** to the nearest millimetre.

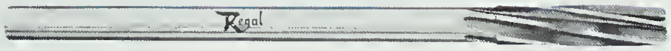
36. a. 7 cm b. 66 mm



37. a. 4 cm b. 43 mm

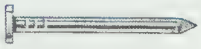


38. a. 7 cm b. 71 mm



Courtesy Beloit Tool Corporation

39. a. 2 cm b. 21 mm



40–51. Draw segments of each length.
40–51. *Drawings*

40. 4 cm
41. 16 cm
42. 48 mm
43. 88 mm
44. 40 mm
45. 160 mm
46. 10 cm
47. 25 cm
48. 7.5 cm
49. 1.3 cm
50. 4.8 cm
51. 8.8 cm

Measuring Activities

Estimate the measurements of three rooms at home and then check your estimates by measuring.

Quick Quiz

Check these skills:

- *multiplying a whole number or decimal by 10, 100, or 1000*
- *dividing a whole number by 10, 100, or 1000*

1–9. Multiply.

1. 14 × 10 140
2. 72 × 100 7200
3. 43 × 1000 43,000
4. 20 × 1000 20 000
5. 1.4 × 10 14
6. 1.4 × 100 140
7. 0.17 × 100 17
8. 2.1 × 1000 2100
9. 8.16 × 10 81.6

10–12. Divide.

10. $\frac{25}{10}$ 2.5
11. $\frac{185}{100}$ 1.85
12. $\frac{25}{1000}$ 0.025

MEASURING ACTIVITIES

1–4. Answers will vary.

1. Suppose you are a painter hired to paint your classroom. You might make a list like the one below.

	ESTIMATE	ACTUAL LENGTH
length of longer wall		
length of shorter wall		
height of ceiling		
height of door(s)		
width of door(s)		
height of window(s)		
width of window(s)		

For your classroom, first estimate each measurement above. Then use a metrestick or a tape measure to check your estimates. Record all these estimates and measurements in a table like that above.

2–4. Make a list of the measurements you would need to make if you were employed to do each job. Then estimate each measurement for your classroom. Check your measurements by actually measuring.

2. carpet the floor

3. install ceiling tiles

4. install window blinds

Quick Quiz

Check these skills:

- *changing improper fractions to mixed numbers*
- *changing mixed numbers to improper fractions*
- *multiplying whole numbers by fractions or mixed numbers*

1–4. Change to mixed numbers.

1. $\frac{3}{2}$ $1\frac{1}{2}$ **2.** $\frac{5}{3}$ $1\frac{2}{3}$ **3.** $\frac{32}{6}$ $5\frac{1}{3}$ **4.** $\frac{54}{12}$ $4\frac{1}{2}$

5–8. Change to fractions.

5. $1\frac{1}{2}$ $\frac{3}{2}$ **6.** $2\frac{1}{3}$ $\frac{7}{3}$ **7.** $2\frac{3}{4}$ $\frac{11}{4}$ **8.** $4\frac{3}{16}$ $\frac{67}{16}$

9–12. Multiply.

9. $12 \times \frac{1}{3}$ 4 **10.** $12 \times \frac{3}{4}$ 9 **11.** $36 \times 1\frac{1}{2}$ 54 **12.** $7\frac{3}{4} \times 12$ 93

Equivalent Lengths 5.4

Proportions are useful in changing from one unit to another. For most uses, all you have to remember is the following:

10 millimetres in 1 centimetre
100 centimetres in 1 metre
1000 millimetres in 1 metre
1000 metres in 1 kilometre

Performance Objective: The student can change a length from one metric unit to another.

Examples:

1. Change 14 cm to mm.

10 mm in 1 cm
? mm in 14 cm

Be sure to list the units in the same order.

$$\frac{10}{n} = \frac{1}{14}$$

Let *n* stand for the number of millimetres.

$$10 \times 14 = n \times 1$$

Find the cross-products.

$$140 = n$$

Multiply.

14 cm is 140 mm.

2. Change 20 km to m.

1000 m in 1 km
n m in 20 km

$$\frac{1000}{n} = \frac{1}{20}$$

$$1000 \times 20 = n \times 1$$

$$20\,000 = n$$

20 km is 20 000 m.

Metric usage recommends space instead of commas in writing measurements with five or more digits.

3. Change 25 mm to cm.

10 mm in 1 cm
25 mm in *n* cm

$$\frac{10}{25} = \frac{1}{n}$$

$$10 \times n = 25 \times 1$$

$$10n = 25$$

$$n = 2.5$$

Divide by 10.

4. Change 185 m to km.

1000 m in 1 km
185 m in *n* km

$$\frac{1000}{185} = \frac{1}{n}$$

$$1000 \times n = 185 \times 1$$

$$1000n = 185$$

$$n = 0.185$$

Exercises

1–22. Change each measurement to complete the following:

1. 1 m is 100 cm.

2. 1 m is 1000 mm.

3. 1 km is 1000 m.

4. 1 cm is 10 mm.

5. 10 cm is 100 mm.

6. 50 cm is 500 mm.

7. 8 km is 8000 m.

8. 19 km is 19 000 m.

9. 29 m is 29 000 mm.

10. 83 m is 83 000 mm.

11. 1.8 cm is 18 mm.

12. 2.5 mm is 0.25 cm.

13. 2.6 km is 2600 m.

14. 8.5 km is 8500 m.

15. 80 mm is 8 cm.

16. 150 mm is 15 cm.

17. 5000 m is 5 km.

18. 10 000 m is 10 km.

19. 17 mm is 1.7 cm.

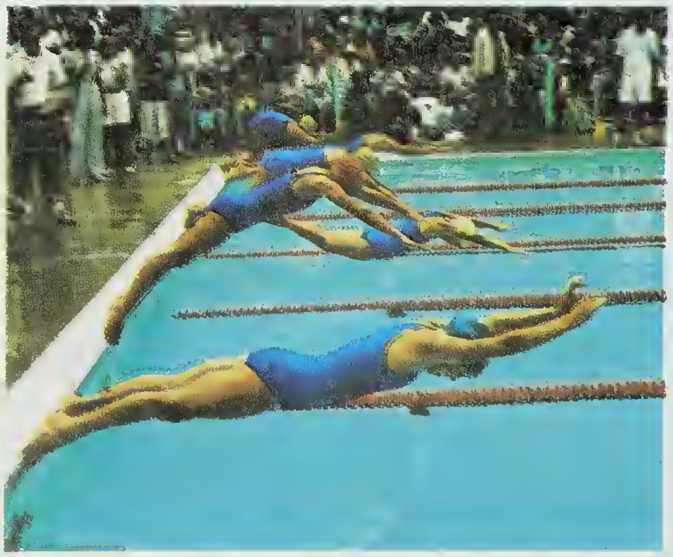
20. 125 mm is 12.5 cm.

21. 170 m is 0.170 km.

22. 155 m is 0.155 km.

23. The length of an Olympic-sized pool is 50 m. In order to swim 1 km, how many lengths would you need to swim? 20

24. The length of one long-distance race in track is 20 000 m. How many kilometres is this? 20



Johnson/Photo Research International

De Wys, Inc.

CAREER CORNER

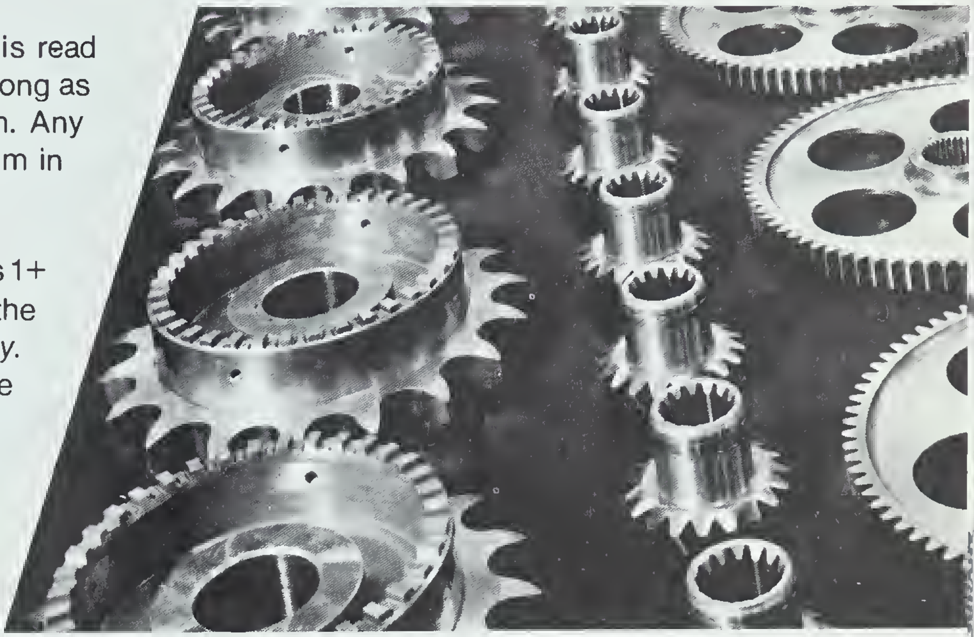
MATH IN INDUSTRY

In industry, the **tolerance** of a part is the amount it can vary from a given size. For example, a steel bolt might be designed to have a length of

1 ± 0.01 cm

The tolerance is ± cm. The symbol ± is read "plus or minus." So the bolt might be as long as 1 + 0.01 cm or as short as 1 - 0.01 cm. Any bolt between 1 - 0.01 cm and 1 + 0.01 cm in length would be acceptable.

Sometimes a measurement is listed as 1+ 0.01 cm or 3 - 0.02 cm. In such cases, the tolerance is allowed in *one direction only.* For 1 + 0.01 cm, the length could be anywhere between 1.00 and 1.01 cm. For 3 - 0.02 cm, the length could be between 2.98 and 3.0 cm.



H. Armstrong Roberts

Example: A metal gear is designed to have a diameter of 0.57 cm with a tolerance of ± 0.005 cm. Find the acceptable upper and lower limits for the diameter.

The upper limit is 0.57 + 0.005 cm or 0.575 cm.
The lower limit is 0.57 - 0.005 cm, or 0.565 cm.

Copy and fill in the table.

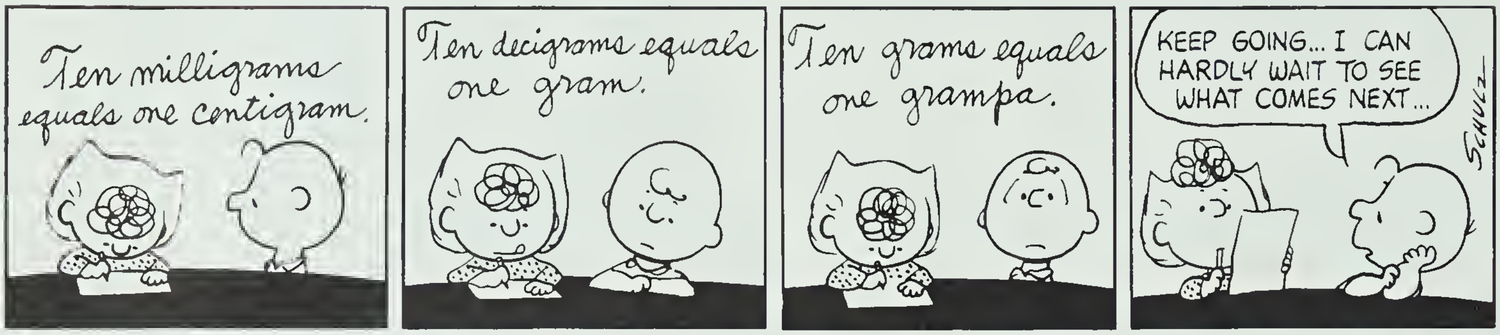
Designed Length	Tolerance	Upper Limit	Lower Limit
0.22 cm	± 0.01 cm	0.23 cm	0.21 cm
0.83 cm	± 0.005 cm	0.835 cm	0.825 cm
7.5 cm	± 0.25 cm	7.75 cm	7.25 cm
8 cm	± 0.006 cm	8.006 cm	7.994 cm
2 cm	+ 0.005 cm	2.005 cm	2 cm
10 cm	- 0.006 cm	10 cm	9.994 cm

5.5

Mass

Performance Objective:
The student can use units of mass and can change from one unit to another.

The main unit of mass in the metric system is the kilogram (kg). The gram (g) is another unit of mass that is commonly used. A paper clip has a mass of about 1 g. A nickel has a mass of about 5 g. A football player would have a mass of approximately 100 kg.


Ten milligrams equals one centigram.
Ten decigrams equals one gram.
Ten grams equals one grampa.
KEEP GOING... I CAN HARDLY WAIT TO SEE WHAT COMES NEXT...
SCHULZ

© (1972) United Feature Syndicate, Inc.

Units of Mass

UNIT	RELATION TO GRAM	SYMBOL
kilogram	1000 g	kg
hectogram	100 g	hg
decagram	10 g	dag
gram	1 g	g
decigram	0.1 g	dg
centigram	0.01 g	cg
milligram	0.001 g	mg

The units of mass contain the same prefixes as the units of length in the chart on page 203. The prefixes express the same numeric value in each situation.

kilo-	thousand	deci-	tenth
hecto-	hundred	centi-	hundredth
deca-	ten	milli-	thousandth

Proportions are useful in changing from one unit of mass to another.

Examples:

1. Change 8.5 kg to grams.

1000 g in 1 kg

n g in 8.5 kg

$$\frac{1000}{n} = \frac{1}{8.5}$$

$$1000 \times 8.5 = n \times 1$$

$$8500 = n$$

8.5 kg is 8500 g.

2. Change 757 g to kilograms.

1000 g in 1 kg

757 g in n kg

$$\frac{1000}{757} = \frac{1}{n}$$

$$1000 \times n = 757 \times 1$$

$$1000n = 757$$

$$n = 0.757$$

Divide by 1000.

757 g is 0.757 kg.

Exercises

1. The number of grams in 1 kg is 1000.

2. Would kilograms or grams be better to measure the mass of a lion? Kilograms

3. Would grams or milligrams be better to measure the mass of an orange? Grams

4–5. What is a short way to write each unit of measure?

4. gram g **5.** kilogram kg

6–14. Complete the following:

6. 29 kg is 29 000 g. **7.** 68 kg is 68 000 g.

8. 127 g is 0.127 kg. **9.** 963 g is 0.963 kg.

10. 8.3 kg is 8300 g. **11.** 39.2 kg is 39 200 g.

12. 18 g is 0.018 kg. **13.** 7 g is 0.007 kg.

14. Which mass is heavier, 999 g or 1 kg? 1 kg

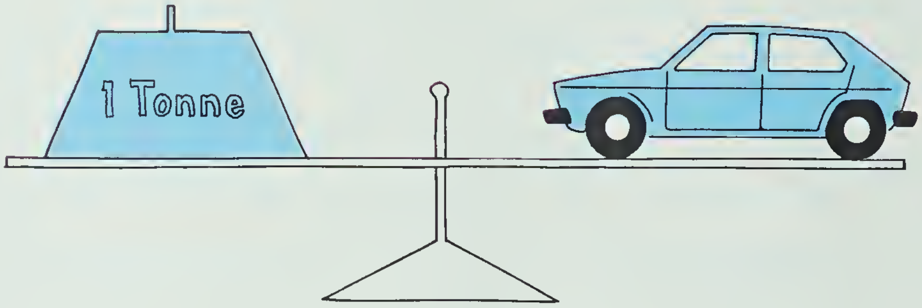
15–18. Medicines are often measured in milligrams. Complete the following.

15. 125 g = 125 000 mg.

16. 75 mg = 0.075 g.

17. 1 mg = 0.001 g.

18. 1085 g = 1 085 000 mg.



19–22. In the metric system very large masses are measured in metric tonnes. A Volkswagen Rabbit would have a mass of approximately 1 tonne (t). Large quantities of fish are measured in metric tonnes. (1 tonne (t) = 1000 kg) Complete the following:

19. 2500 kg = 2.5 t

20. 5 t = 5000 kg

21. 46800 kg = 4.68 t

22. 7.2 t = 7200 kg

23. A nickel weighs 5 g. How many nickels would it take to weigh 1000 g (1 kg)? 200

24. The largest animal in the world is the blue whale. In 1931 a blue whale with a mass of 177 t was captured. How much is 177 t in kilograms? (1000 kg in 1 t) 177 000 kg

25. Each person on an overseas flight is allowed 20 kg of luggage. On a certain flight, the excess baggage cost is \$6.50/kg. If a person has 22.3 kg of luggage, how much extra will this person pay? \$14.95

26. The most expensive substance in the world is a chemical element called Californium 252 (Element 98). It sells for \$100 per tenth of a *microgram*. (1 000 000 micrograms in 1 g) How much would a gram of Californium 252 cost? \$1 000 000 000

27. Diamonds are measured by a unit of mass called the *carat.* (5 carats in 1 g) Diamonds used for engagement rings range from 1/4 to 1 carat in most cases. In 1905 a rough diamond having a mass of 3106 carats was found in South Africa. How much is this in grams? 621.2 g

Courtesy Diamond Information Center

28. A moving company charges $28.80/50 kg to move 2500 kg of household belongings from St. John's to Halifax. There is also a ferry charge of $4.05/50 kg. What will the total cost be? $1642.50

29. If 2500 kg of household belongings are to be moved from Winnipeg to Vancouver, the charges would be $36.10/50 kg. Calculate the cost of the move. Can you speculate why the charges from Winnipeg to Vancouver are greater than the charges from St. John's to Halifax?
$1805; The distance is greater.

30. A provincial Motor Vehicle Branch bases its vehicle registration fees on mass. If the fee charged this year is $1.70/100 kg, how much would it be for a 2300 kg vehicle? $39.10

Measuring Activities

1—4. Use a balance scale to find the mass of the following objects in grams:
1–4. *Answers will vary.*

1. a quarter
2. a wrist watch
3. a gym shoe
4. a granola bar

5. For four people in your class, estimate the mass of each person in kilograms. Measure each person's mass in kilograms. Express the mass of each in grams.

5.6 Capacity

The amount a container will hold is called its capacity. The main unit to measure capacity in the metric system is the litre. Smaller amounts of liquid are measured in millilitres

Performance Objective:
The student can use units of capacity and can change from one unit to another.

UNIT	RELATION TO LITRE	SYMBOL
kilolitre	1000 L	kL
hectolitre	100 L	hL
decalitre	10 L	daL
litre	1 L	L
decilitre	0.1 L	dL
centilitre	0.01 L	cL
millilitre	0.001 L	mL

Again you can use proportions to change from one unit of measure to another.

Examples:

1. Change 3.2 L to millilitres

1000 mLin 1 litre

n mLin 3.2 L

$$\frac{1000}{n} = \frac{1}{3.2}$$

$$1000 \times 3.2 = n \times 1$$

$$3200 = n$$

3.2 L is 3200 mL.

2. Change 876 mL to litres

1000 mL in 1 litre

876 mL in n L

$$\frac{1000}{876} = \frac{1}{n}$$

$$1000 \times n = 876$$

$$0.876 = n$$

876 mL is 0.876 L

1–4. What is a short way to write each unit of measure?

1. litre L
2. kilolitre kL
3. millilitre mL
4. tonne t

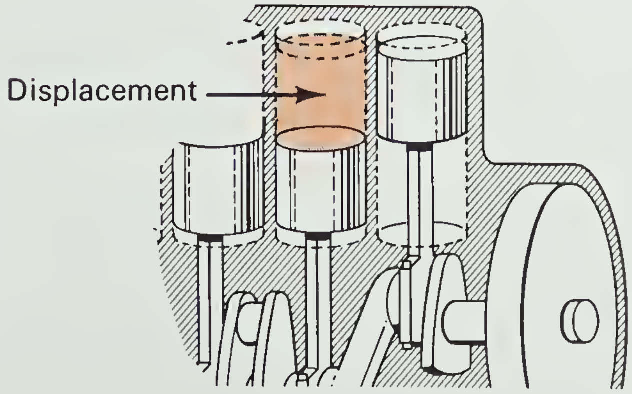
5–14. Complete the following:

5. 2.2 L = 2200 mL
6. 1500 mL = 1.5 L
7. 456 kL = 456 000 L
8. 8.3 L = 8300 mL
9. 855 mL = 0.855 L
10. 921 kL = 921 000 L
11. 9865 L = 9.865 kL
12. 7.05 L = 7050 mL
13. 0.4 L = 400 mL
14. 98764 mL = 98.764 L

15. The engine displacement of a car is often measured in litres.

	Total Engine Displacement	Number of Cylinders
Chevrolet Chevette	1.4 L	4
Ford Pinto	2.3 L	6
Buick Electra	7.5 L	8

For each car above, find the displacement per cylinder.



Chevette, 0.35 L
Pinto, 0.383 . . . or
approx 0.38 L
Electra, 0.9375 L

16. Bill had 1 L of water for his plants. He used 30 mL on a geranium.
(a) How many millilitres did he have for the rest of the plants? 970 mL
(b) What percent of the water did he have for the other plants? 97%

17. If 18 L of paint has a mass of 15 kg, what is the approximate mass of 1 L of paint? 0.8 kg

18. The most expensive perfume in the world is a type called Adoration. It is made in Spain and sells for $185 for 16 mL. If a litre bottle of this perfume were available, how much would it cost? $11,500.00

Y Dojc/Miller Services

19. A 300 mL bottle of pop costs 40¢ and a 750 mL bottle costs 75¢. What would a litre cost at each price? Which size is a better buy—300 mL or 750 mL? $1.33, $1.00, 750 mL

Measuring Activities

1–2. *Answers will vary.*

1. Find three large pails and fill each with water. Estimate the number of litres of water in each pail. Using a litre container measure the capacity of each pail.

2. Estimate and then measure each of the following in millilitres:
   (a) a soft drink bottle
   (b) a juice can
   (c) a coffee cup
   (d) a mixing bowl

Time 5.7

Performance Objective: The student can change from one unit of time to another and can add or subtract intervals of time.

Units of time are just units of measure.

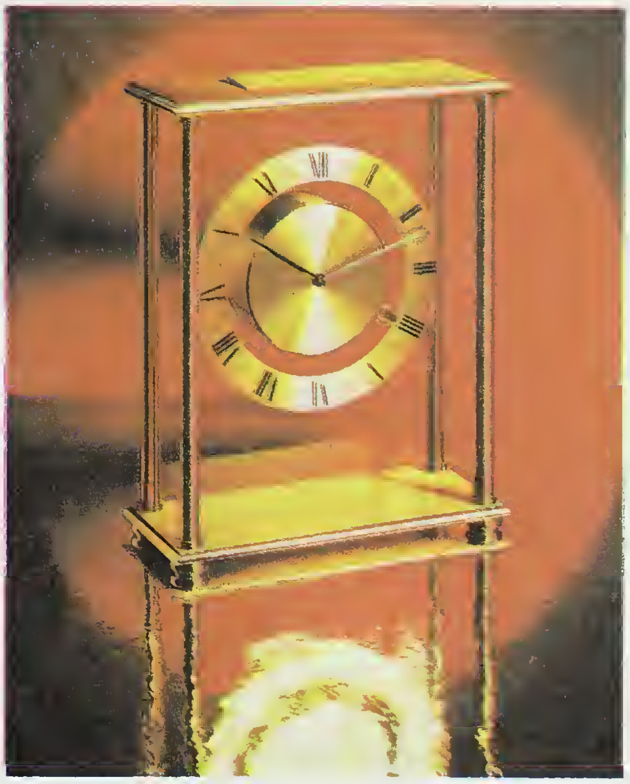
60 seconds (s) in 1 minute (min)

60 minutes (min) in 1 hour (h)

24 hours in 1 day

365 days in 1 year

52 weeks in 1 year



Courtesy Hamilton Watch Company

In problems involving addition and subtraction of time, you must often change the units. In both addition and subtraction, always add or subtract the smallest units first.

Examples:

1.

$$\begin{array}{r} 1\phantom{\text{ min 23 s}} \\ 7\text{ min }23\text{ s} \\ +\,58\text{ min }50\text{ s} \\ \hline 66\text{ min }73\text{ s} \\ 13\phantom{\text{ s}} \\ \\ 1\text{ h }6\text{ min }13\text{ s} \end{array}$$

① Add the seconds.

② Change 73 s to 1 min 13 s, since 73 = 60 + 13.

③ Add the minutes.

④ Change 66 min to 1 h 6 min and remember to include the seconds.

2.

$$\begin{array}{r} \overset{5}{\cancel{6}}\text{ h }\overset{73}{\cancel{13}}\text{ min} \\ -3\text{ h }45\text{ min} \\ \hline 2\text{ h }28\text{ min} \end{array}$$

Since you cannot subtract 45 min from 13 min, change 6 h 13 min to 5 h 73 min. Then subtract.

In this country, each twenty-four hour day is divided into two twelve hour parts—a.m. (before noon) and p.m. (after noon). Sometimes this fact must be remembered in addition and subtraction problems involving time.

Example 3: A bus leaves Regina at 7:15 a.m. and arrives in Saskatoon at 8:45 a.m. How long does the trip take?

8:45 a.m.
− 7:15 a.m.
1:30

Subtract the time it leaves from the time it arrives.

The trip takes 1 h 30 min.

Example 4: Joyce Kee started work at 3:30 p.m. and stopped at 8:15 p.m. How long did she work?

7.75
Time stopped ~~8:15~~ p.m.
Time started − 3:30 p.m.
4:45

Since you cannot subtract 30 min from 15 min, change 8:15 to 7:75

Joyce worked 4 h 45 min.

Example 5: A car came into a parking lot at 9:10 a.m. and went out at 3:20 p.m. How long was it parked?

15
Time out ~~3:20~~ p.m.
Time in − 9:10 a.m.
6:10

Since one time is p.m. and the other a.m. add 12:00 to 3:20

The car was parked 6 h 10 min.

Exercises

1. 365 days in 1 year
2. 52 weeks in 1 year
3. 60 min in 1 h
4. 60 s in 1 min
5. 1 min 5 s in 65 s
6. 1 h 40 min in 100 min
7. 85 s in 1 min 25 s
8. 63 min in 1 h 3 min

9—32. Add or subtract.

9. 6 min 15 s
+ 15 min 32 s
21 min 47 s

10. 8 h 28 min
+ 2 h 31 min
10 h 59 min

11. 28 min 48 s
- 12 min 44 s
16 min 4 s

12. 7 h 40 min
- 1 h 28 min
6 h 12 min

13. 7 min 42 s
+ 6 min 30 s
14 min 12 s

14. 4 h 10 min
+ 3 h 80 min
8 h 30 min

15. 6 min 28 s
- 2 min 42 s
3 min 46 s

16. 15 h 33 min
- 8 h 48 min
6 h 45 min

17. 7:25 p.m.
-3:15 p.m.
4:10

18. 11:20 a.m.
- 3:04 a.m.
8:16

19. 8:31 a.m.
- 3:25 a.m.
5:06

20. 12:50 a.m.
- 5:32 a.m.
7:18

21. 5:20 a.m.
- 3:40 a.m.
1:40

22. 10:07 p.m.
- 8:22 p.m.
1:45

23. 9:15 a.m.
- 5:40 a.m.
3:35

24. 4:07 a.m.
- 1:32 a.m.
2:35

25. 2:00 p.m.
- 8:00 a.m.
6:00

26. 5:10 p.m.
- 11:00 a.m.
6:10

27. 3:40 p.m.
- 7:23 a.m.
8:17

28. 1:40 p.m.
- 9:05 a.m.
4:35

29. 3:15 p.m.
- 6:25 a.m.
8:50

30. 2:14 p.m.
- 11:30 a.m.
2:44

31. 8:23 p.m.
- 12:40 a.m.
7:43

32. 9:50 p.m.
- 10:55 a.m.
10:55

33. A person punched the time clock at 7:40 a.m. and punched out at 4:20 p.m. How long did the person work? 8 h 40 min

34. A train leaves Calgary at 4:15 p.m. and arrives at Banff at 6:20 p.m. How long does the trip take? 2 h 05 min

35. A car parked in a parking garage at 9:10 a.m. and left at 1:32 p.m. How long was the car parked? 4 h 22 min

36. A boat cruise on the Assiniboine leaves at 10:30 a.m. and returns at 3:15 p.m. How long is the cruise? 4 h 45 min

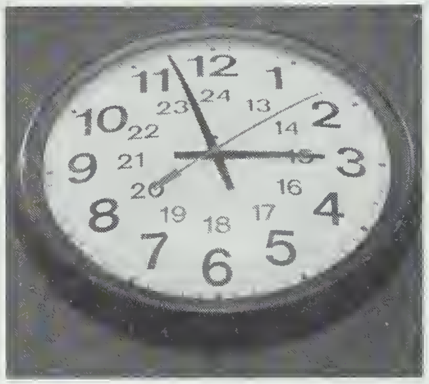
Performance Objective:
The student can use the 24-hour clock and add and subtract time.

5.8 24-Hour Clock

The 24-hour clock eliminates the need to designate a time as being a.m. or p.m. Four digits are used to express the time of the day.

5:30 a.m.	05 30
12:00 a.m. (noon)	12 00
4:08 p.m.	16 08
11:35 p.m.	23 35
12:00 p.m. (midnight)	24 00 or 00 00

The first two digits represent the number of hours since midnight and the second two digits represent the number of minutes passed in the hour. A space (or 1/2 space) is used to separate the hours from the minutes. You will notice the hours after 12:00 a.m. (noon) are added to 12 00



The use of the 24-hour clock simplifies airline, train and bus schedules. It also makes it easier to do calculations involving time.

Example 1: Bill punched in his time card at 08 00 and punched out at 16 30. How many hours did he work that day?

Time stopped	16 30
Time started	08 00
	8 30

Bill worked 8 hours 30 minutes or $8\frac{1}{2}$ hours.

Example 2: Larry arrived at work at 16 53 and left at 23 38. How many hours did he work?

Time stopped $\overset{22\ 98}{\cancel{23\ 38}}$
Time started 16 53
6 45

Since you cannot subtract 53 minutes from 38 minutes, change 23 38 to 22 98.

Larry worked 6 hours 45 minutes or 6 $\frac{3}{4}$ hours.

Exercises

1—6. Write the time using the 24-hour clock.

1. 3:50 p.m. 15 50
2. 11:20 a.m. 11 20
3. 7:14 a.m. 07 14
4. 12:00 p.m. 24 00 or 00 00
5. 8:47 p.m. 20 47
6. 0:49 a.m. 00 49

7. An airplane departs Sydney at 09 40 and arrives in Fredericton at 11 30. How long did the trip take? 1h 50 min

8—11. On the following time cards, calculate the hours worked.

8.
In 07 56
Out 17 02
9 h 6 min

9.
In 19 26
Out 03 31
8 h 5 min

10.
In 00 10
Out 08 14
8 h 4 min

11.
In 09 28
Out 17 17
7 h 49 min

12. A bus leaves at 08 40 and it takes 5 hours and 50 minutes to reach Regina. At what time would the bus arrive? 14 30

13. A flight departs Toronto at 19 30 (Eastern Standard Time) and arrives in Winnipeg at 20 55 (Central Standard Time). What is the duration of the flight? (Remember Central Standard Time is one hour behind Eastern Standard Time.) 2 h 25 min

14. The ferry from Sydney to Port-aux-Basques is an overnight trip. If you leave at 23 55 and arrive at 06 05 the next morning, how long was the ferry trip? 6 h 10 min

Performance Objective: A student can work with fractions of time and calculate payments based on time.

5.9

Hourly Payments

To establish wages, parking charges and rentals, periods of time must be calculated.

$1\text{ h} = 60\text{ min}$	$\frac{1}{12}\text{ h} = 5\text{ min}$
$\frac{1}{2}\text{ h} = 30\text{ min}$	$\frac{1}{30}\text{ h} = 2\text{ min}$
$\frac{3}{4}\text{ h} = 45\text{ min}$	$\frac{2}{3}\text{ h} = 40\text{ min}$

Example 1: Frank punched in at 07 55 and punched out at 16 15. He gets paid \$9.30 an hour. How much did he earn? How many hours did he work?

$$\begin{array}{lr} & 15\ \ 75 \\ \text{Time stopped} & \cancel{16\ \ 15} \\ \text{Time started} & \underline{07\ \ 55} \\ & \underline{08\ \ 20} \end{array}$$

He worked 8 h 20 min or $8\frac{1}{3}$ h.

$$\begin{aligned} \text{Earnings} &= \text{no. of hours worked} \times \text{earnings/h} \\ &= 8\tfrac{1}{3} \times 9.30 \\ &= \tfrac{25}{3} \times 9.30 \\ &= 25 \times 3.10 \\ &= \$77.50 \end{aligned}$$

He earned \$77.50

Exercises

1—8. Express the following minutes as fractions of an hour.

1. 15 min = $\frac{1}{4}$ h
2. 40 min = $\frac{2}{3}$ h
3. 60 min = 1 h
4. 12 min = $\frac{1}{5}$ h
5. 48 min = $\frac{4}{5}$ h
6. 35 min = $\frac{7}{12}$ h
7. 23 min = $\frac{23}{60}$ h
8. 1 min = $\frac{1}{60}$ h

9–16. Express the following fractions of an hour as minutes.

9. $\frac{1}{6}$ h 10 min

10. $\frac{1}{9}$ h 5 min

11. $\frac{2}{3}$ h 40 min

12. $\frac{5}{12}$ h 25 min

13. $\frac{7}{60}$ h 7 min

14. $\frac{2}{5}$ h 24 min

15. $\frac{5}{6}$ h 50 min

16. $\frac{17}{30}$ h 34 min

17–20. Solve each problem by using fractions of an hour.

17. A person punched in at 07 28 and punched out at 16 32. If the person is paid $8.60 an hour, how much was earned that day? $77.97

18. A car entered a parking lot at 09 36 and left at 11 44. The lot charges 30¢/$\frac{1}{2}$ h or any fraction of $\frac{1}{2}$ h. What were the parking charges? $1.50

19. A plumber is paid $19.50/h. The plumber arrives at work at 08 01 and leaves at 17 00. How much does the plumber earn? $175.18

20. On Saturdays and Sundays, a plumber is paid double time. Using question 19, how much would be earned working Saturday from 09 35 to 14 20? $185.25

21. The charges at a downtown parking lot are 45¢/$\frac{1}{2}$ h or any fraction thereof or $4.50/day maximum. If a car entered the lot at 10 33 and left at 16 58, what was the parking charge? Why? $4.50; the $\frac{1}{2}$ h rate would total $5.85 (more than the maximum)

22. A car can be rented for $18.95/day. A car is checked out at 15 10 on June 10 and returned at 17 45 on June 11. What will the total cost be if the company charges $4.00/h for every hour or fraction of an hour over the 24 h? If the odometre read 9365 when the car was rented and read 9779 when the car was returned, what would the charge be at a rate of 12¢/km? $30.95; $49.68

CAREER CORNER

MATH IN BUILDING TRADES

Many different workers are in the building trades. There are carpenters, plasterers, lathers, tile setters, glaziers, electricians, plumbers, pipe fitters, roofers, painters, ... The list goes on and on.

Each of these workers uses measurements just about every working day.

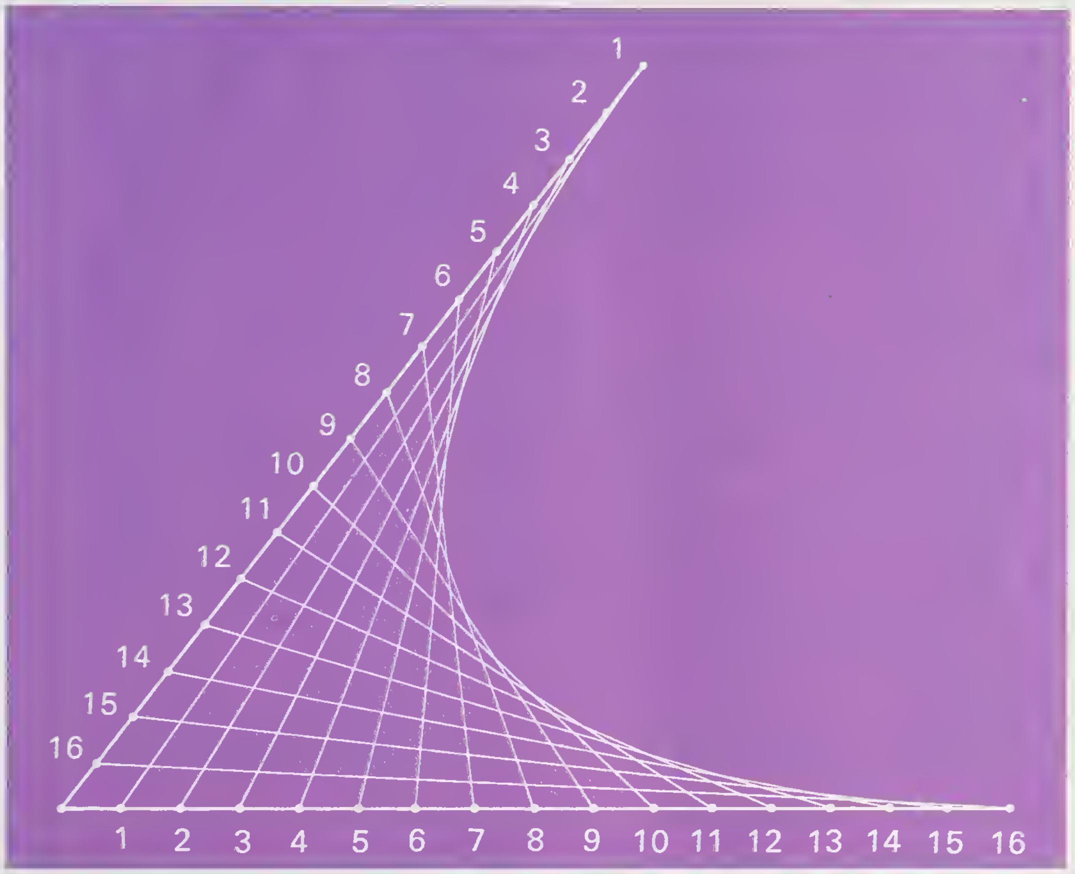
1. *Plumber:* Can three pieces of pipe measuring 90 cm, 60 cm, and 45 cm all be cut from one 2 m length of pipe? Yes
2. *Roofer:* A roll of roofing material has a mass of 50 kg. How many rolls can be carried on a $\frac{3}{4}$ t truck? 15
3. *Painter:* One litre of paint covers 15 m^2. How many litres are needed to paint 180 m^2 of wall area? 12
4. *Carpenter:* A folding-door unit 200 cm wide must be placed in an opening 196 cm wide. There are 4 doors in the unit. How much should be trimmed off the width of each door if the doors are to be of equal width? 1 cm

Zefa

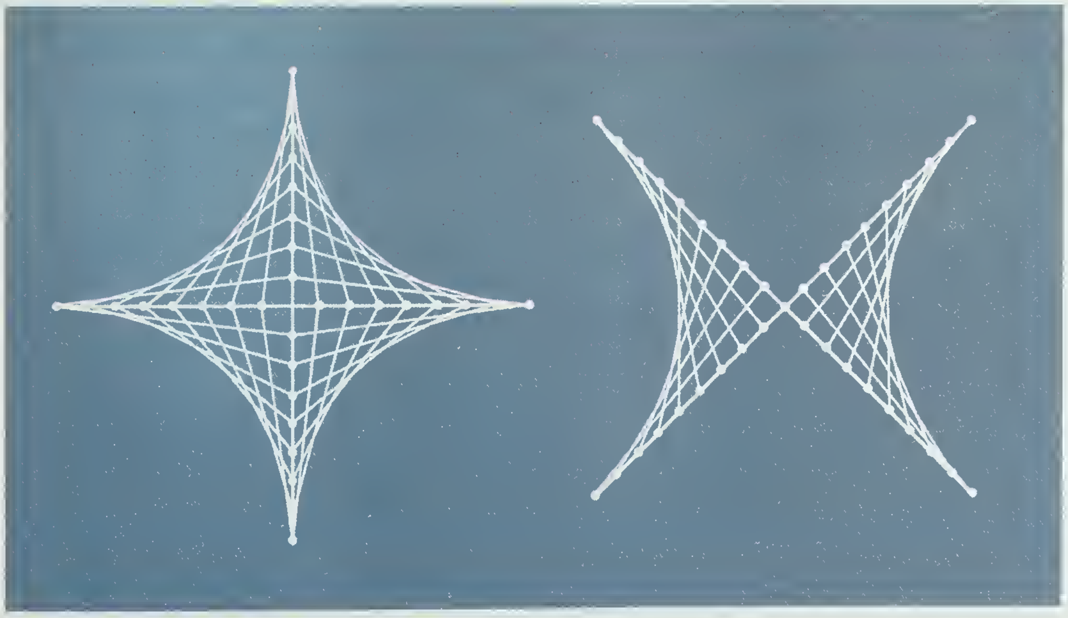
Phillips/Durango Mendoza

LINE ART

Draw two straight lines that meet at a point. Mark off 16 points 5 mm apart on each line. Label the points as shown. Then use a ruler to join 1 to 1, 2 to 2, and so on. The resulting "curve" is formed by straight lines.

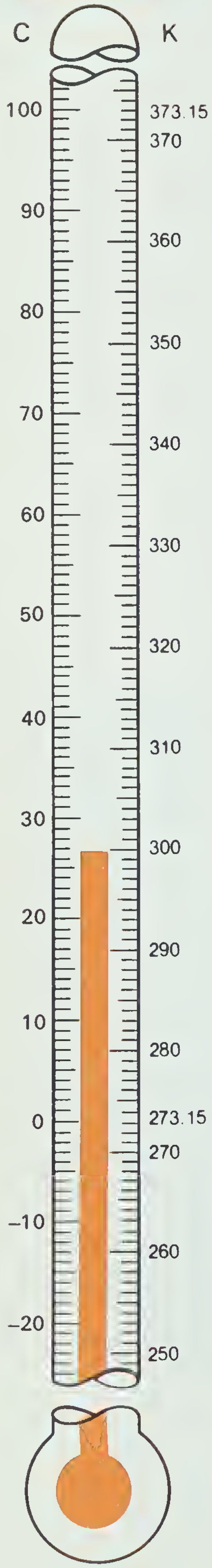

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

This idea can be used to create many interesting designs. Two examples are shown below. See if you can make up some new ones.



5.10 Temperature

Performance Objective:
The student can use Kelvin and Celsius scales and can change from one scale to the other.



Temperature can be measured on the **Celsius scale**. The table below lists temperatures on both scales. The **Kelvin scale,** (sometimes called the absolute scale), is used in certain fields of science.

	Kelvin Scale	Celsius Scale
boiling point of water	373 K	100°C
freezing point of water	273 K	0°C
recommended room-air temperature	293 K	20°C
highest recorded air temperature (Libya)	321 K	58°C
lowest recorded air temperature (Antarctica)	185 K	-88°C *
normal body temperature	310 K	37°C
normal January temperature in Toronto, Ont.	270 K	-3°C
normal June temperature in Montreal, Qué.	292 K	19°C
normal January temperature in Whitehorse, N.W.T.	249 K	-24° C

*-88°C means 88° below zero.

Exercises

1–12. Use the thermometer shown at the left to change each given temperature to the other scale.

1. 100 K -173°C
2. 40 K 233°C
3. 20°C 293 K
4. 60°C 333 K
5. 273 K 0°C
6. 373 K 100°C
7. 10°C 283 K
8. 30°C 303 K
9. 0 K -273 K
10. 10 K − 263°C
11. -10°C 263 K
12. -5°C 268 K

13–24. For each question, give the answer in both Kelvin and Celsius temperatures. Refer to the table on page 228.

13. What is the normal body temperature? 37°C, 310K

14. What is the highest recorded air temperature? 58°C, 321K

15. What is the lowest recorded air temperature? -88°C, 185K

16. What is the freezing point of water? 0°C, 273K

17. What is the difference between the boiling point of water and the freezing point of water? 100°C, 100K

18. What is the difference between the recommended room-air temperature and the normal body temperature? 17°C, 17K

19. What is the difference between the boiling point of water and the highest recorded air temperature? 42°C, 42K

20. What is the difference between the normal June temperature in Montreal and the freezing point of water? 19°C, 19K

21. What is the difference between the lowest recorded air temperature and a normal January temperature in Whitehorse? 64°C, 64K

22. What is the difference between the normal January temperatures in Toronto and Whitehorse? 21°C, 21K

23. What is the difference between the highest recorded air temperature and the lowest recorded air temperature? 146°C, 146K

24. What is the difference between a normal January temperature in Toronto and the lowest recorded temperature in Antarctica? 85°C, 85K

Measuring Activities

1–2. Use both a Kelvin thermometer and a Celsius thermometer to measure the following:
1–2. *Answers will vary.*

1. the temperature of your classroom

2. the temperature of water from a faucet

5.10

Measuring Angles

Performance Objective: The student can use a protractor to measure angles and to draw angles.

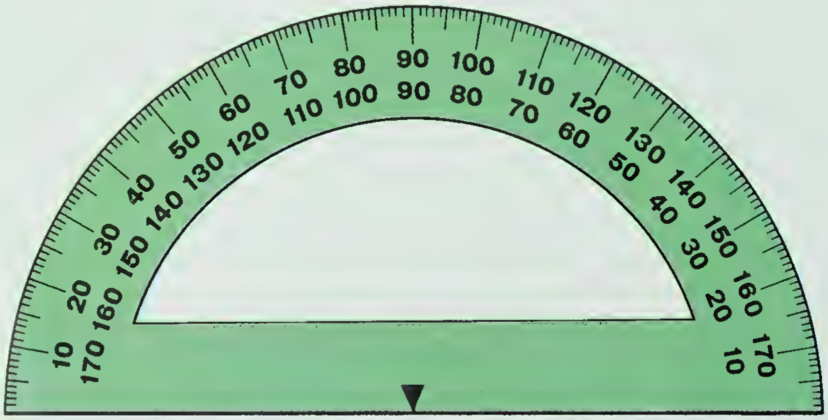
Angles occur in designs, buildings, and instructions, as in the pictures below. An **angle** has two sides that meet at a point. The sides can extend from their common point as far as necessary.

Courtesy Impact Public Relations

Grant Heilman

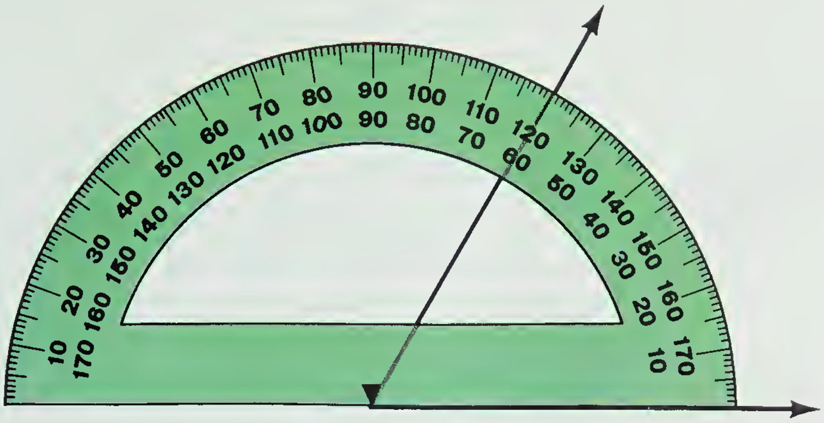
Camerique

Often an angle must be measured. Most of the time, you measure angles with a **protractor.** The unit on the protractor is the **degree.** Many protractors look like the one shown below.


10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170
170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10

There are 180 degrees (written 180°) on the protractor. Think of drawing lines from the center of the protractor to each mark on the outer edge. Then 180 unit angles would be formed. This gives a hint about how we measure with a protractor.

Example 1: Measure with a protractor.


10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170
170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10

a. Place the center of the protractor at the point where the sides of the angle meet.

b. Lay the bottom edge of the protractor along one side of the angle. Extend the sides of the angle if needed.

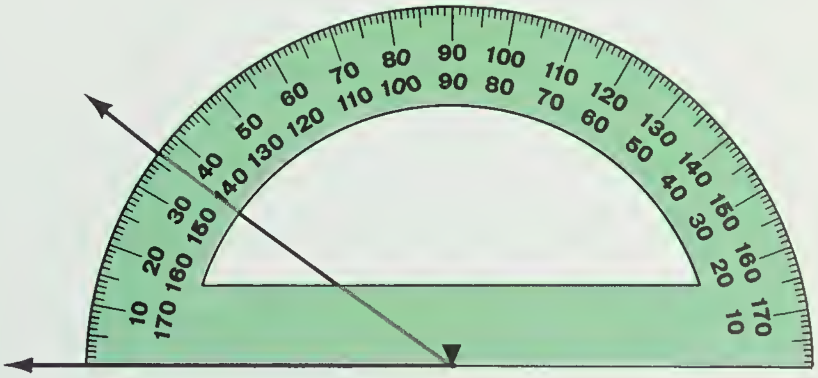
c. Read the number of degrees from the scale that has its 0° mark on a side of the angle. The measure of the angle is 60°. (In some cases, you will read from the other scale as in the next example.)

In deciding which scale to use, the idea of a **right angle** is useful. The corner of this page forms a right angle. A right angle has a measure of 90°. Therefore an angle that is larger than a right angle has a measure of more than 90°. An angle that is smaller than a right angle has a measure less than 90°.

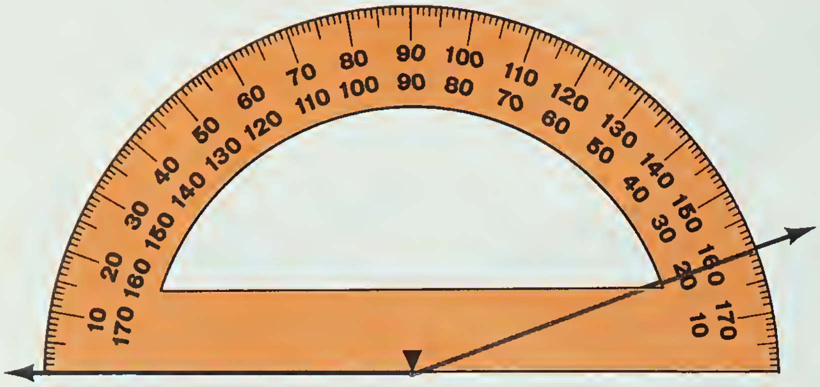
Example 2: Measure with a protractor.

The angle is smaller than a right angle. So its measure is less than 90°.

Use the scale that has its 0° mark on a side of the angle. The measure is 36°.


10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170
170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10

Example 3: Measure with a protractor.



The angle is larger than a right angle. So the measure is more than 90°. The measure is 160°.

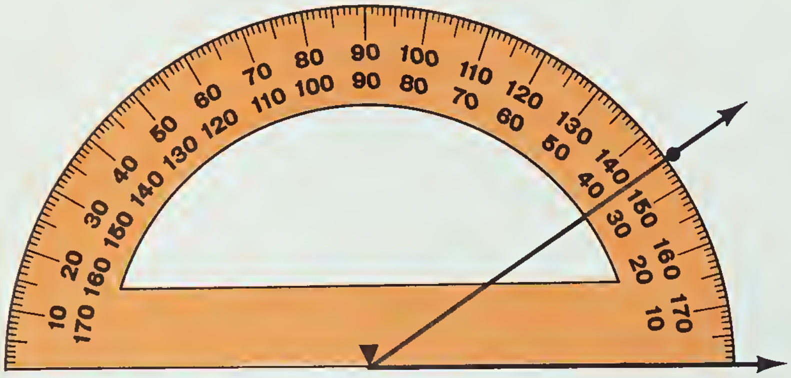
Example 4: Draw an angle with a measure of 35°.

a. First draw a horizontal line.

b. Place the bottom edge of the protractor on the line so that the center of the protractor is at the endpoint of the line.

c. On the circular edge of the protractor, locate the 35° mark of the scale that has its 0° mark on the horizontal line. Place a point there at 35°.

d. Draw a line from the end of the horizontal line to the point for 35°.



Exercises

1—4. Complete each sentence.

1. An Angle has two sides that meet at a point.

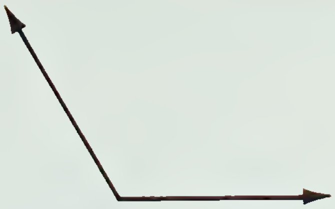
2. Angles are measured with a Protractor.

3. There are 180 degrees on a protractor.

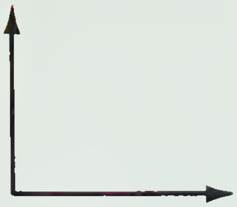
4. There are 90 degrees in a right angle.

5–6. Refer to the angles below.

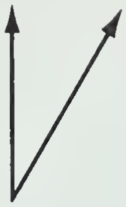
a.



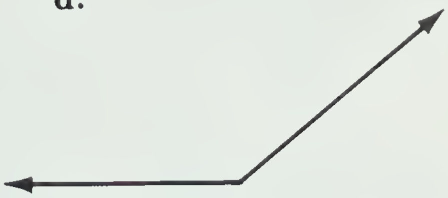
b.



c.



d.

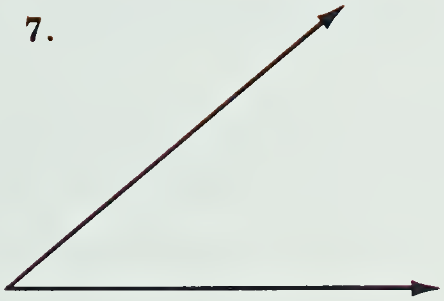


5. Which angle is a right angle? b

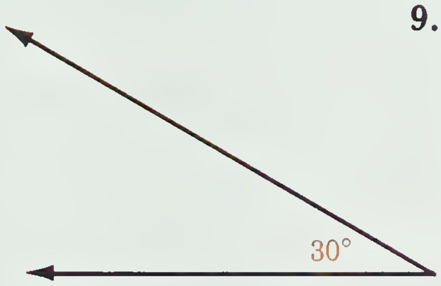
6. Which angles are larger than a right angle? a, d

7–14. Measure each angle with a protractor.

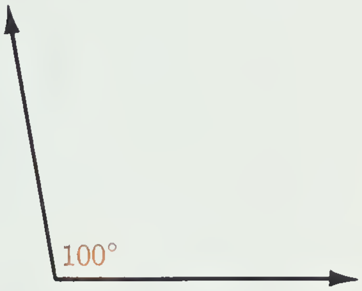
7.



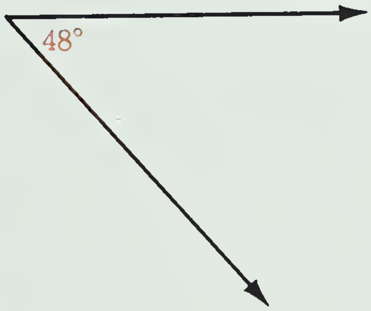
8.



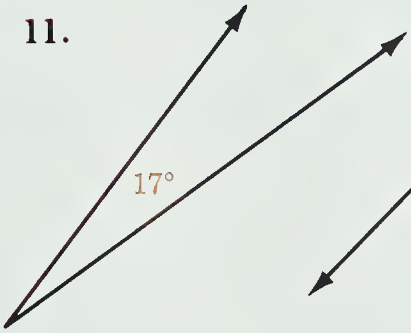
9.



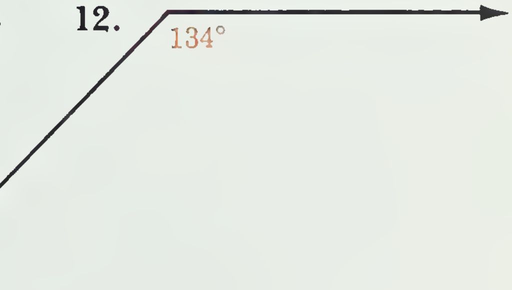
10.



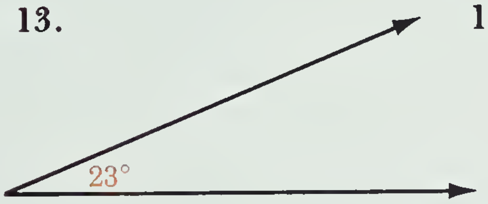
11.



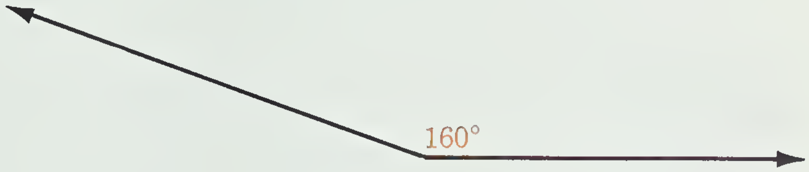
12.



13.



14.



15–22. Draw an angle having each measure.

15–22. *Drawings*

15. 45°

16. 30°

17. 120°

18. 150°

19. 52°

20. 83°

21. 163°

22. 98°

23. What angle do the hands of a clock make at 2:00? At 10:00? At 4:00? At 7:00? 60°, 60°, 120°, 150°

Test for Chapter 5

Multiple choice.

1. The (gram, truckload) is not a standard unit of measure. (5.1)

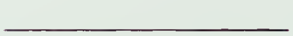
2. The (fathom, hand) is an ancient unit that is measured between the oustretched arms. (5.2)

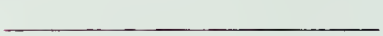
Complete the following: (5.3)

3. 15 m is 1500 cm

4. 180 mm is 18 cm

Measure each segment to the nearest centimetre. (5.4)

5.  3 cm

6.  4 cm

Complete the following:

7. 2800 g is 2.8 kg.

8. 2.5 kg is 2500 g. (5.5)

9. 12 L is 12 000 mL.

10. 4200 kL is 4.2 L. (5.6)

Add or subtract. (5.7)

11. 14 min 48 s
+9 min 18 s
24 min 6 s

12. 9:40 a.m.
−3:48 a.m.
5:52

13. 2:35 p.m.
−8:25 a.m.
6:10

Write the time using the 24 hour clock.

14. 2:15 a.m. 02 15

15. 5:40 p.m. 17 40 (5.8)

Complete the following:

16. 40 min. is 2/3 h

17. 3/10 h is 18 min (5.9)

18. What is the freezing point of water on the Celsius scale? 0°C (5.10)

Use a protractor to draw an angle having each measure. (5.11)
19–20. *Drawings*

19. 68°

20. 125°

RATE YOURSELF

Number Correct	
19–20	Superior
17–18	Excellent
15–16	Good
11–14	Fair
0–10	Poor

Chapter 5 Review

Name a unit of measure that could be used to measure each of the following: (5.1)

Ex. 1–4: Typical answers

1. your mass kilogram
2. your waist measurement centimetre
3. your age Year
4. your body temperature Degree Celsius

Which are standard units of measure?

5. a gram Yes
6. a whole lot No
7. a little while No
8. a kilometre Yes

Each ancient unit can be described by what other name? (5.2)

9. the width of the outstretched hand Handspan
10. the distance between the outstretched arms Fathoms
11. the distance from the elbow to the tip of the middle finger Cubit
12. the distance from the tip of the nose to the end of the outstretched arm Yard

Measure each segment to the nearest millimetre (5.3)

13. ____________________ 36 mm
14. ____________________________ 52 mm
15. __________________________ 48 mm
16. ______________ 25 mm

17–20. Measure each segment of the above to the nearest centimetre. (5.4)

17. 4 cm 18. 5 cm 19. 5 cm 20. 3 cm

Change each measurement.

21. 22 cm to millimetres 220 mm
22. 4 m to millimetres 4000 mm
23. 480 cm to metres 4.8 m
24. 15 km to metres 15 000 m

Draw a segment of each length.
21-22. *Drawings*

25. 16 cm

26. 88 mm

Complete the following:

27. 7.2 t is 7200 kg.

28. 8 kg is 8000 g. (5.5)

29. 65 mg is 0.065 g.

30. 5000 g is 5 kg.

31. 3.8 L is 3800 mL

32. 25 kL is 25 000 L (5.6)

33. 1125 mL is 1.125 L.

34. 0.7 L is 700 mL

Add or subtract. (5.7)

35.
3 h 50 min
+ 1 h 15 min
5 h 5 min

36.
8 min 35 s
-6 min 53 s
1 min 42 s

37.
8:10 p.m.
-4:20 p.m.
3:50

38.
3:50 p.m.
-7:30 a.m.
8:20

Write the time using the 24-hour clock. (5.8)

39. 8:55 a.m. 08 55

40. 2:44 p.m. 14 44

Complete the following: (5.9)

41. 2 min is $\frac{1}{30}$ h

42. 2/3 h is 40 min

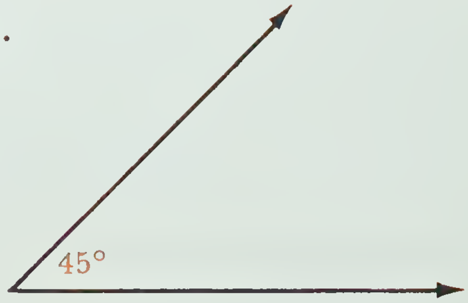
Use the thermometers on page 228 to change each temperature to the other scale. (5.10)

43. 10°C 283 K

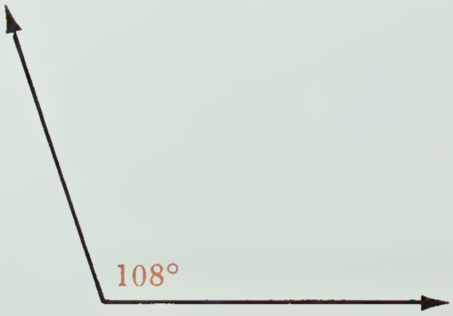
49. 100 K 173°C

Measure each angle with a protractor (5.11)

45.



46.



Preview Test for Chapter 6

1. How many right angles does a rectangle have? 4 (6.1)

2. How many sides does a hexagon have? 6

Find the perimeter of each polygon. (6.2)

3. rectangle with length 63 m and width 47 m 220 m

4. square with sides 2.5 m 10 m

How many square units are in each figure? (6.3)

5. rectangle with 8 rows of 6 square units each 48

6. square with 9 rows of 9 square units each 81

Find the area of each polygon. (6.4)

7. rectangle with length 8.2 cm and width 2.6 cm 21.32 cm^2

8. square with sides 8.2 m 67.24 m^2

Change each measurement as follows. (6.5)

9. 2 m^2 to cm^2 20 000 cm^2

10. 4 000 000 m^2 to km^2 4 km^2

Find the area of each parallelogram. (6.6)

11. base, 12 m; height, 9 m 108 m^2

12. base, 93 mm; height, 78 mm 7254 mm^2

Find the area of each triangle. (6.7)

13. base, 36 cm; height, 20 cm 360 cm^2

14. base, 14 m; height, 26 m 182 m^2

15. Find the area of a trapezoid with a height of 25 mm and bases of 10 mm and 12 mm. 275 mm^2 (6.8)

Find each square root. (6.9)

16. $\sqrt{36}$ 6

17. $\sqrt{16}$ 4

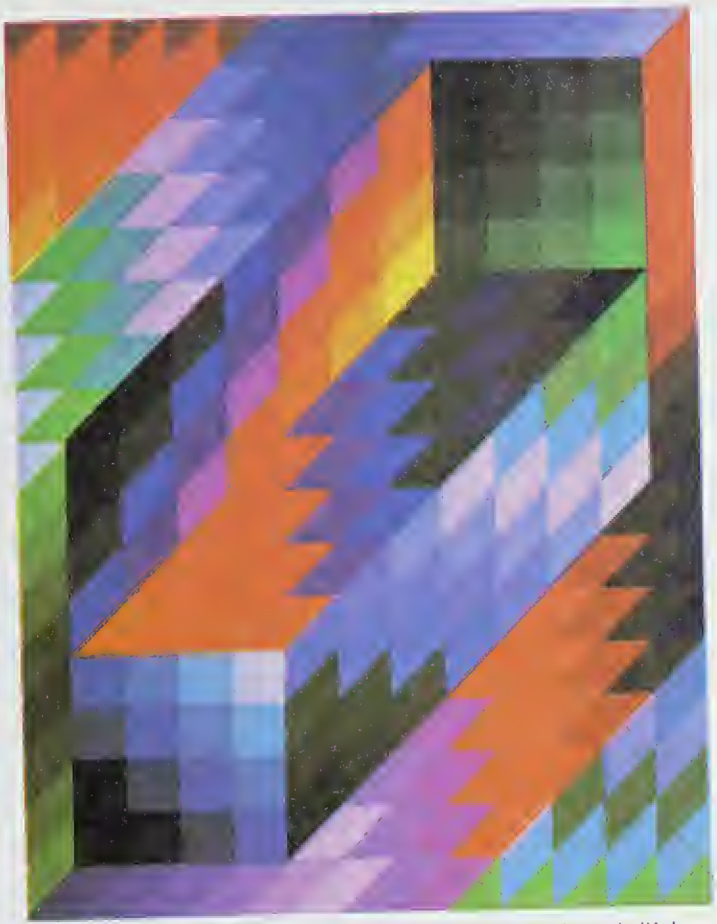
18. Find the circumference of a circle with a diameter of 18 m. Use $\frac{22}{7}$ for π. $56\frac{4}{7}$ m (6.10)

19. Find the area of a circle with a radius of 8 m. Use 3.14 for π. 200.96 m^2 (6.11)

Grant Heilman

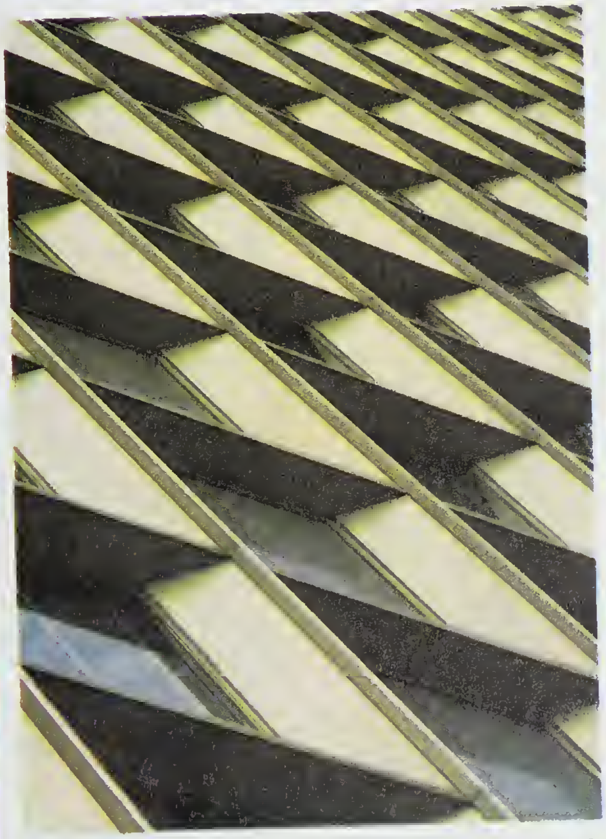
6 PLANE FIGURES

Artstreet



Gestalt III by Victor Vasarely/Kultura

PICTURE PREVIEW: Plane figures such as squares, triangles, and circles occur often in everyday life. These pictures show a few places where you might see plane figures.



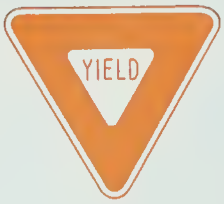
Dr. E. R. Degginger

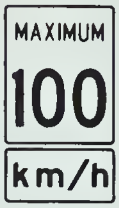
Artstreet

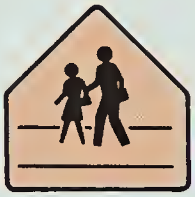
6.1 Polygons

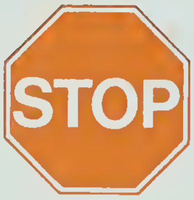
Performance Objective: The student can identify types of polygons, as well as identify polygons when given certain properties.

The road signs below have shapes that are very common. Each one is roughly a **polygon**.

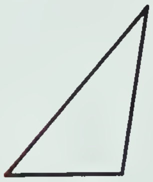


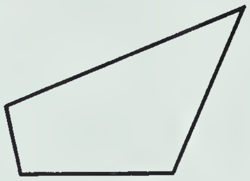


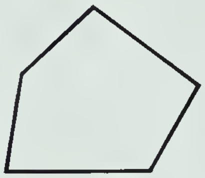


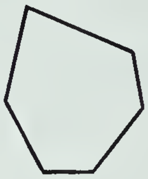


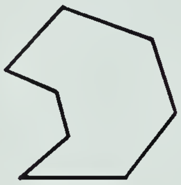
A polygon has three or more sides. All the sides are line segments. Special names are given to polygons according to the number of sides. A few are shown below.


triangle
(3 sides)


quadrilateral
(4 sides)

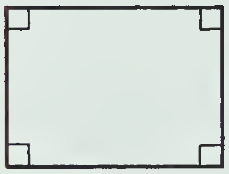

pentagon
(5 sides)

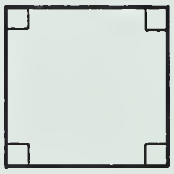

hexagon
(6 sides)

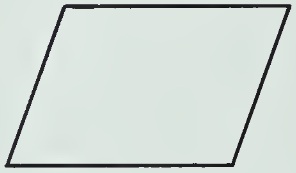

octagon
(8 sides)

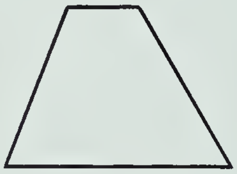
There are four special quadrilaterals (4-sided polygons) that will be used in this chapter. The following exercises review some things about them.

┌ is the symbol for a right angle.


rectangle


square


parallelogram


trapezoid

Exercises

1–6. How many sides does each polygon have?

1. triangle 3
2. square 4
3. rectangle 4
4. parallelogram 4
5. trapezoid 4
6. pentagon 5

7–8. How many right angles does each quadrilateral have?

7. rectangle 4
8. square 4

9–12. Tell how many sides each traffic sign has. Then name the polygon with that number of sides.

9. stop sign 8, octagon

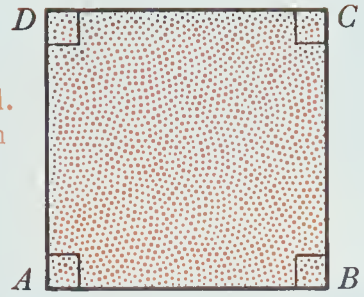
10. yield sign 3, triangle

11. school-crossing sign 5, pentagon

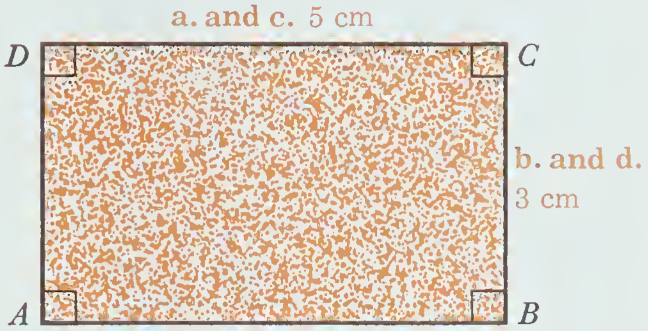
12. speed-limit sign 4, quadrilateral

13–16. For each figure measure **a.** $\overline{AB}$, **b.** $\overline{BC}$, **c.** $\overline{CD}$, and **d.** $\overline{DA}$ to the nearest centimetre.

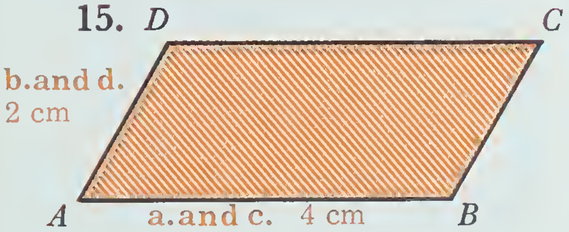
13.



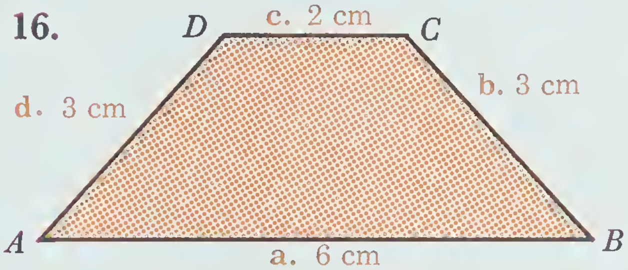
14.



15.



16.



17–18. Refer to your measurements from Exercises **13–16.**

17. Which polygons have opposite sides of equal length? 13, 14, 15

18. Which polygon has all sides of equal length? 13

19–20. Look at the figures in Exercises **13–16.**

19. For which polygons will opposite sides never meet, no matter how far they are extended? 13, 14, 15

20. For which polygon will one pair of opposite sides meet when extended? 16

21–26. The following statements are true. Draw a figure to illustrate each statement.

21–26. *Drawings*

21. A triangle can have one right angle.

22. The three sides of a triangle can be equal in length.

23. A rectangle can have four sides equal in length. (A square is a rectangle.)

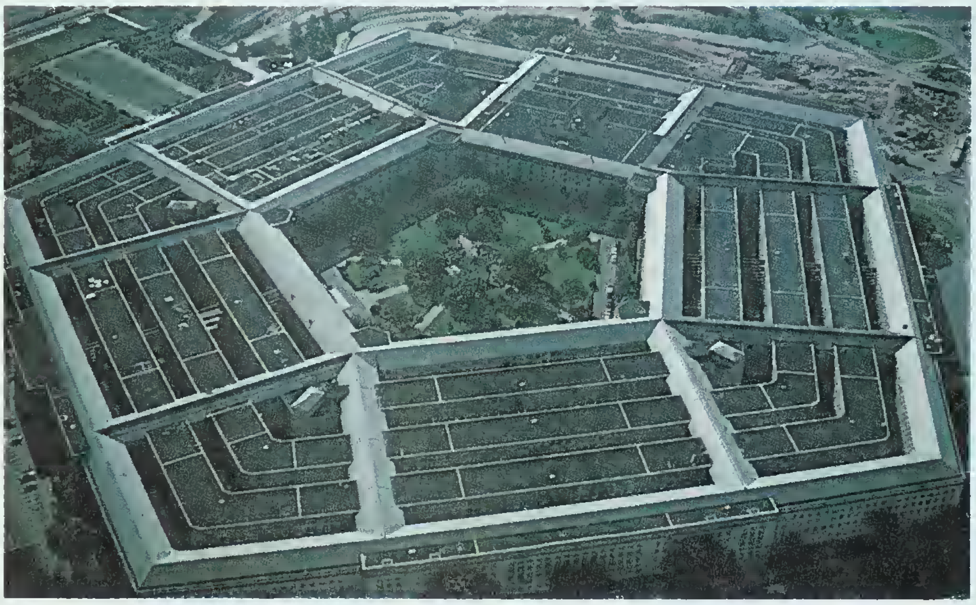
24. A parallelogram can have four right angles. (A rectangle is a parallelogram.)

25. A trapezoid can have two right angles.

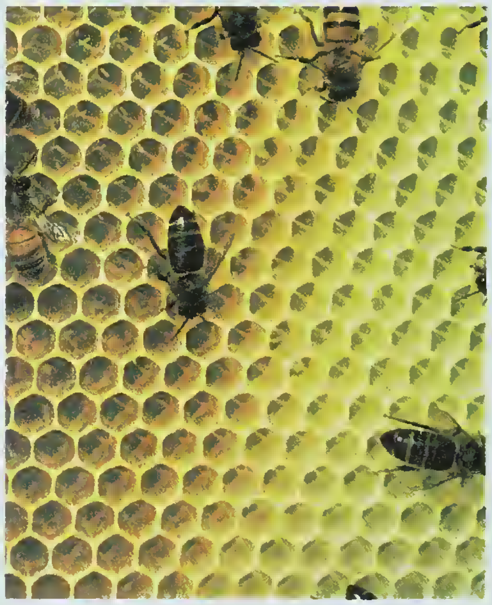
26. A parallelogram can have four sides of equal length and four right angles. (A square is a parallelogram.)

27. The building in the picture below gets its name from its shape. What do you think its name is? Pentagon building

28. What kind of polygons do you see in the honeycomb shown below? Hexagons



Johnson/De Wys, Inc.



Zefa

Quick Quiz

Check these skills:

- *adding two or more mixed numbers or decimals*
- *multiplying a decimal or mixed number by a whole number*

1–6. Add.

1. $6\frac{3}{4} + 1\frac{1}{4}$ 8

2. $3\frac{1}{2} + 4\frac{1}{4} + 5\frac{1}{8}$ $12\frac{7}{8}$

3. $2\frac{1}{6} + 3\frac{1}{12} + 5\frac{1}{2} + 1\frac{5}{12}$ $12\frac{1}{6}$

4. $1.8 + 6.3$ 8.1

5. $1.75 + 3.82 + 1.47$ 7.04

6. $3.72 + 0.8 + 6.1 + 0.124$ 10.744

7–12. Multiply.

7. $2(6.2)$ 12.4

8. $2(3.8)$ 7.6

9. $4(99.5)$ 398

10. $2(6\frac{1}{4})$ $12\frac{1}{2}$

11. $4(3\frac{1}{2})$ 14

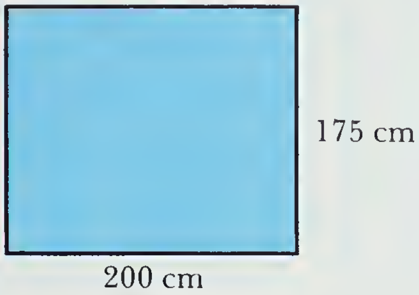
12. $4(6\frac{1}{4})$ 25

Performance Objective: The student can find the perimeters of polygons.

Perimeter

6.2

An old quilt measures 200 cm long by 175 cm wide. It needs a new border. How many centimetres of binding tape are needed to border the quilt?



$175 + 200 + 175 + 200 = 750$

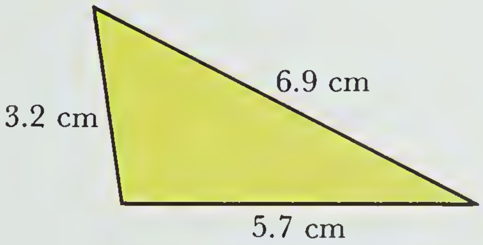
750 cm of tape are needed.

Kuechmann/Bedford Photographic

This problem involves **perimeter**—the distance around a closed figure. You find the perimeter of a polygon by adding the lengths of the sides.

Examples: Find the perimeter of each polygon. Let P stand for perimeter.

1.

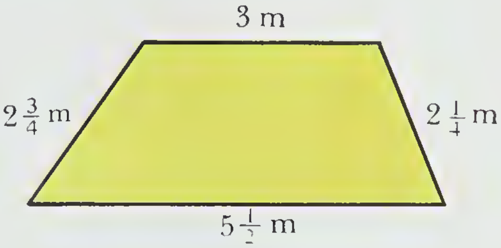


$P = 3.2 + 5.7 + 6.9$

$P = 15.8$

The perimeter is 15.8 cm.

2.



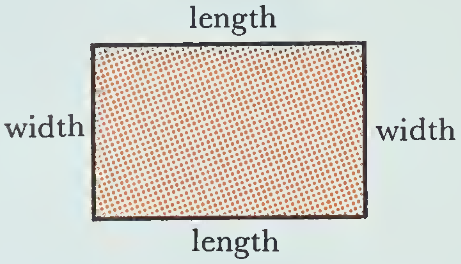
$P = 5\frac{1}{2} + 2\frac{1}{4} + 3 + 2\frac{3}{4}$

$P = 13\frac{1}{2}$

The perimeter is $13\frac{1}{2}$ m.

Formulas are useful for finding perimeters of rectangles and squares.

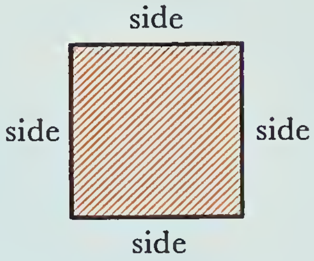
RECTANGLE



Perimeter = 2(length) + 2(width)

$$P = 2\ell + 2w$$

SQUARE



Perimeter = 4(side)

$$P = 4s$$

Examples:

3. What is the perimeter of a rectangle with a length of 6.2 m and a width of 3.8 m?

$P = 2\ell + 2w$

$P = 2(6.2) + 2(3.8)$

$P = 12.4 + 7.6$ Multiply.

$P = 20.0$ Add.

or

$P = 2\ell + 2w$

$P = 2(6.2) + 2(3.8)$

$P = 2(6.2 + 3.8)$

$P = 2(10.0)$ Add.

$P = 20.0$ Multiply.

The perimeter is 20 m.

4. What is the perimeter of a square with a side of $2\frac{3}{4}$ cm?

$P = 4s$

$P = 4(2\frac{3}{4})$

$P = 4(\frac{11}{4})$

$P = 11$ Multiply.

The perimeter is 11 cm.

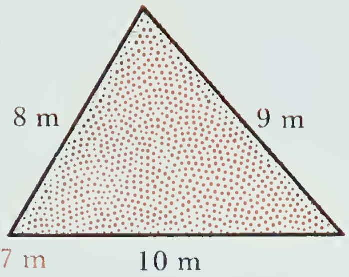
Exercises

1. What is the formula for the perimeter of a rectangle? What does each letter stand for? $P = 2\ell + 2w$, perimeter, length, width

2. What is the formula for the perimeter of a square? $P = 4s$

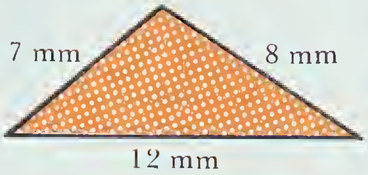
3–14. Find the perimeter of each polygon.

3.



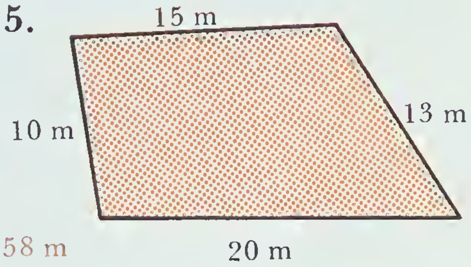
27 m

4.


7 mm
8 mm
12 mm

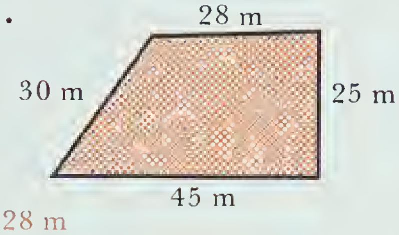
27 mm

5.


15 m
10 m
13 m
20 m

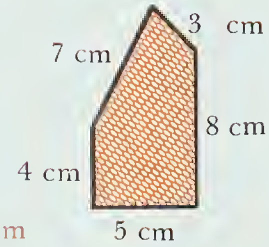
58 m

6.


28 m
30 m
25 m
45 m

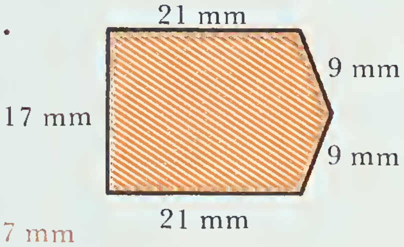
128 m

7.


3 cm
7 cm
8 cm
4 cm
5 cm

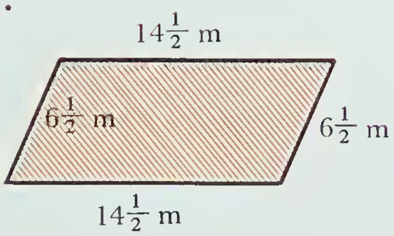
27 cm

8.


21 mm
9 mm
17 mm
9 mm
21 mm

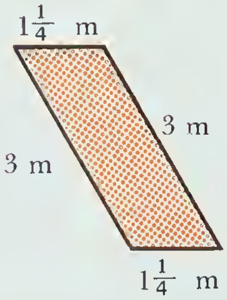
77 mm

9.


$14\frac{1}{2}$ m
$6\frac{1}{2}$ m
$6\frac{1}{2}$ m
$14\frac{1}{2}$ m

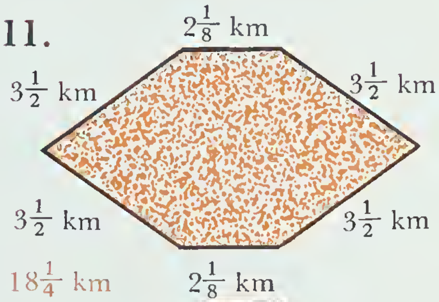
42 m

10.


$1\frac{1}{4}$ m
3 m
3 m
$1\frac{1}{4}$ m

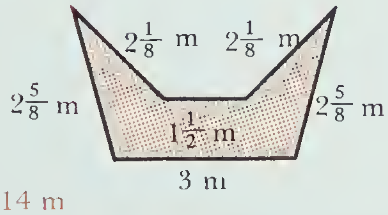
$8\frac{1}{2}$ m

11.


$2\frac{1}{8}$ km
$3\frac{1}{2}$ km
$3\frac{1}{2}$ km
$3\frac{1}{2}$ km
$3\frac{1}{2}$ km
$2\frac{1}{8}$ km

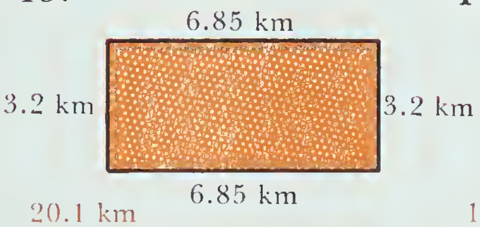
$18\frac{1}{4}$ km

12.


$2\frac{1}{8}$ m
$2\frac{1}{8}$ m
$2\frac{5}{8}$ m
$2\frac{5}{8}$ m
$1\frac{1}{2}$ m
3 m

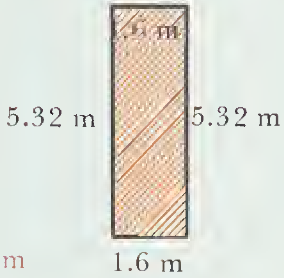
14 m

13.


6.85 km
3.2 km
3.2 km
6.85 km

20.1 km

14.


1.6 m
5.32 m
5.32 m
1.6 m

13.84 m

15–20. Find the perimeter of each polygon having the given sides.

15. $2\frac{1}{2}$ m, $3\frac{1}{4}$ m, 5 m, $2\frac{1}{2}$ m, $1\frac{1}{2}$ m $14\frac{3}{4}$ m

16. $5\frac{3}{4}$ cm, $7\frac{1}{8}$ cm, $10\frac{1}{2}$ cm, $12\frac{3}{8}$ cm $35\frac{3}{4}$ cm

17. 72.4 cm, 93.7 cm, 11.5 cm, 43.15 cm, 99.4 cm, 84.8 cm 404.95 cm

18. 7.47 mm, 1.3 mm, 8.549 mm, 8.43 mm 25.749 mm

19. $3\frac{1}{12}$ m, $7\frac{1}{6}$ m, $8\frac{1}{2}$ m, $9\frac{5}{12}$ m $28\frac{1}{6}$ m

20. $10\frac{1}{2}$ m, $15\frac{3}{4}$ m, $21\frac{5}{12}$ m, $30\frac{1}{3}$ m 78 m

21–28. Find the perimeter of each rectangle with the given length and width.

21. 28 m, 23 m 102 m

22. 31 cm, 17 cm 96 cm

23. $2\frac{1}{2}$ m, 3 m 11 m

24. 6 cm, $4\frac{1}{2}$ cm 21 cm

25. 327 m, 105 m 864 m

26. 241 cm, 129 cm 740 cm

27. 5.28 m, 7.39 m 25.34 m

28. 83.4 cm, 71.2 cm 309.2 cm

29–36. Find the perimeter of each square with the given side.

29. $3\frac{1}{2}$ m 14 m

30. $6\frac{1}{4}$ cm 25 cm

31. 179 m 716 m

32. 747 mm 2988 mm

33. 99.5 m 398 m

34. 87.6 cm 350.4 cm

35. $7\frac{3}{8}$ cm $29\frac{1}{2}$ cm

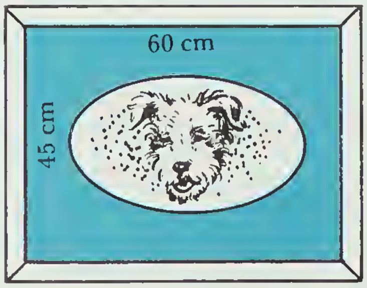
36. $23\frac{1}{6}$ m $92\frac{2}{3}$ m

37. A wallpaper border is to be applied around a room that is 6.5 m long by 4.5 m wide. How many metres of border are needed? 22 m

38. A garden is to be fenced completely by a chain-link fence. The garden is 38.2 m long by 28.1 m wide. How many metres of fencing are needed? 132.6 m

39. A square pen will be built as a rabbit cage. The pen is 2.5 m on each side. How many metres of fencing are needed? 10 m

40. A fringe is to be sewn around the outer edge of a bedspread that is 215 cm by 150 cm. How many centimetres of fringe are needed? If fringe sells for \$0.98/m and is only sold in whole metres, how much will the fringe cost? 730 cm, \$7.84



41. A picture that is 60 cm by 45 cm is to be framed with a 4 cm wide frame. How many centimetres of frame are needed to surround the picture? HINT: Find the outside perimeter. 242 cm

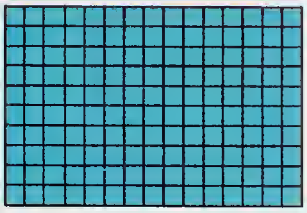
Performance Objective: The student can find the areas of polygons by using unit squares.

Measuring with Squares

6.3

A tile setter needs to tile a floor in a supermarket that is 15 m long by 10 m wide. If each tile is a 1-m square, how many tiles are needed to cover the floor?

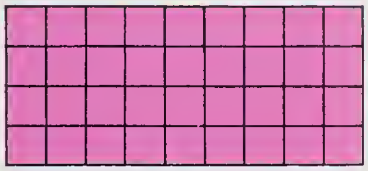
Zefa



10 rows of 15 tiles each is 150 tiles.

It is very common to cover a surface with square units. For example, you might cover a wall with mirror squares.

Example 1: A wall is to be covered with 1 m mirror squares. If the wall is 4 m high by 9 m wide, how many squares are needed?

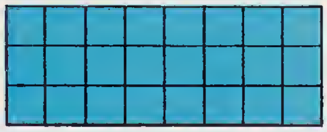


4 rows of 9 squares each is 36 squares.

Of course, nobody finds the number of squares in a rectangle by actually counting them. Everybody uses a shortcut. They multiply the number of rows by the number of squares in each row.

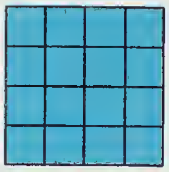
Examples: How many square units are in each rectangle?

2.



3 rows of 8 squares each is 24 squares.

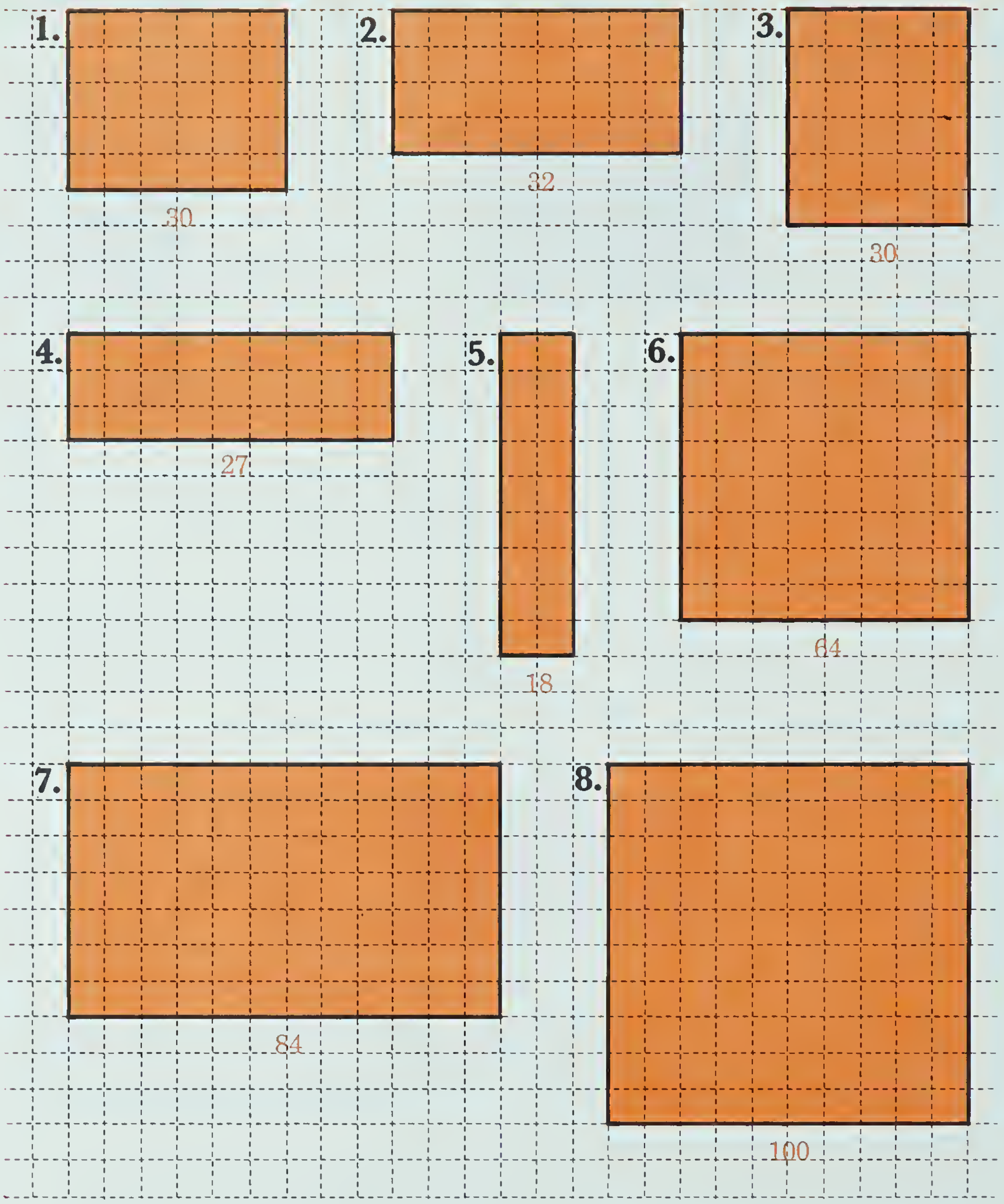
3.



4 rows of 4 squares each is 16 squares.

Exercises

1–8. How many square units are in each rectangle?


1.
2.
3.
30
32
30
4.
5.
6.
27
18
64
7.
8.
84
100

9–12. Draw rectangles having the following lengths and widths. Then draw in the square units to show the number of square centimetres in each rectangle.

9–12. *Drawings*

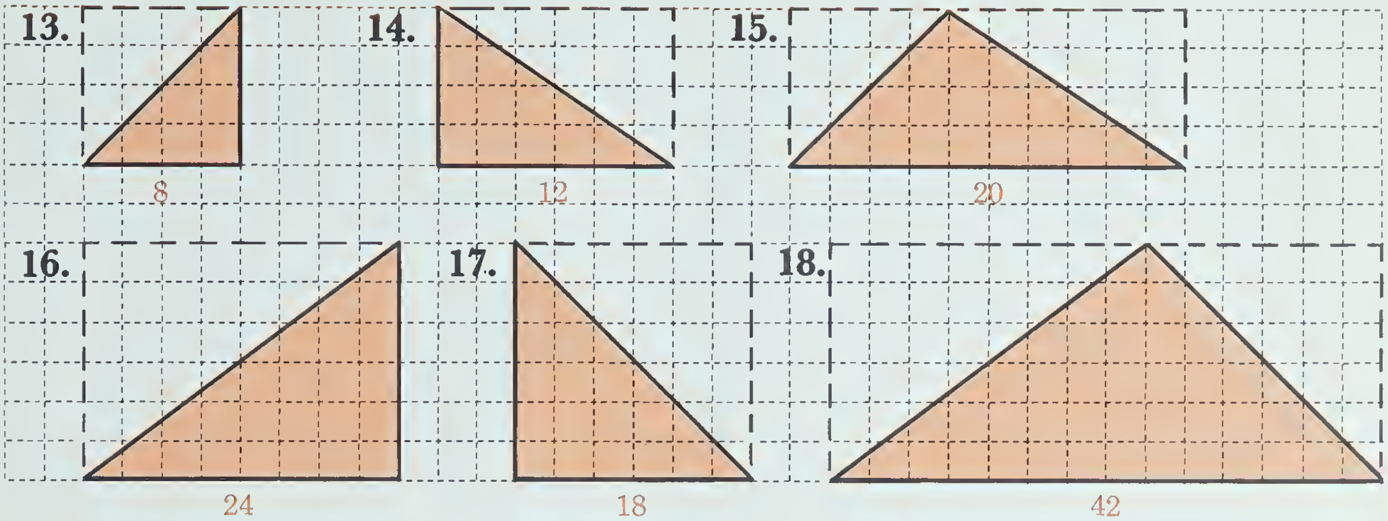
9. 4 cm, 2 cm

10. 5 cm, 3 cm

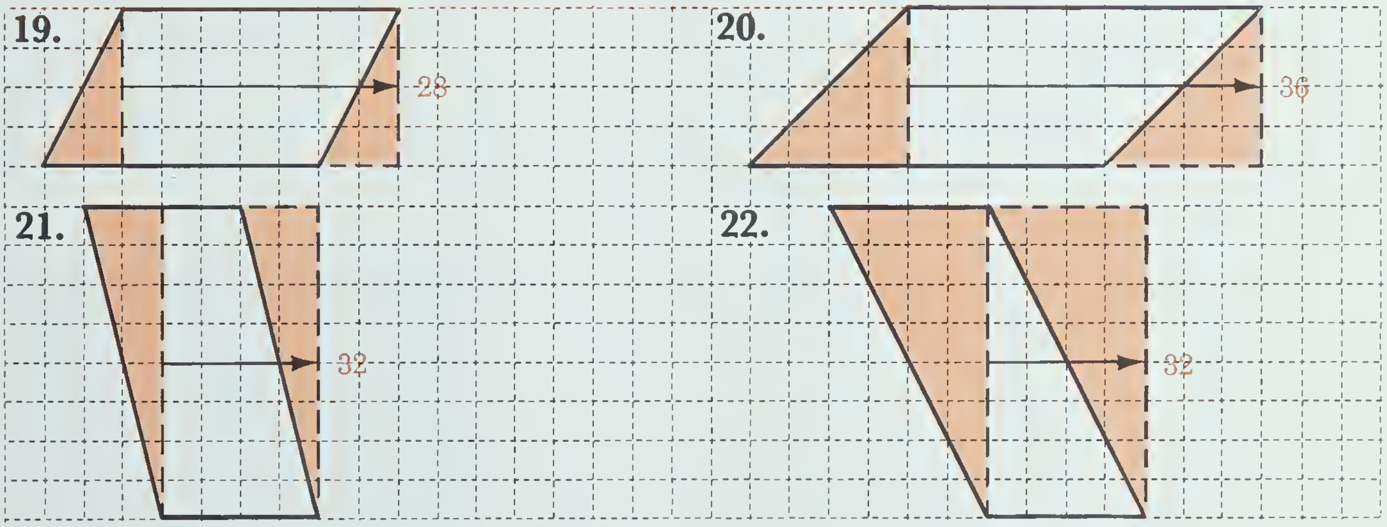
11. 6 cm, 4 cm

12. 5 cm, 4 cm

13–18. How many square units are in each triangle? HINT: Each triangle is half of a rectangle.



19–22. How many square units are in each parallelogram? HINT: Each parallelogram becomes a rectangle if the shaded part is slid as shown.



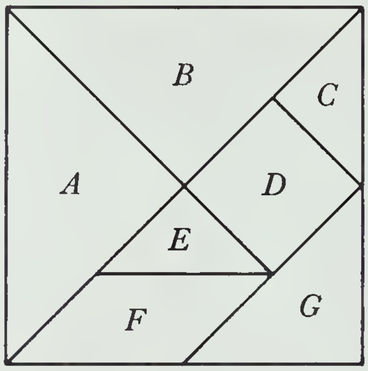
23. A ceiling is being covered with 1 m-square tiles. The ceiling is 5 m long by 4 m wide. How many tiles are needed? 20

24. A wall that is 3 m high by 7 m long is being paneled with cork tiles. How many tiles are needed? Each tile is a 1 m square. 21

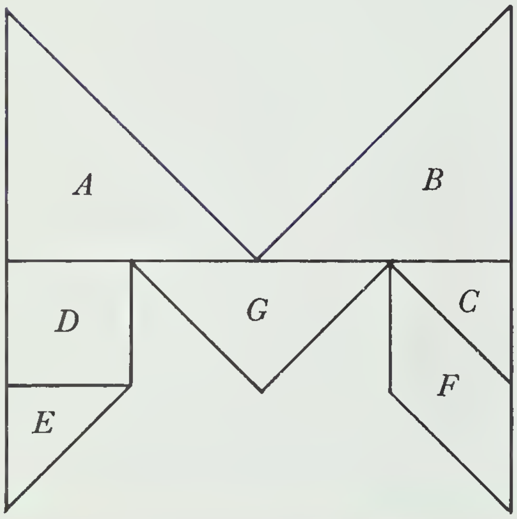
25. A tabletop is 80 cm long by 66 cm wide. It is being covered with ceramic tiles that are 1 cm squares. How many tiles are needed? 5280

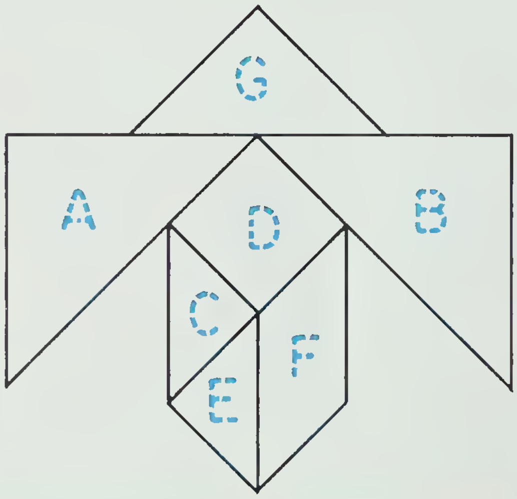
TANGRAMS

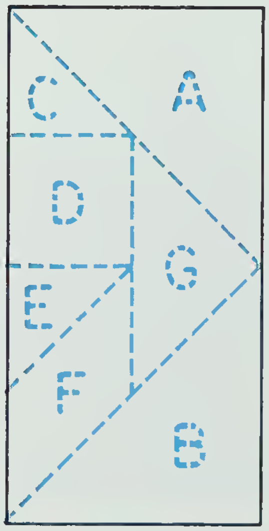
A tangram is an ancient Chinese puzzle made of seven pieces, called ***tans***, which form a square. The seven pieces consist of 5 triangles, a square, and a parallelogram.



With these seven pieces, hundreds of shapes can be formed. Use heavy paper to cut out a set of tangram pieces. Use these pieces to make each figure below. Then invent some shapes of your own.







Quick Quiz

Check this skill:

- *multiplying decimals or mixed numbers*

1–8. Multiply.

1. 1.5(2.3) 3.45

2. 2.25(3.5) 7.875

3. 93.9(79.9) 7502.61

4. 4.3(0.005) 0.0215

5. $1\frac{1}{2}(3\frac{1}{2})$ $5\frac{1}{4}$

6. $7\frac{1}{4}(2\frac{1}{2})$ $18\frac{1}{8}$

7. $1\frac{1}{4}(1\frac{1}{4})$ $1\frac{9}{16}$

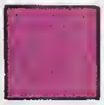
8. $8\frac{3}{4}(6\frac{1}{4})$ $54\frac{11}{16}$

Area

6.4

The **area** of a figure is the amount of surface it covers. Usually area is measured in square units. A common unit of area is shown below. Other units are the *square millimetre, square metre, square kilometre,* and so on.

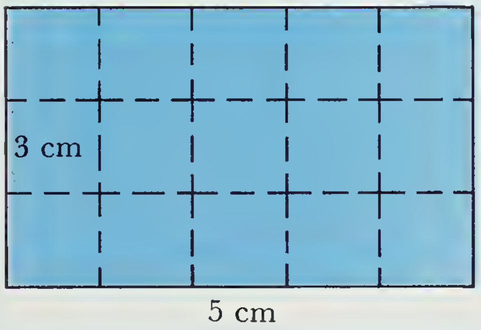
Performance Objective: The student can find the areas of squares and rectangles.



One square centimetre (cm^2)

Area could be measured by other units besides squares. However, squares are the easiest to use. In the section before this, square units were used to measure figures. Actually, you were finding areas.

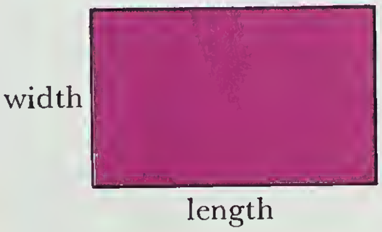
Example 1: What is the area of a rectangle that is 5 cm by 3 cm?



The dashed squares show that there are 15 *square centimetres* in the rectangle

You can find the area of a rectangle without drawing a picture, though.

To find the area of a rectangle, multiply the length by the width. (Both the length and the width must be in the same units.)



Area = length × width

$$A = \ell w$$

Example 2: What is the area of a rectangle having a length of 14 cm and a width of 6 cm?

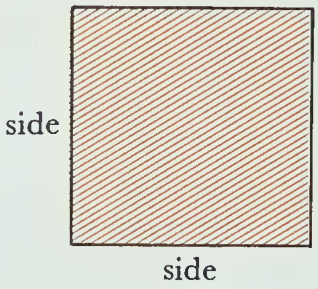
$$A = \ell w$$

$$A = 14(6)$$

$$A = 84$$ Multiply.

The area is 84 cm^2

To find the area of a square, multiply the side times itself.



$$\text{Area} = \text{side} \times \text{side}$$

$$\text{Area} = (\text{side})^2$$

$$A = s^2$$

Example 3: What is the area of a square having a side of 7 m?

$$A = s^2$$ s^2 is read "s squared."

$$A = 7^2$$ 7^2 is read "7 squared."

$$A = 49$$ $7^2 = 7 \times 7 = 49$

The area is 49 m^2.

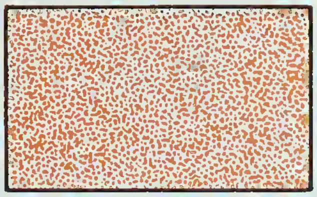
Exercises

1. Name three common units of area. Typical answers: square millimetre, square centimetre, square metre
2. What is the formula for the area of a rectangle? What does each letter stand for? $A = \ell w$, area, length, width
3. What is the formula for the area of a square? $A = s^2$
4. If the lenth and width of a rectangle are 9 cm and 6 cm, the area is 54 $\underline{\text{cm}^2}$.
5. If the length and width of a rectangle are 9 m and 6 m, the area is 54 $\underline{\text{m}^2}$.

6. If the side of a square is 4m, its area is 16 m^2.

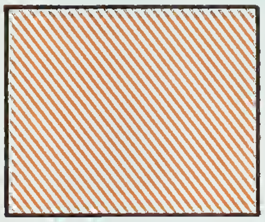
7–12. Find the area of each rectangle.

7.



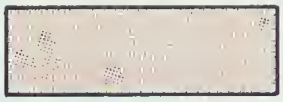
60 cm^2

8.



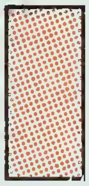
56 m^2

9.



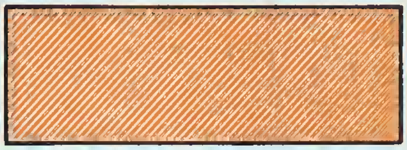
27 cm^2

10.



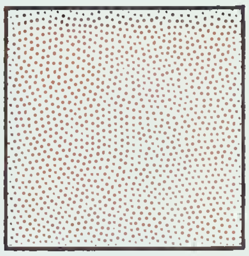
36 m^2

11.



140 m^2

12.



64 mm^2

13–30. Find the area of each rectangle having the given length and width.

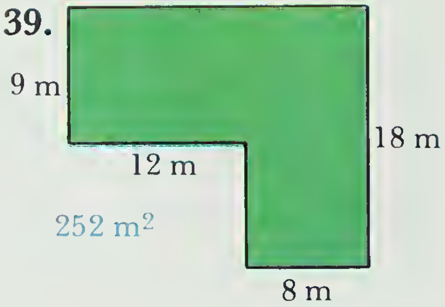
13. 16 m, 15 m 240 m^2
14. 25 cm, 13 cm 325 cm^2
15. 14 m, 18 m 252 m^2
16. 17 mm, 36 mm 612 mm^2
17. 783 m, 125 m 97 875 m^2
18. 155 cm, 397 cm 61 535 cm^2
19. 1.5 km, 2.3 km 3.45 km^2
20. 3.2 m, 6.3 m 20.16 m^2
21. $1\frac{1}{2}$ m, $3\frac{1}{2}$ m $5\frac{1}{4}$ m^2
22. $7\frac{1}{4}$ cm, $2\frac{1}{2}$ cm $18\frac{1}{8}$ cm^2
23. $3\frac{1}{4}$ cm, $2\frac{1}{4}$ cm $7\frac{5}{16}$ cm^2
24. $8\frac{3}{4}$ cm, $6\frac{1}{4}$ cm $54\frac{11}{16}$ cm^2
25. 2.25 km, 3.5 km 7.875 km^2
26. 6.39 m, 7.85 m 50.1615 m^2
27. 75.4 km, 25.3 km 1907.62 km^2
28. 93.9 cm, 79.9 cm 7502.61 cm^2
29. 9.875 m, 1.5 m 14.8125 m^2
30. 4.3 cm, 0.055 cm 0.2365 cm^2

31–38. Find the area of each square having the given side.

31. 28 m 784 m^2
32. 87 mm 7569 mm^2
33. $2\frac{1}{2}$ m $6\frac{1}{4}$ m^2
34. 1.25 cm 1.625 cm^2
35. 7.2 cm 51.84 cm^2
36. 1.8 mm 3.24 mm^2
37. 2.25 m 5.0625 m^2
38. 8.43 km 71.0649 km^2

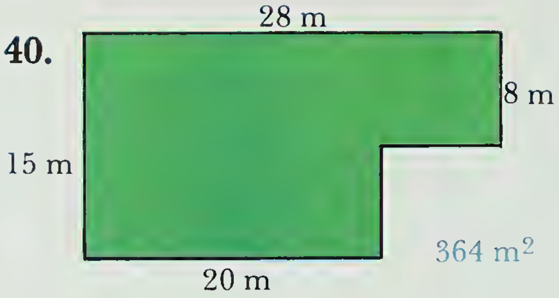
39—40. Find the area of each figure in square metres. HINT: Each figure can be divided into two rectangles.

39.



252 m²

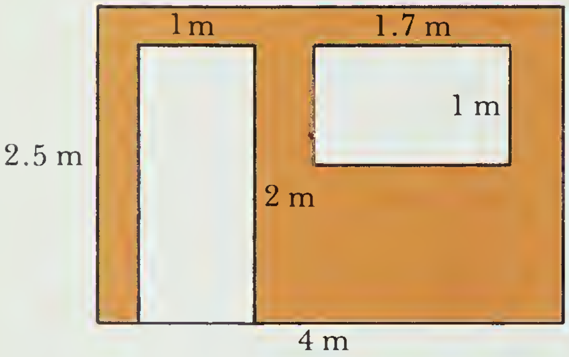
40.



364 m²

41–42. Assume each figure in Exercises **39–40** is the floor plan of a room. The floor is to be covered with tiles that cost \$.99/m². How much will it cost to tile each room? 41. \$249.48 42. \$360.36

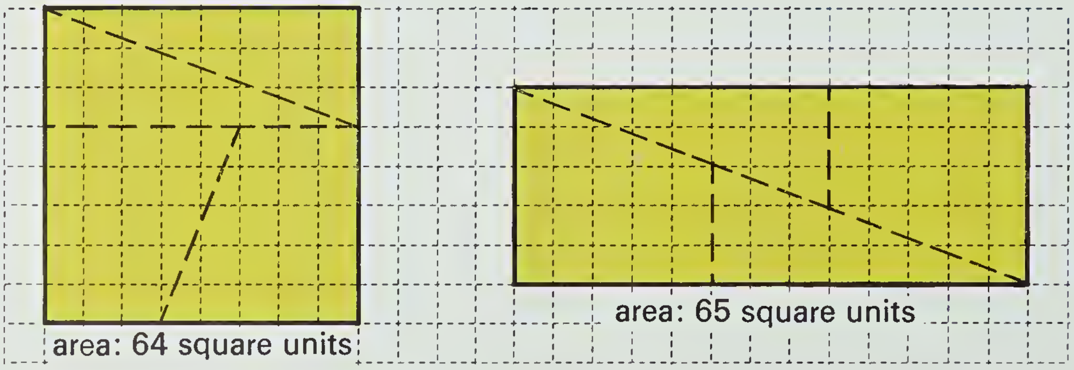
43. A gymnasium floor is to be covered with a sealer that covers 10 m²/L. If the floor is 24 m by 38 m, how many litres are needed? 92



44. The drawing at the left shows a wall with a door and a window. What is the area of the window? What is the area of the door? If the wall is to be painted, how many square metres of surface need to be covered? (The door and window won't be painted.) 1.7 m², 2 m², 6.3 m²

The Extra Square Unit

If you cut the square below along the dashed lines, the parts can be rearranged to form the rectangle on the right.



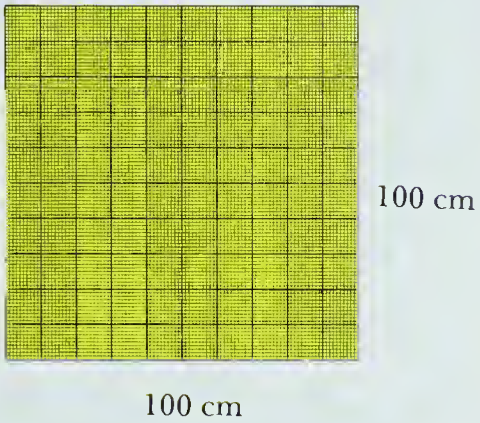
The rectangle is made from exactly the same pieces as the square. The areas should be the same. Why are the two areas different? When the pieces are rearranged, there is a narrow gap along the diagonal line that has an area of 1 square unit.

Performance Objective: The student can change a given area to an equivalent area in the same system.

Equivalent Areas

6.5

Let the drawing below stand for one square metre.



Because the square metre is 100 cm long by 100 cm wide, its area is 10 000 cm². Therefore, 10 000 cm² and 1 m² are **equivalent areas.**

10 000 cm² in 1 m²

Courtesy Armstrong Cork Company

Example 1: Change 25 000 cm² to square metres.

10 000 cm² in 1 m²

25 000 cm² in n m²

Let n stand for the number of square metres.

$$\frac{10\ 000}{25\ 000} = \frac{1}{n}$$

$$10\ 000 \times n = 25\ 000 \times 1$$

$$10\ 000\,n = 25\ 000$$

$$\frac{10\ 000n}{10\ 000} = \frac{25\ 000}{10\ 000}$$

$$n = 2.5$$

There are 2.5 m² in 25 000 cm²

Example 2: Change 4 m² to square centimetres.

10 000 cm² in 1 m²

n cm² in 4m²

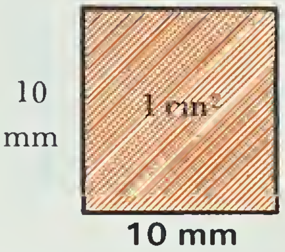
$$\frac{10\ 000}{n} = \frac{1}{4}$$

$$10\ 000 \times 4 = 1 \times n$$

$$40\ 000 = n$$

There are 40 000 cm² in 4 m².

Sometimes you need to change an area from cm² to mm². Since 1 cm² has a length of 10 mm and a width of 10 mm, its area is 10 × 10 or 100 mm².



$$A = s^2$$

$$A = 10^2$$

$$A = 100$$

100 mm² in 1 cm²

Exercises

1. 1 m² is equivalent to __10 000__ square centimetres.

2. 1 cm² is equivalent to __100__ square millimetres.

3. Which covers more area—1 m² or 5 000 cm²?

4. Which covers less area — 90 mm² or 1 cm²?

5—24. Change each measurement as follows.

5. 1.5 m² to cm² 15 000 cm²

6. 7.5 m² to cm² 75 000 cm²

7. 45 000 cm² to m² 4.5 m²

8. 25 000 cm² to m² 2.5 m²

9. 400 mm^2 to cm^2 4 cm^2

10. 950 mm^2 to cm^2 9.5 cm^2

11. 6.4 cm^2 to mm^2 640 mm^2

12. 12.2 cm^2 to mm^2 1220 mm^2

13. 120 m^2 to cm^2 1 200 000 cm^2

14. 290 m^2 to cm^2 2 900 000 cm^2

15. 75 cm^2 to mm^2 7500 mm^2

16. 110 cm^2 to mm^2 11 000 mm^2

17. 0.2 m^2 to cm^2 2000 cm^2

18. 0.9 m^2 to cm^2 9000 cm^2

19. 0.45 cm^2 to mm^2 45 mm^2

20. 0.58 cm^2 to mm^2 58 mm^2

21. 1 000 000 cm^2 to m^2 100 m^2

22. 2 300 000 cm^2 to m^2 230 m^2

23. 963 000 cm^2 to m^2 96 m^2

24. 764 000 cm^2 to m^2 76.4 m^2

25. A room is 8 m long by 4 m wide. What is its floor area in square metres? How much would it cost to carpet the room with carpeting that costs \$9.95/m^2? 32 m^2, \$318.40

26. A tabletop is 1 m long by 1.5 m wide. What is its area in square metres? In square centimetres? How much would it cost to tile the tabletop with ceramic tiles that sell for \$0.04/10 cm^2? 1.5 m^2, 15 000 cm^2, \$60.00

27. How many square centimetres are in 1 m^2? 10 000

28. How many square metres are in 1 km^2? 1 000 000

29. How many square millimetres are in 1 m^2? 1 000 000

30—35. There are 10 000 m^2 in a hectare. For each field whose length and width are given, find the area in square metres. Then tell if the field is larger or smaller than a hectare.

30. 300 m, 180 m 54 000 m^2, larger

31. 40 m, 125 m 5000 m^2, smaller

32. 279 m, 198 m 55 242 m^2 larger

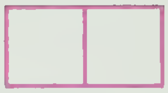
33. 319 m, 124 m 39 556 m^2, larger

34. 460 m, 32 m 13 120 m^2, larger

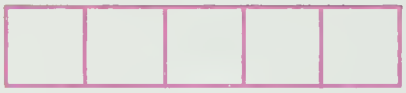
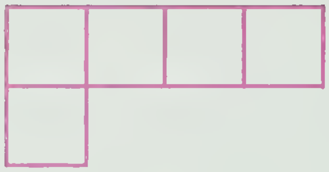
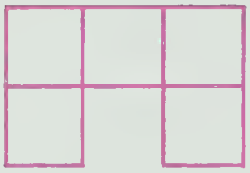
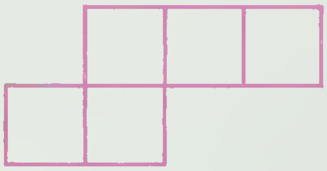
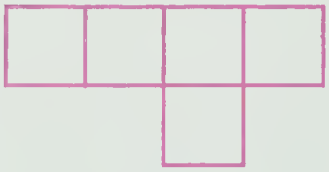
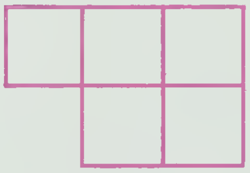
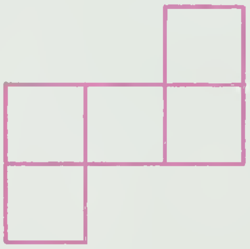
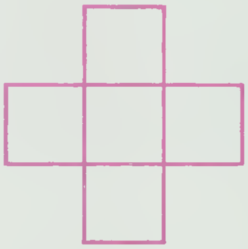
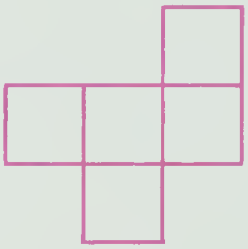
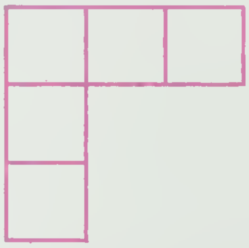
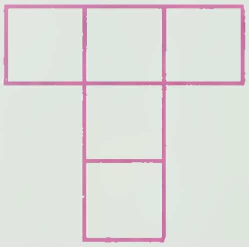
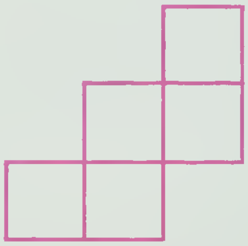
35. 125 m, 75 m 9375 m^2, smaller

PATTERNS WITH SQUARES

Dominoes are made of two squares and have only one arrangement.



Pentominoes have five squares and twelve arrangements.

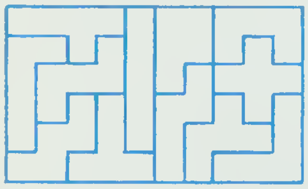
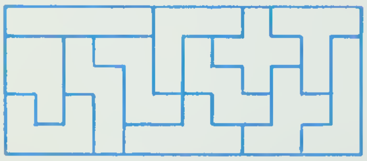
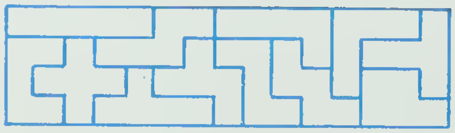
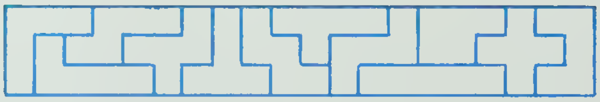













1. Use 1 cm squares to draw twelve pentominoes like those above. Then cut them out.

2. The twelve pieces can be fit together to form a 6 × 10 rectangle. Try it.

3. It is also possible to form 5 × 12, 4 × 15, and 3 × 20 rectangles with pentomino pieces. Try to form each one. (The 3 × 20 is *very difficult* since there are only two possible ways to do it.)

2–3: *Typical answers*

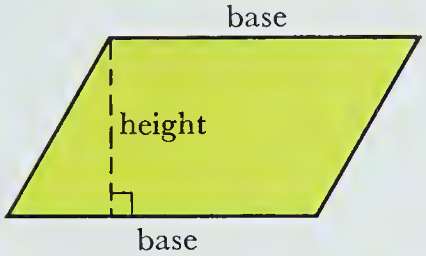





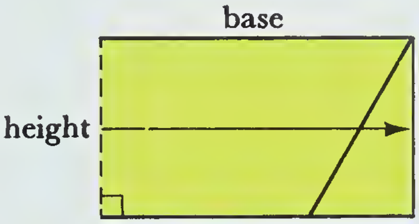
Parallelograms

6.6

In the parallelogram at the left below, two of the opposite sides are labeled **base**. The distance between the bases is called the **height**.

Performance Objective: The student can find the areas of parallelograms.





Suppose you want to find the area of the parallelogram. Think of cutting along the dashed line. If the resulting triangle is slid as shown above, a rectangle is formed.

For the rectangle that is formed, the length is labeled *base* and the width is labeled *height*. Therefore,

$$\text{Area} = \text{base} \times \text{height}.$$

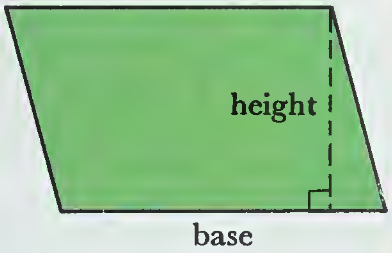
Any parallelogram can be made into a rectangle as above. If we let b stand for the base and h for height, we have

$$A = bh.$$

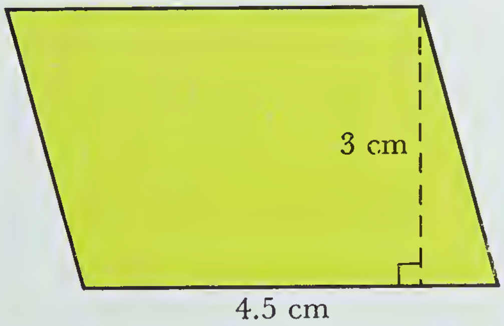
To find the area of a parallelogram, multiply the base by the height.

$$\text{Area} = \text{base} \times \text{height}$$

$$A = bh$$



Example: What is the area of the parallelogram below?



$$A = bh$$

$$A = 4.5(3)$$

$$A = 13.5$$

The area is 13.5 cm^2.

Exercises

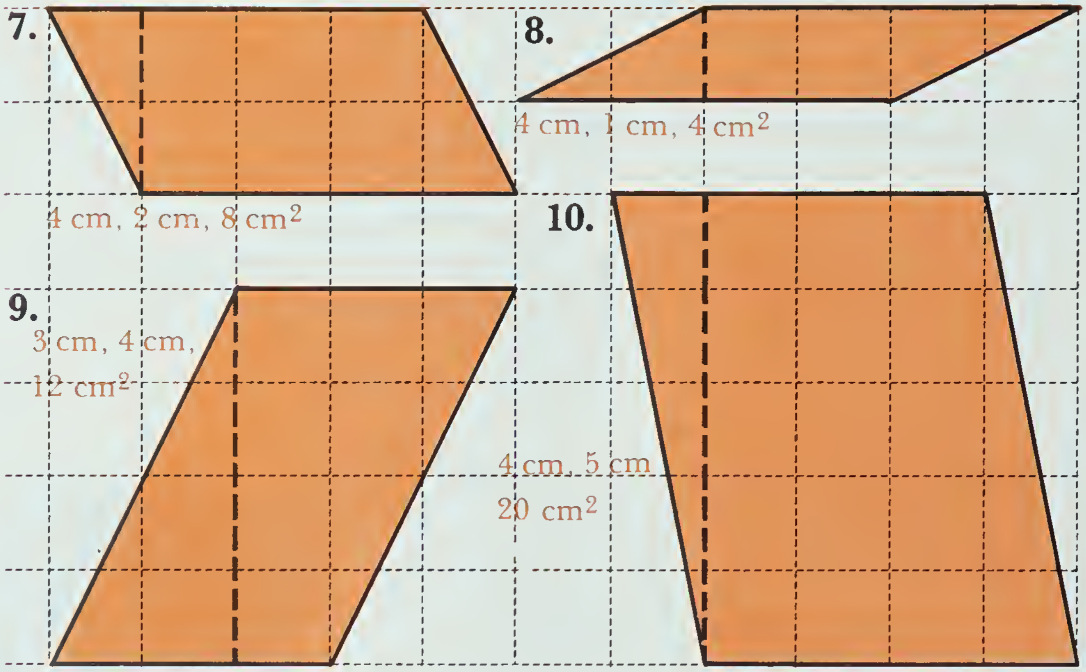
1. The distance between the bases of a parallelogram is called the Height.

2. What is the formula for the area of a parallelogram? What does each letter stand for? $A = bh$, area, base, height

3–6. What is the area of each parallelogram with the given base and height?

3. 7 m, 9 m 63 m²
4. 4 cm, 7 cm 28 cm²
5. 10 mm, 8 mm 80 mm²
6. 6 m, 8 m 48 m²

7–10. Give the base and height for each parallelogram. Then find the area. Each square is 1 square centimetre.



7. 4 cm, 2 cm, 8 cm²

8. 4 cm, 1 cm, 4 cm²

9. 3 cm, 4 cm, 12 cm²

10. 4 cm, 5 cm 20 cm²

11–14. Trace each parallelogram in Exercises **7–10** on another sheet of paper. Cut each parallelogram along the dashed line. Then rearrange the parts to form a rectangle. 11–14. *Student work*

15–26. Find the area of each parallelogram with the given base and height.

15. $4\frac{1}{4}$ cm, 8 cm 34 cm²
16. $1\frac{1}{2}$ km, 5 km $7\frac{1}{2}$ km²
17. $2\frac{1}{3}$ m, $1\frac{1}{2}$ m $3\frac{1}{2}$ m²
18. $9\frac{1}{2}$ mm, $5\frac{1}{8}$ mm $48\frac{11}{16}$ mm²

19. 1.5 cm, 3.5 cm 5.25 cm^2

20. 8.2 m, 9.3 m 76.26 m^2

21. 8.53 km, 1.73 km 14.7569 km^2

22. 6.33 m, 9.35 m 59.1855 m^2

23. 7.25 m, 3.4 m 24.65 m^2

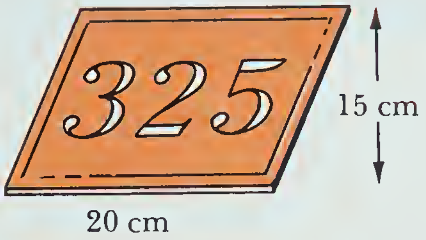
24. 5.47 km, 7.85 km 42.9395 km^2

25. 788 dm, 567 dm 446 796 dm^2

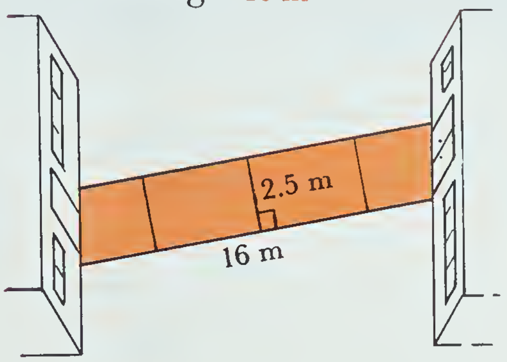
26. 885 mm, 520 mm 460 200 mm^2

27–30. Find each area.

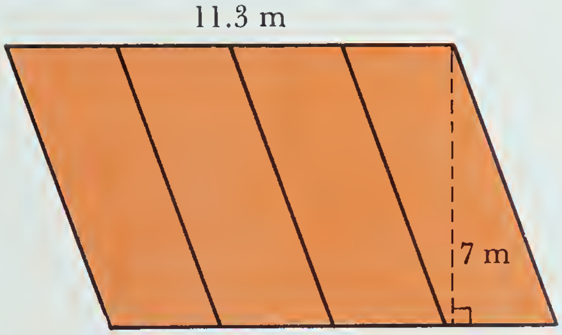
27. an address plate 300 cm^2



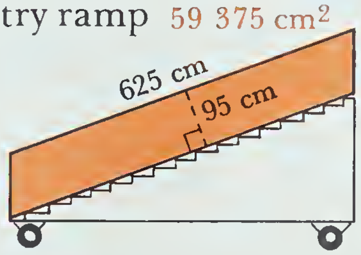
28. a sidewalk between two buildings 40 m^2



29. a parking lot 79.1 m^2



30. the side of an airplane entry ramp 59 375 cm^2



1–6. Multiply.

1. $\frac{1}{2}(8)(10)$ 40

2. $\frac{1}{2}(11)(5)$ $27\frac{1}{2}$

3. $\frac{1}{2}(80)(58)$ 2320

4. $\frac{1}{2}(6.8)(1.2)$ 4.08

5. $\frac{1}{2}(3\frac{1}{2})(4)$ 7

6. $\frac{1}{2}(10)(2\frac{1}{4})$ $11\frac{1}{4}$

7–9. Find the answer.

7. $\frac{1}{2}(6)(7 + 11)$ 54

8. $\frac{1}{2}(9)(8 + 12)$ 90

9. $\frac{1}{2}(14)(28 + 36)$ 448

Quick Quiz

Check these skills:

- *finding the product of $\frac{1}{2}$ and two other numbers*
- *doing multiple operations indicated with parentheses*

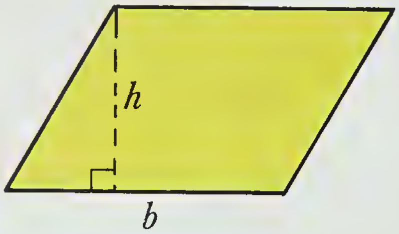
6.7 Triangles

Performance Objective: The student can find the areas of triangles.

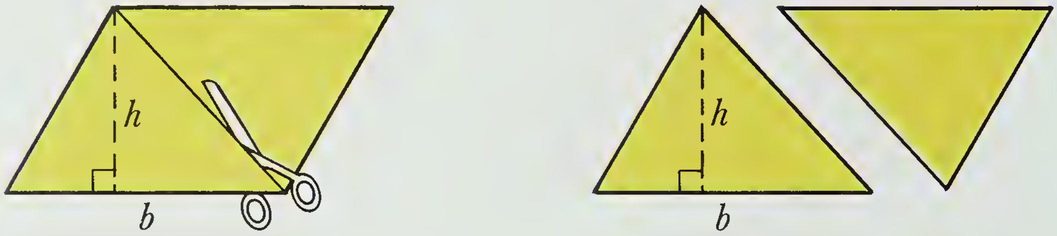
Artstreet

Artstreet

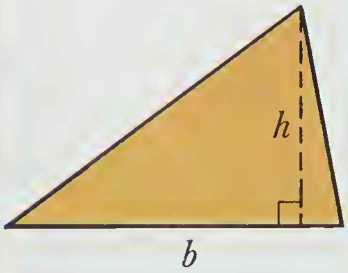
To find the area of a parallelogram, multiply the base by the height. The formula is $A = bh$.



Suppose you have to find the area of a triangle. Think of cutting a parallelogram in half.



Two triangles of the same size are formed. Each is **half** of the parallelogram. The formula for the area of the parallelogram is $A = bh$. So the formula for the area of one of the triangles is $A = \frac{1}{2}bh$.

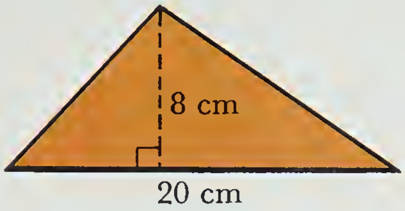


To find the area of a triangle, multiply $\frac{1}{2}$ times the base times the height.

$$\text{Area} = \tfrac{1}{2} \times \text{base} \times \text{height}$$

$$A = \tfrac{1}{2}bh$$

Example: What is the area of the triangle shown?



$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2}(20)(8)$$
$$A = \tfrac{20}{2}(8)$$
$$A = (10)(8)$$
$$A = 80$$

The area is 80 cm^2.

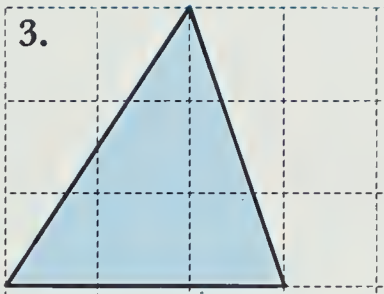
Exercises

1. What is the formula for the area of a triangle? What does each letter stand for? $A = \frac{1}{2}bh$, area, base, height

2. Draw a triangle. Then trace it on another sheet of paper. Cut out both triangles and put them together to form a parallelogram. *Student work*

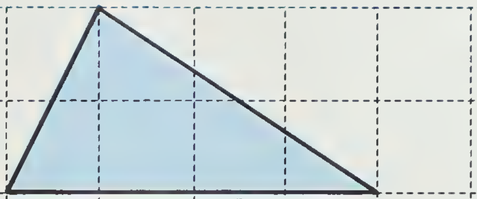
3–8. Give the base and height of each triangle. Then find the area. Each square is one square centimetre.

3.



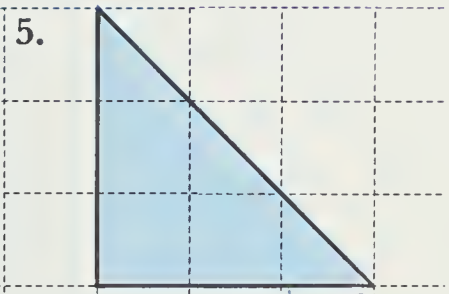
3 cm, 3 cm, $4\frac{1}{2}$ cm^2

4.



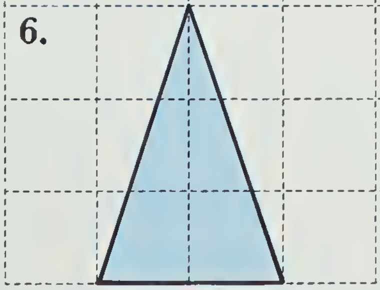
4 cm, 2 cm, 4 cm^2

5.



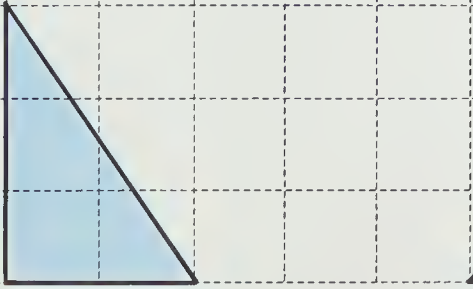
3 cm, 3 cm, $4\frac{1}{2}$ cm^2

6.



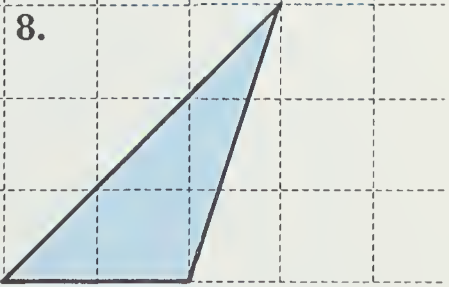
2 cm, 3 cm, 3 cm^2

7.



2 cm, 3 cm, 3 cm^2

8.



2 cm, 3 cm, 3 cm^2

9–24. Find the area of each triangle with the given base and height.

9. 28 cm, 18 cm 252 cm²

10. 74 cm, 62 cm 2294 cm²

11. 37 dm, 79 dm 1461½ dm²

12. 93 mm, 76 mm 3534 mm²

13. 1.5 cm, 10 cm 7.5 cm²

14. 20 m, $3\frac{1}{2}$ m 35 m²

15. 2.5 m, 2.5 m 3.125 m²

16. $1\frac{1}{4}$ cm, $1\frac{1}{4}$ cm $\frac{25}{32}$ cm²

17. 6.8 cm, 30 cm 102 cm²

18. 8.5 m, 42 m 178.5 m²

19. 8.3 m, 7.5 m 31.125 m²

20. 1.2 km, 3.6 km 2.16 km²

21. 3.3 dm, 6.5 dm 10.725 dm²

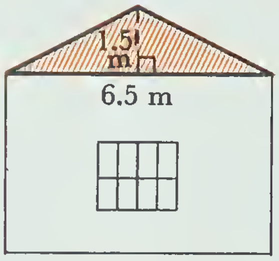
22. $2\frac{1}{2}$ m, $8\frac{1}{2}$ m $10\frac{5}{8}$ m²

23. 7.38 km, 5.28 km 19.4832 km²

24. 5.25 m, 3.70 m 9.7125 m²

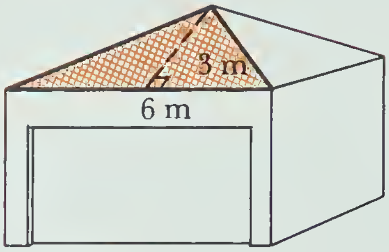
25–30. Find each area shown in orange.

25.



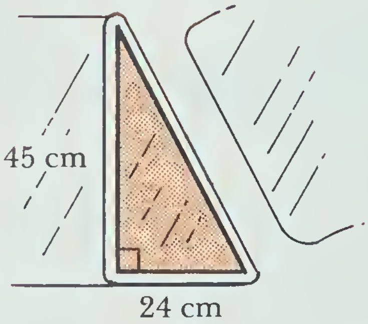
4.875 m²

26.



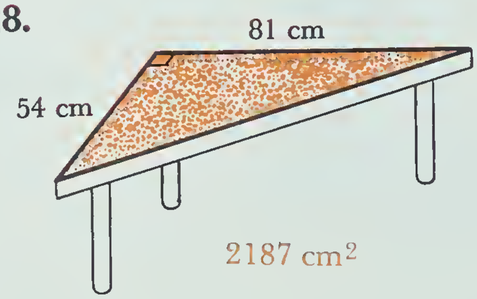
9 m²

27.



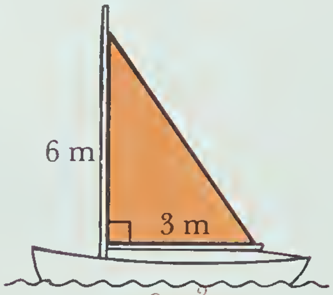
540 cm²

28.



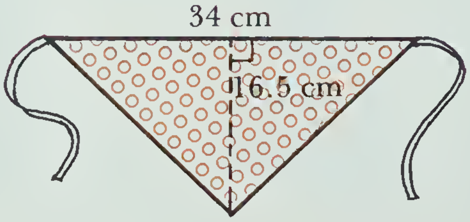
2187 cm²

29.



9 m²

30.

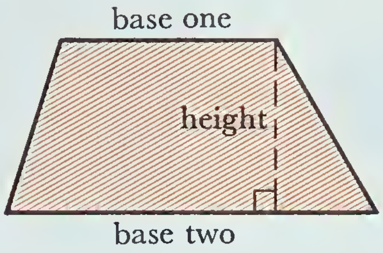


280.5 cm²

Trapezoids 6.8

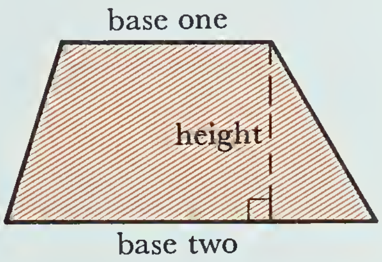
Performance Objective: The student can find the areas of trapezoids.

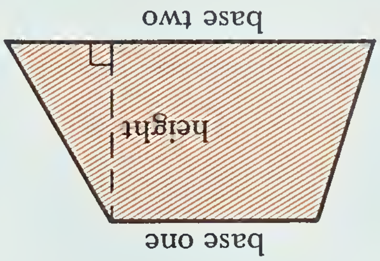
The sides of a trapezoid that would never meet are called **bases**.


base one
height
base two

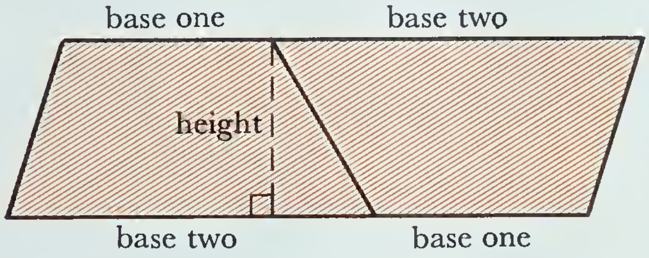
The **height** of a trapezoid is the distance between the bases. Suppose you have to find the area of this trapezoid.

Think of another trapezoid the same size, but turned upside down.


base one
height
base two


base two
height
base one

If the trapezoids are put together, a parallelogram is formed.


base one
base two
height
base two
base one

The base of this parallelogram is (base one + base two). So

Area of parallelogram = height × (base one + base two).

But we want the area of the trapezoid that is only **half** the area of the parallelogram. Therefore,

Area of trapezoid = $\frac{1}{2}$ × height × (base one + base two).

More briefly,

$$A = \tfrac{1}{2}h(b_1 + b_2)$$

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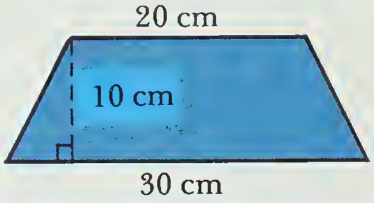
Example: What is the area of the trapezoid?



$$A = \tfrac{1}{2}h(b_1 + b_2)$$
$$A = \tfrac{1}{2}(10)(20 + 30)$$
$$A = \tfrac{1}{2}(10)(50)$$
$$A = \tfrac{10}{2}(50)$$
$$A = 5(50)$$
$$A = 250$$

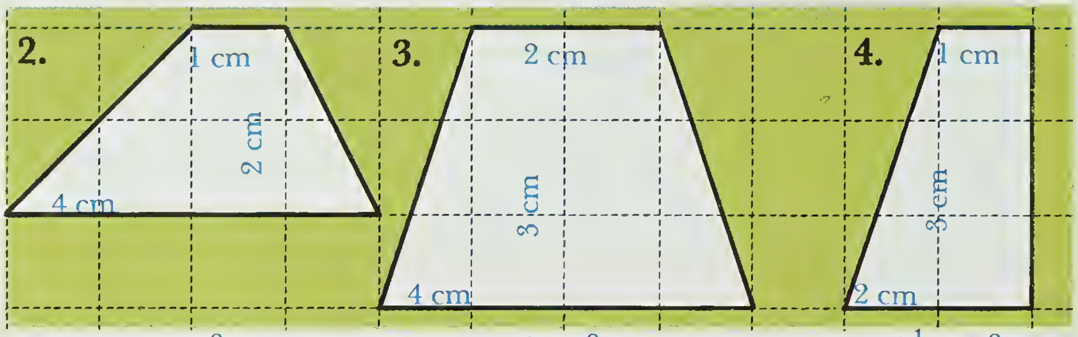
The area is 250 cm^2.

Exercises

1. What is the formula for the area of a trapezoid? What does each letter stand for?

$A = \frac{1}{2}h(b_1 + b_2)$, area, height, base one, base two

2—4. Give the height and the bases for each trapezoid. Then find the area. Each square is 1 square centimetre.



5 cm^2 9 cm^2 $4\frac{1}{2}$ cm^2

5—12. Find the area of each trapezoid.

5. $h = 16$ cm, $b_1 = 10$ cm, $b_2 = 20$ cm 240 cm^2

6. $h = 50$ m, $b_1 = 60$ m, $b_2 = 40$ m 2500 m^2

7. $h = 150$ dm, $b_1 = 315$ dm, $b_2 = 471$ dm 58 950 dm^2

8. $h = 95$ m, $b_1 = 176$ m, $b_2 = 315$ m 23 322.5 m^2

9. $h = 16$ mm, $b_1 = 6.5$ mm, $b_2 = 7.5$ mm 112 mm^2

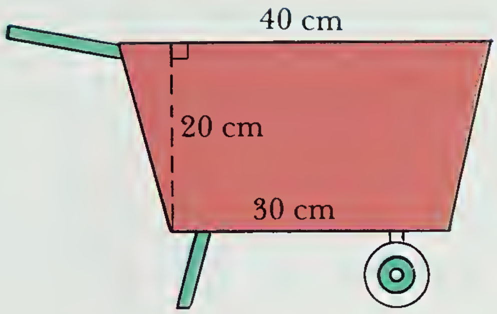
10. $h = 8$ km, $b_1 = 3.34$ km, $b_2 = 4\frac{1}{4}$ km 32 km^2

11. $h = 6$ m, $b_1 = 8.5$ m, $b_2 = 6.3$ m 44.4 m^2

12. $h = 4$ cm, $b_1 = 3.6$ cm, $b_2 = 4.6$ cm 16.4 cm^2

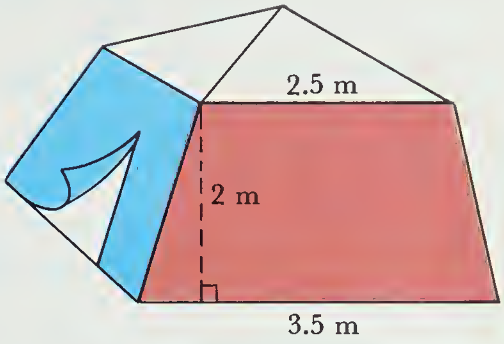
13–18. Find each area shown in red.

13. 700 cm²

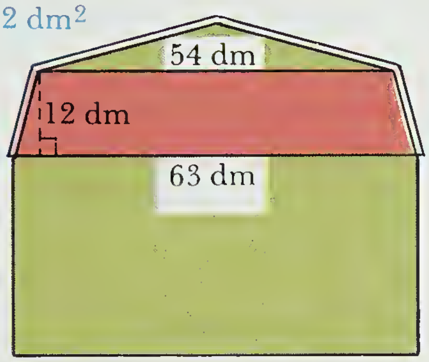


40 cm
20 cm
30 cm

14. 6 m²

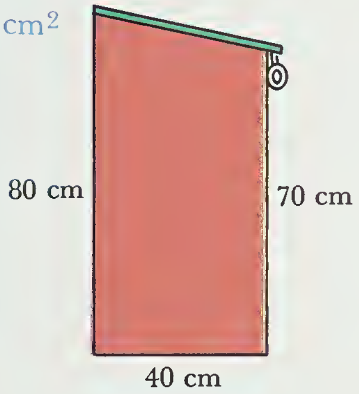


15. 702 dm²



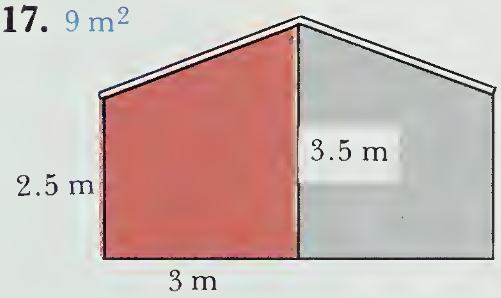
54 dm
12 dm
63 dm

16. 3000 cm²

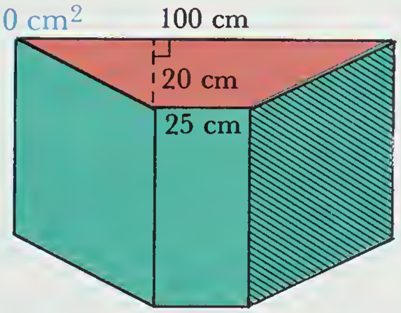


80 cm
70 cm
40 cm

17. 9 m²



18. 1250 cm²



100 cm
20 cm
25 cm

1–6. Find the answer.

1. 3^2 9

2. 5^2 25

3. 10^2 100

4. 7^2 49

5. 4^2 16

6. 6^2 36

Quick Quiz

Check this skill:

- *squaring a whole number*

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CAREER CORNER

AREA OR PERIMETER?

Area and perimeter are two ideas that are often confused. Yet people who work in many different jobs use area and perimeter.

For each problem below, tell whether you need to find the area or the perimeter. Then work the problem.

1. *Carpenter:* How much baseboard is needed for a room that is 6 m long by 4 m wide? (Neglect the amount wasted for doorways.) Perimeter, 20 m
2. *Tilesetter:* How many 1 dm square tiles are needed to tile a floor that is 16 dm long by 13 dm wide? Area, 208 dm^2
3. *Farmer:* How much fencing is needed to enclose a field that is 80 m long by 50 m wide? Perimeter, 260 m
4. *Painter:* How much paint is needed to cover a solid wall that is 2.5 m high and 4 m long? A litre covers about 10 m^2 Area, 1 L
5. *Artist:* How much wood framing 5 cm wide is needed to frame a canvas that is 82 cm long by 63 cm wide? Perimeter, 330 cm

Grant Heilman

Shackman/Monkmeyer Press Photo Servi

Square Roots

6.9

If the area of a square is known, then the length of its sides can be found. Suppose a square has an area of 36 cm^2.

Performance Objective: The student can find the square roots of perfect squares and can use the table of square roots.

Since $6^2 = 36$, the side of the square is 6 cm.

6 is called the **square root** of 36.

(square root of a number) × (square root of a number) = the number

For example, 2 is the square root of 4 because $2 \times 2 = 4$.

The idea *square root* is represented by the symbol $\sqrt{\ }$.

$$\begin{array}{rll} \sqrt{9} = 3 & & \text{because } 3 \times 3 = 9. \\ \sqrt{16} = 4 & & \text{because } 4 \times 4 = 16. \\ \sqrt{1} = 1 & & \text{because } 1 \times 1 = 1. \\ \sqrt{25} = 5 & & \text{because } 5 \times 5 = 25. \\ \sqrt{64} = 8 & & \text{because } 8 \times 8 = 64. \\ \sqrt{10} = ? & & \end{array}$$

The last number, 10, does not have a whole number square root. The other numbers are called *perfect squares* because their square roots are whole numbers.

Notice the list of square roots below.

$$\begin{array}{l} \sqrt{9} = 3 \\ \left.\begin{array}{l} \sqrt{10} \\ \sqrt{11} \\ \sqrt{12} \\ \sqrt{13} \\ \sqrt{14} \\ \sqrt{15} \end{array}\right\} \quad \text{These square roots are all between 3 and 4.} \\ \sqrt{16} = 4 \end{array}$$

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When a number is a perfect square, you can often find the square root mentally. For numbers like $\sqrt{10}$, though, you would have to use a table of square roots. Such a table is listed on page 272.

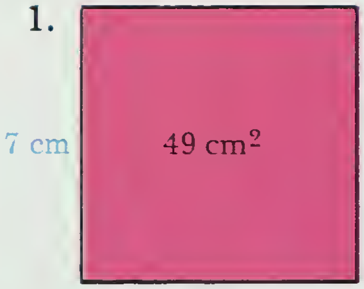
If you look up $\sqrt{10}$, you find 3.162. Since 10 is not a perfect square number, 3.162 is not an exact square root. Actually,

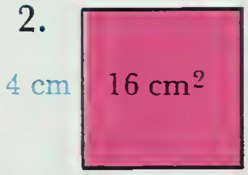
$$3.162 \times 3.162 = \underbrace{9.998\,244}_{\text{This number is very close to 10.}}$$

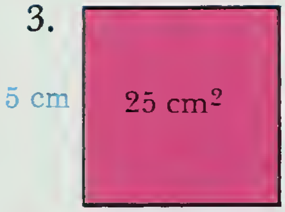
The table on page 272 also lists the squares of numbers from 1 through 150. Remember that n^2 means $n \times n$.

Exercises

1–3. For each square whose area is shown, find the length of each side.

1.


2.


3.


4. Which numbers are perfect squares?

10 $\underline{16}$ 20 $\underline{25}$ 30 $\underline{36}$

5. Which numbers are whole numbers?

$\sqrt{10}$ $\underline{\sqrt{16}}$ $\sqrt{20}$ $\underline{\sqrt{25}}$ $\sqrt{30}$ $\underline{\sqrt{36}}$

6. 9^2 means $\underline{\quad 9 \quad}$ × $\underline{\quad 9 \quad}$.

7. Since $11 \times 11 = 121$, $\sqrt{121} = \underline{\quad 11 \quad}$.

8. Since $12 \times 12 = 144$, $\sqrt{144} = \underline{\quad 12 \quad}$.

9. $\sqrt{25} = 5$ and $\sqrt{36} = 6$, so $\sqrt{30}$ is between (25 and 36, $\underline{5 \text{ and } 6}$).

10. $\sqrt{49} = 7$ and $\sqrt{64} = 8$, so $\sqrt{53}$ is between (49 and 64, $\underline{7 \text{ and } 8}$).

11–22. Use the table of square roots on page 272 to find each square root.

11. $\sqrt{5}$ 2.236 **12.** $\sqrt{30}$ 5.477 **13.** $\sqrt{10}$ 3.162

14. $\sqrt{27}$ 5.196 **15.** $\sqrt{7}$ 2.646 **16.** $\sqrt{48}$ 6.928

17. $\sqrt{80}$ 8.944 **18.** $\sqrt{60}$ 7.746 **19.** $\sqrt{41}$ 6.403

20. $\sqrt{3}$ 1.732 **21.** $\sqrt{17}$ 4.123 **22.** $\sqrt{37}$ 6.083

23–28. Find each number by using the table on page 272.

23. 34^2 1156 **24.** 19^2 361 **25.** 42^2 1764

26. 80^2 6400 **27.** 73^2 5329 **28.** 99^2 9801

29. You have 144 one-decimetre-square concrete patio stones. How large a square patio can you make? 12 dm by 12 dm

30. You want a square garden of 196 m^2. How large a square should it be? 14 m by 14 m

31. A solar heating panel has to have a 49 m^2 panel exposed to the sunlight. It will be square in shape. How large a square should it be? 7 m by 7 m

SQUARES AND SQUARE ROOTS*

x	x^2	$\sqrt{x}$	x	x^2	$\sqrt{x}$	x	x^2	$\sqrt{x}$
1	1	1.000	**51**	2601	7.141	**101**	10 201	10.050
2	4	1.414	**52**	2704	7.211	**102**	10 404	10.100
3	9	1.732	**53**	2809	7.280	**103**	10 609	10.149
4	16	2.000	**54**	2916	7.348	**104**	10 816	10.198
5	25	2.236	**55**	3025	7.416	**105**	11 025	10.247
6	36	2.449	**56**	3136	7.483	**106**	11 236	10.296
7	49	2.646	**57**	3249	7.550	**107**	11 449	10.344
8	64	2.828	**58**	3364	7.616	**108**	11 664	10.392
9	81	3.000	**59**	3481	7.681	**109**	11 881	10.440
10	100	3.162	**60**	3600	7.746	**110**	12 100	10.488
11	121	3.317	**61**	3721	7.810	**111**	12 321	10.536
12	144	3.464	**62**	3844	7.874	**112**	12 544	10.583
13	169	3.606	**63**	3969	7.937	**113**	12 769	10.630
14	196	3.742	**64**	4096	8.000	**114**	12 996	10.677
15	225	3.873	**65**	4225	8.062	**115**	13 225	10.724
16	256	4.000	**66**	4356	8.124	**116**	13 456	10.770
17	289	4.123	**67**	4489	8.185	**117**	13 689	10.817
18	324	4.243	**68**	4624	8.246	**118**	13 924	10.863
19	361	4.359	**69**	4761	8.307	**119**	14 161	10.909
20	400	4.472	**70**	4900	8.367	**120**	14 400	10.954
21	441	4.583	**71**	5041	8.426	**121**	14 641	11.000
22	484	4.690	**72**	5184	8.485	**122**	14 884	11.045
23	529	4.796	**73**	5329	8.544	**123**	15 129	11.091
24	576	4.899	**74**	5476	8.602	**124**	15 376	11.136
25	625	5.000	**75**	5625	8.660	**125**	15 625	11.180
26	676	5.099	**76**	5776	8.718	**126**	15 876	11.225
27	729	5.196	**77**	5929	8.775	**127**	16 129	11.269
28	784	5.292	**78**	6084	8.832	**128**	16 384	11.314
29	841	5.385	**79**	6241	8.888	**129**	16 641	11.358
30	900	5.477	**80**	6400	8.944	**130**	16 900	11.402
31	961	5.568	**81**	6561	9.000	**131**	17 161	11.446
32	1024	5.657	**82**	6724	9.055	**132**	17 424	11.489
33	1089	5.745	**83**	6889	9.110	**133**	17 689	11.533
34	1156	5.831	**84**	7056	9.165	**134**	17 956	11.576
35	1225	5.916	**85**	7225	9.220	**135**	18 225	11.619
36	1296	6.000	**86**	7396	9.274	**136**	18 496	11.662
37	1369	6.083	**87**	7569	9.327	**137**	18 769	11.705
38	1444	6.164	**88**	7744	9.381	**138**	19 044	11.747
39	1521	6.245	**89**	7921	9.434	**139**	19 321	11.790
40	1600	6.325	**90**	8100	9.487	**140**	19 600	11.832
41	1681	6.403	**91**	8281	9.539	**141**	19 881	11.874
42	1764	6.481	**92**	8464	9.592	**142**	20 164	11.916
43	1849	6.557	**93**	8649	9.644	**143**	20 449	11.958
44	1936	6.633	**94**	8836	9.695	**144**	20 736	12.000
45	2025	6.708	**95**	9025	9.747	**145**	21 025	12.042
46	2116	6.782	**96**	9216	9.798	**146**	21 316	12.083
47	2209	6.856	**97**	9409	9.849	**147**	21 609	12.124
48	2304	6.928	**98**	9604	9.899	**148**	21 904	12.166
49	2401	7.000	**99**	9801	9.950	**149**	22 201	12.207
50	2500	7.071	**100**	10 000	10.000	**150**	22 500	12.247

* Square roots are given to nearest thousandth.

CONSUMER CORNER

3 ROOMS OF CARPETING ONLY $199

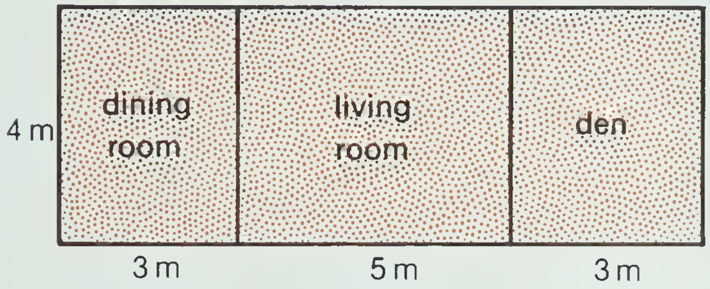
SPECIAL SALE

Maximum: 25 m^2

A BARGAIN?

Advertisements are sometimes misleading. The carpet advertisement says, "3 rooms of carpeting for $199." Notice, however, that the small print limits the amount of carpeting at this price to 25 m^2.

Suppose a family wants to carpet the three rooms shown below.



dining room:	12 m^2
living room:	20 m^2
den:	12 m^2
	44 m^2

So 25 m^2 is not enugh to carpet the three rooms shown. The amount really needed would cost more than $199. What other advertisements have you seen recently that place a maximum on the quantity you can purchase?

Activities

1–4. *Answers will vary.*

1. Measure your classroom. Find out how many square metres of carpeting are needed to cover the floor.
2. Repeat Exercise 1 for your room at home.
3. Measure any 3 rooms at home. Find out if you could carpet all 3 rooms with 25 m^2, as in the advertisement above.
4. Look in the newspaper or in a catalog and find the prices of two different kinds of carpeting. Then find how much it would cost to carpet your room with the one you like best.

A NUMBER CALLED PI

Circles are common figures in everyday life. You see circles in wheels, jar lids, plates, and many other objects.

Photo Research International

Robert Brown

Hamilton/De Wys, Inc.

As an activity, find three circular objects of different sizes. Mark a point on the outer edge of each one. Then roll each circle along a metre-stick for one complete turn. This distance that a circle travels in one complete turn is called its **circumference.** Record the circumference of each circular object to the nearest millimetre.

Next measure the **diameter** of each object to the nearest millimetre. The diameter is the distance across a circle (through its center).

Make a table like the following.
Answers will vary.

	circumference	diameter	circumference ÷ diameter
first object			
second object			
third object			

In the last column, you should get about the same number each time. What is it? 3.14

For any circle, the circumference divided by the diameter is always the same number. This number is approximately 3.14, or $\frac{22}{7}$. However, since this number cannot be represented exactly by a decimal or a fraction, we use the Greek letter π (called pi).

Performance Objective: The student can find the circumferences of circles.

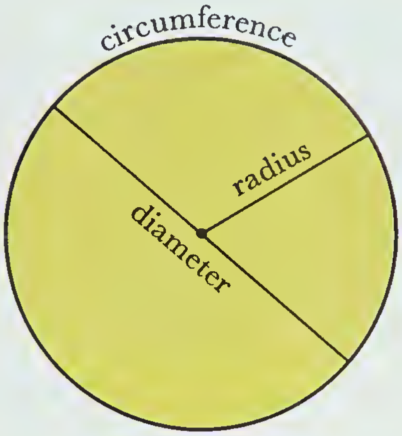
Circumference

6.10

For all circles

circumference ÷ diameter = the same number, π.

the distance around the circle



The number π is close to 3.14. If you let C stand for circumference and d for diameter, then

$$C \div d = \pi \qquad \text{or} \qquad \frac{C}{d} = \pi$$

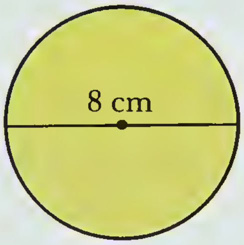
Now multiply both sides by d.

$$\frac{C}{d}(d) = \pi d$$

$$C = \pi d$$

This is a formula you can use to find the circumference of a circle when you know the diameter.

Example 1: Find the circumference of a circle having a diameter of 8 cm. (Use 3.14 for π.)



$$C = \pi d$$

$$C = (3.14)(8)$$

$$C = 25.12$$

The circumference is 25.12 cm.

In a circle, the **radius** is the distance from the center to the outer edge. Notice that the diameter is twice as long as the radius of a circle.

$$\text{diameter} = 2(\text{radius})$$

$$d = 2r$$

Therefore, if you replace d with $2r$ in $C = \pi d$, you get $C = \pi(2r)$ or $C = 2\pi r$. So another formula for finding the circumference of a circle is

$$C = 2\pi r$$

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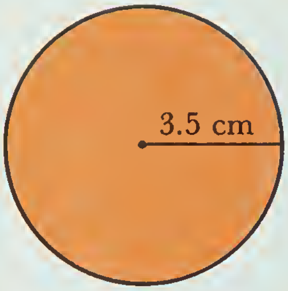
Example 2: Find the circumference of a circle having a radius of 3.5 cm.



$$C = 2\pi r$$

$$C = 2(3.14)(3.5)$$

$$C = 21.98$$

The circumference is 21.98 cm.

Exercises

1. What is the number that results when the circumference of a circle is divided by the diameter? Pi

2. The distance from the center of a circle to the outer edge is called the Radius.

3. For a circle, the diameter is (twice, half) the radius, and the radius is (twice, half) the diameter.

4. What decimal is approximately equal to π? 3.14

5. What fraction is approximately equal to π? $\frac{22}{7}$

6. Give two different formulas for finding the circumference of a circle. $C = \pi d$, $C = 2\pi r$

7–10. For each circle whose radius is given, find the diameter.

7. 18 mm 36 mm
8. 25 m 50 m
9. 52 cm 104 cm
10. 41 cm 82 cm

11–14. For each circle whose diameter is given, find the radius.

11. 28 mm 14 mm
12. 62 cm 31 cm
13. 15.5 m 7.75 m
14. 80 m 40 m

15–22. Find the circumference of each circle having the given diameter. Use 3.14 for π.

15. 6 cm 18.84 cm
16. 15 m 47.10 m
17. 2.8 cm 8.792 cm
18. 5.9 m 18.526 m
19. 3.63 mm 11.3982 mm
20. 7.28 m 22.8592 m
21. 22.1 mm 69.394 mm
22. 38.5 cm 120.89 cm

23–30. Find the circumference of each circle having the given radius. Use $\frac{22}{7}$ for π.

23. 14 m 88 m
24. 35 mm 220 mm
25. 28 mm 176 mm
26. 70 m 440 m
27. 21 m 132 m
28. 63 cm 396 cm
29. 49.7 cm 312.4 cm
30. 19.6 m 123.2 m

31–35. Use 3.14 for π.

31. A wheel has a diameter of 28 cm. How far does the wheel roll in one complete turn? 87.92 cm

32. A circular rug is 2 m in diameter. How many metres of fringe are needed to border it? 6.3 m

33. A circular pool is to be surrounded by a low fence. If the radius of the pool is 2 m, how many feet of fence are needed? 12.6 m

34. The largest four-faced clock in the world has a 6 m long minute hand. How far does the tip of the minute hand travel in one hour (or one complete turn)? 37.7 m

35. The famous Big Ben in London has clock faces that are 7 m across. What is the circumference of each face? 21.98 m

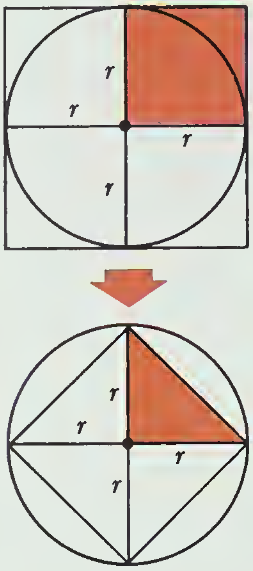
Courtesy Allen-Bradley Company

Courtesy British Tourist Authority

6.11 Area of a Circle

Performance Objective: The student can find the areas of circles.

Suppose you want to find the area of a circle with a radius r.



area of red square $= r^2$

areas of 4 such squares $= 4r^2$

So area of the circle is less than $4r^2$.

area of red triangle $= \frac{1}{2}r^2$

area of 4 such triangles $= 2r^2$

So area of the circle is more than $2r^2$.

Take the average of $2r^2$ and $4r^2$.

$$\frac{2r^2 + 4r^2}{2} = \frac{6r^2}{2}$$

$$= 3r^2$$

This should be very close to the area of the circle.

Actually,

$$A = \pi r^2$$

π is about 3.14.

To find the area of a circle with radius r, use the formula

$$A = \pi r^2$$

Example: Find the area of a circle with radius 6 cm.

$A = \pi r^2$

$A = 3.14(6)(6)$

$A = 113.04$

The area is 113.04 cm.^2

Exercises

1. What is the formula for the area of a circle? $A = \pi r^2$

2. In the formula $A = \pi r^2$, r^2 means $\underline{\ r\ }$ × $\underline{\ r\ }$.

3. If the radius of a circle is 3 cm, its area is 3.14(3)(3), or 28.26 ____. cm^2

4. If the radius of a circle is 4 m, its area is $\underline{\ 16\ }$ × 3.14 m^2.

5–8. Estimate the area of each circle whose radius is given. Use 3 for π.

5. 5 cm 75 cm^2
6. 6 mm 108 mm^2
7. 10 m 300 m^2
8. 8 km 192 km^2

9–20. Find the area of each circle whose radius is given. Use 3.14 for π.

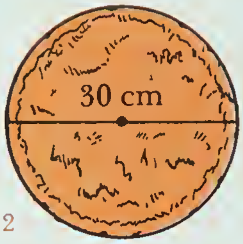
9. 20 m 1256 m^2
10. 14 dm 615.44 dm^2
11. 46 cm 6644.24 cm^2
12. 29 mm 2640.74 mm^2
13. 6.2 km 120.7016 km^2
14. 3.8 m 45.3416 m^2
15. 1.4 m 6.1544 m^2
16. 2.8 cm 24.6176 cm^2
17. 10.2 cm 326.6856 cm^2
18. 16.1 mm 813.9194 mm^2
19. 123 mm 47 506.06 mm^2
20. 461 km 667 315.94 km^2

21–30. Find the area of each circle whose diameter is given. Remember to find the radius first. Use $\frac{22}{7}$ for π.

21. 14 mm 154 mm^2
22. 28 m 616 m^2
23. 70 cm 3850 cm^2
24. 84 mm 5544 mm^2
25. 142 m 15 843$\frac{1}{7}$ m^2
26. 56 cm 2464 cm^2
27. 2.8 cm 6.16 cm^2
28. 8.4 mm 55.44 mm^2
29. 11.2 km 98.56 km^2
30. 15.4 m 186.34 m^2

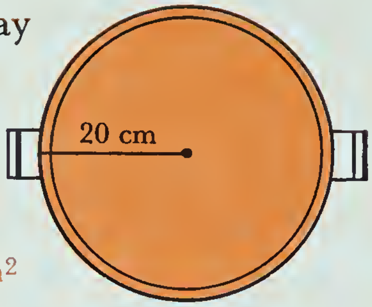
31–36. Find each area shown in orange. Use 3.14 for π.

31. pizza



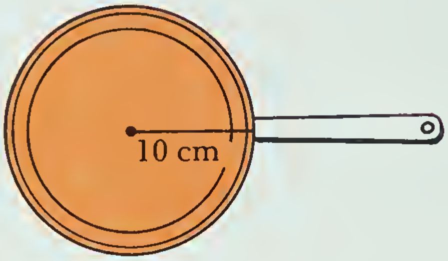
706.5 cm^2

32. tray



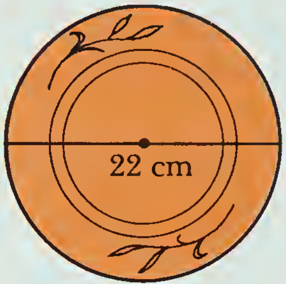
1256 cm^2

33. skillet



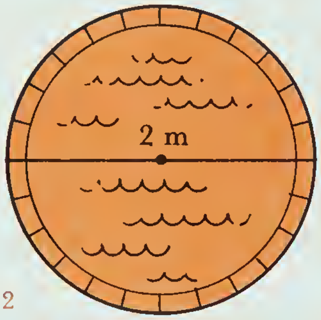
314 cm^2

34. plate



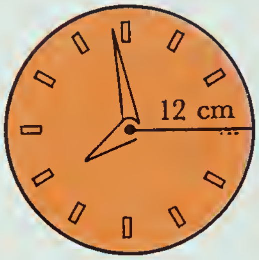
379.94 cm^2

35. fountain



3.14 m^2

36. clock



452.16 cm^2

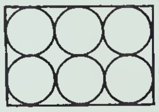
37–40. Use 3.14 for π.

37. A sewer cover has a diameter of 46 cm. What is its area? 1661.06 cm^2

38. A jar lid has a radius of 3.5 cm. What is its area? 38.465 cm^2

39. A circular windowpane, 100 cm in diameter, is cut from a square piece of glass that is 100 cm on a side. How much glass is wasted? 2150 cm^2

40. Six circular discs are cut from a piece of sheet metal that is 30 cm long by 20 cm wide. Each disc is 10 cm in diameter. How much sheet metal is wasted? 129 cm^2



CONSUMER CORNER

FOR PIZZA FREAKS ONLY

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"You the guy who ordered the Grand Slam pizza?"

1. A small (30 cm) sausage pizza costs \$3.50. A large (35 cm) sausage pizza costs \$4.25. Both are round (circles). Which is the better buy? HINT: You can find the cost per square centimetre of pizza. 35 cm

2. The largest pizza ever baked was 640 cm across. What was its area? (Source: *The Guinness Book of World Records.*) 321 536 cm^2

3. The Super Pizza in Exercise **2** had a mass of almost 1.4 kg/cm^2. What was its mass? About 450 150.4 kg

4. Find the cost of the Super Pizza. Use the rate for the 35 cm pizza in Exercise **1.** \$1414.76

5. SOMETHING TO DO: Get a menu from your favourite pizza place. Find out which is a better buy—a large or a small sausage pizza.

Test for Chapter 6

1. How many sides does a trapezoid have? 4 (6.1)

2. How many right angles does a square have? 4

Find the perimeter of each polygon. (6.2)

3. rectangle with length 41 mm and width 28 mm 138 mm

4. square with sides 1.25 cm 5 cm

How many square units are in each figure? (6.3)

5. rectangle with 7 rows of 5 square units each 35

6. square with 6 rows of 6 square units each 36

Find the area of each polygon (6.4)

7. rectangle with length 7.3 m and width 4.1 m 29.93 m^2

8. square with sides $6\frac{1}{2}$ cm $42\frac{1}{4} \text{ cm}^2$

Change each measurement as follows. (6.5)

9. 5 m^2 to cm^2 $50\,000 \text{ cm}^2$

10. 432 mm^2 to cm^2 4.32 cm^2

Find the area of each parallelogram. (6.6)

11. base, 21 cm; height, 12 cm 252 cm^2

12. base, 15 m; height, 12 m 180 m^2

Find the area of each triangle. (6.7)

13. base, 28 cm; height, 18 cm 252 cm^2

14. base, 80 m; height, 60 m 2400 m^2

15. Find the area of a trapezoid with a height of 18 cm and bases of 21 cm and 19 cm. 360 cm^2 (6.8)

Find each square root. (6.9)

16. $\sqrt{81}$ 9

17. $\sqrt{49}$ 7

18. A circle has radius 6 m. Find its circumference. Use $\frac{22}{7}$ for π. $37\frac{5}{7}$ m (6.10)

19. Find the area of a circle with diameter 10 m. Use 3.14 for π. 78.5 m^2 (6.11)

RATE YOURSELF

Number Correct	
18–19	Superior
16–17	Excellent
14–15	Good
10–13	Fair
0–9	Poor

Chapter 6 Review

How many sides does each polygon have? (6.1)

1. pentagon 5 **2.** quadrilateral 4

For the given polygon are all four sides always equal?

3. square Yes **4.** rectangle No

Find the perimeter of each polygon. (6.2)

5. square with a side of 16 cm 64 cm

6. rectangle with length 4½ m and width 3 m 15 m

7. triangle with sides 6.5 cm, 4.2 cm, and 3.8 cm 14.5 cm

8. pentagon with sides 18 m, 19 m, 20 m, 17 m, and 10 m 84 m

How many square units are in each figure? (6.3)

9. rectangle with 7 rows of 9 square units each 63

10. square with 10 rows of 10 square units each 100

Find the area of each rectangle having the given length and width. (6.4)

11. 40 cm, 18 cm 720 cm^2 **12.** 17 m, 15 m 255 m^2

Find the area of each square having the given side.

13. 16 m 256 m^2 **14.** 28 cm 784 cm^2

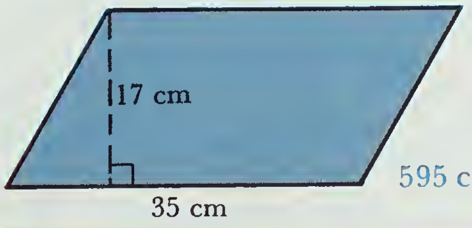
Change each measurement as follows. (6.5)

15. 10 cm^2 to mm^2 1000 mm^2 **16.** 10 800 m^2 to km^2 0.0108 km^2

17. 45 mm^2 to cm^2 0.45 cm^2 **18.** 5.5 km^2 to m^2 5 500 000 m^2

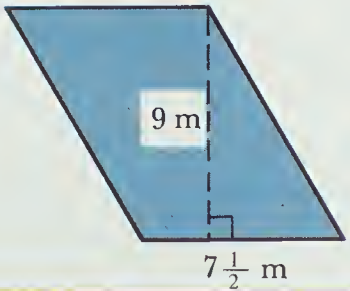
Find the area of each parallelogram. (6.6)

19.



595 cm^2

20.



67 $\frac{1}{2}$ m^2

Find the area of each triangle with the given base and height. (6.7)

21. 34 cm, 10 cm 170 cm^2

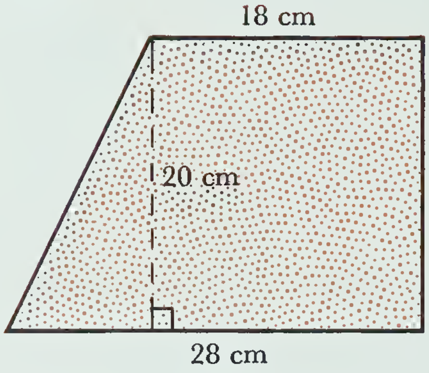
22. 40 mm, 18 mm 360 mm^2

23. 3.8 m, 2 m 3.8 m^2

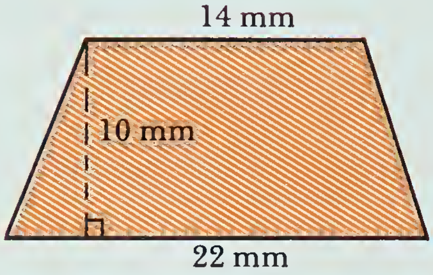
24. 2.5 m, 5 m 6.25 m^2

Find the area of each trapezoid. (6.8)

25. 460 cm^2



26. 180 mm^2



Find each square root. (6.9)

27. $\sqrt{9}$ 3

28. $\sqrt{25}$ 5

29. $\sqrt{64}$ 8

30. $\sqrt{100}$ 10

Use the table on page 272 to find each square root.

31. $\sqrt{41}$ 6.403

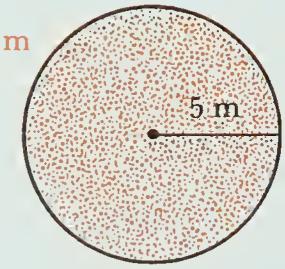
32. $\sqrt{17}$ 4.123

33. $\sqrt{20}$ 4.472

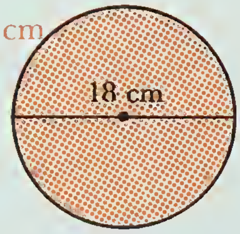
34. $\sqrt{10}$ 3.162

Find the circumference of each circle. Use 3.14 for π. (6.10)

35. 31.4 m



36. 56.52 cm



Find the area of each circle. Use $\frac{22}{7}$ for π. (6.11)

37. The radius is 35 cm. 3850 cm^2

38. The diameter is 42 mm. 1386 mm^2

Name a unit of measure that could be used to measure each of the following: (5.1)

Ex. 1–4: Typical answers

1. your height centimetre
2. mass of a frog gram
3. time to boil an egg minute
4. length of the Red River kilometre

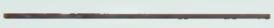
Is the given unit a standard unit of measure?

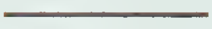
5. kilometre Yes
6. glassful No
7. long time No
8. kilogram Yes

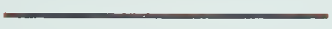
Which ancient unit is close to each following length? (5.2)

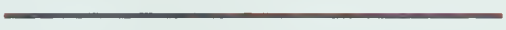
9. 2 m Fathom
10. 45 cm Cubit
11. 10 cm Hand
12. 20 cm Handspan

Measure each segment to the nearest millimetre. (5.3)

13. 
29 mm

14. 
22 mm

15. 
35 mm

16. 
54 mm

17–20. Measure each segment in Exercises **13–16** to the nearest centimetre. (5.4)

3 cm 2 cm 4 cm 5 cm

Complete the following:

21. 7 km is ____ m. 7000
22. 9 m is ____ mm. 9000
23. 14 000 km is ____ m. 14 000 000
24. 140 mm is ____ m. 0.140
25. 2 kg is 2000 g.
26. 64 t is 64 000 kg. (5.5)
27. 3000 g is 3 kg.
28. 1975 mg is 1.925 g.
29. 2350 mL is 2.35 L
30. 1.4 L is 1400 mL. (5.6)
31. 3800 L is 3.8 kL
32. 5 kL is 5 000 000 mL.
33. 0.04 L is 40 mL
34. 0.006 kL is 6 L.

Add or subtract. (5.7)

35. 1 h 23 min
+ 4 h 43 min
6 h 6 min

36. 9 min 12 s
-2 min 38 s
6 min 34 s

37. 5:15 p.m.
-3:30 p.m.
1:45

38. 4:18 p.m.
-10:00 a.m.
6:18

Write the time using the 24-hour clock. (5.8)

39. 1:52 a.m. 01 52

40. 11:10 p.m. 23 10

Subtract

41. 24 00
15 55
8 05

42. 12 31
05 36
6 55

Complete the following: (5.9)

43. 16 min is 4/15 h.

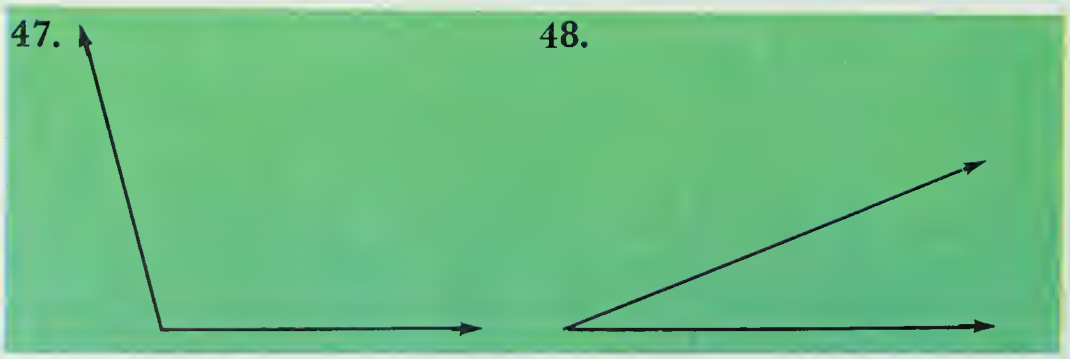
44. 5/12 h is 25 min.

Use the thermometer on page 228 to change each temperature to the other scale. (5.10)

45. 25°C 298 K

46. 90 K -153°C

Measure each angle with a protractor. (5.11)



105° 22°

How many sides does each polygon have? (6.1)

49. parallelogram 4

50. octagon 8

Does the given polygon always have 4 right angles?

51. trapezoid No

52. rectangle Yes

Find the perimeter of each polygon. (6.2)

53. square with a side of 22 cm 88 cm

54. rectangle with a length of 2.5 cm and a width of 3.5 cm 12 cm

55. triangle with sides 3.8 m, 5.4 m, and 3.6 m 12.8 m

56. pentagon with sides 28 mm, 42 mm, 17 mm, 34 mm, 13 mm 134 mm

How many square units are in each figure? (6.3)

57. rectangle with 4 rows of 9 square units each 36

58. square with 7 rows of 7 square units each 49

Find the area of each rectangle having the given length and width. (6.4)

59. 73 cm, 41 cm 2993 cm^2

60. 43 mm, 27 mm 1161 mm^2

Find the area of each square having the given side.

61. 14 mm 196 mm^2

62. 63 m 3969 m^2

Change each measurement as follows. (6.5)

63. 12 m^2 to cm^2 120 000 cm^2

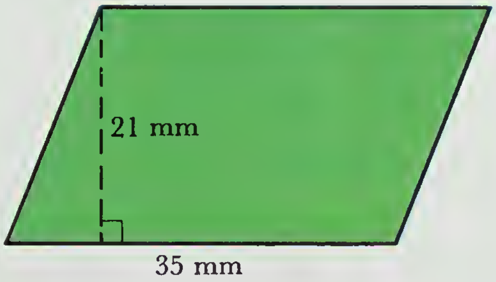
64. 30 000 cm^2 to m^2 3 m^2

65. 3 cm^2 to mm^2 300 mm^2

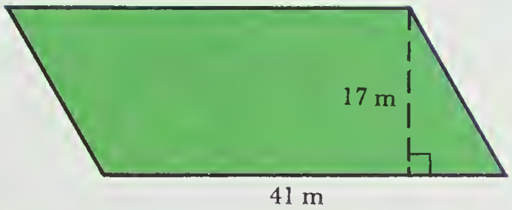
66. 630 mm^2 to cm^2 6.3 cm^2

Find the area of each parallelogram. (6.6)

67. 735 mm^2



68. 697 m^2



Find the area of each triangle with the given base and height. (6.7)

69. 24 m, 16 m 192 m^2

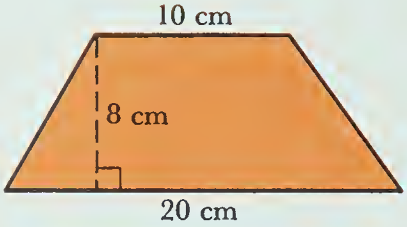
70. 30 m, 18 m 270 m^2

71. 9.4 mm, 4.2 mm 19.74 mm^2

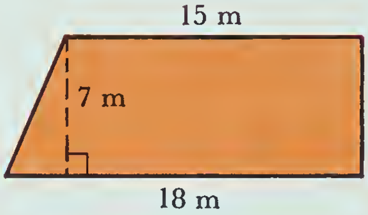
72. 3.25 cm, 6 cm 9.75 cm^2

Find the area of each trapezoid. (6.8)

73. 120 cm^2



74. 115.5 m^2



Find each square root. (6.9)

75. $\sqrt{4}$ 2

76. $\sqrt{16}$ 4

77. $\sqrt{25}$ 5

78. $\sqrt{81}$ 9

Use the table on page 272 to find each square root.

79. $\sqrt{7}$ 2.646

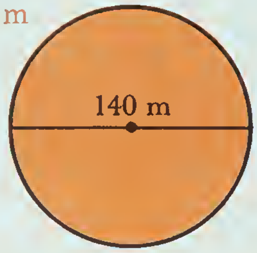
80. $\sqrt{14}$ 3.742

81. $\sqrt{43}$ 6.557

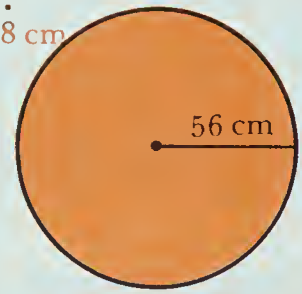
82. $\sqrt{92}$ 9.592

Find the circumference of each circle. Use 3.14 for π. (6.10)

83. 439.6 m



84. 351.68 cm



Find the area of each circle. Use $\frac{22}{7}$ for π. (6.11)

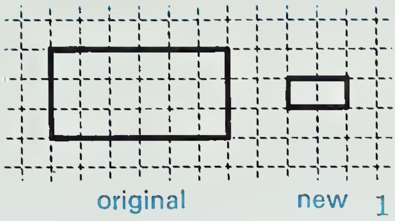
85. The radius is 21 cm. 1386 cm^2

86. The diameter is 49 m. 1886.5 m^2

Preview Test for Chapter 7

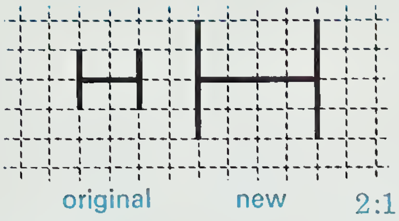
Give each ratio, new figure: original figure. (7.1)

1.



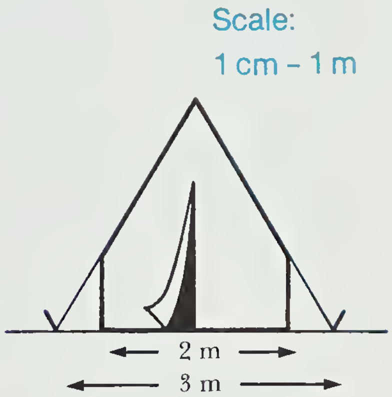
1:3

2.



2:1

Find each measurement of the real tent. (7.2)



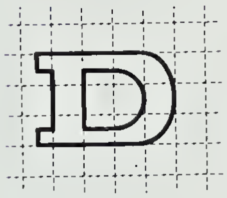
3. width of tent 2 m

4. height of tent 2.5 m

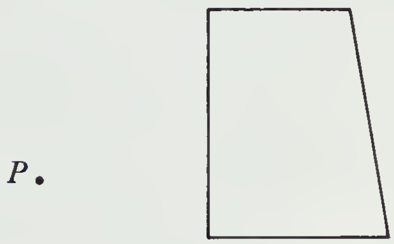
5. distance between pegs 3 m

6. height of door 1.5 m

7–8. Drawings



7. Use the grid method to make a 3:2 enlargement of the figure at the left. (Use a grid with 2 squares/cm). (7.3)



8. Use the point method to make a 3:1 enlargement of the figure at the right. (7.4)

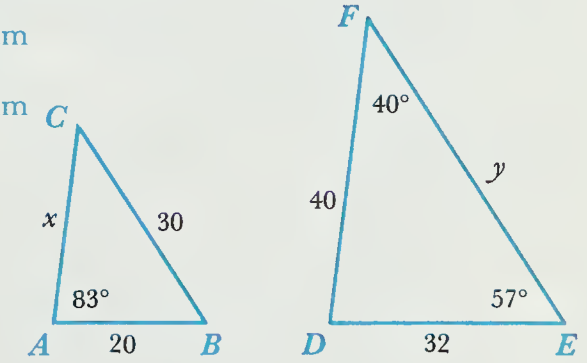
A map has a scale of 1 cm = 32 km. Find the real distance for each map distance. (7.5)

9. 2 cm 64 km

10. 2.5 cm 80 km

11. 10 cm 320 km

12. 3$\frac{5}{8}$ cm 116 km



Triangle *ABC* is similar to triangle *DEF*.

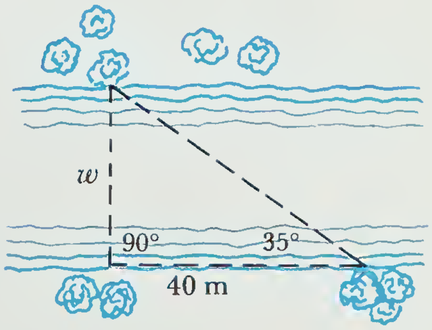
13. Give all 6 pairs of corresponding parts. (7.6)
$\angle A \leftrightarrow \angle D$, $\angle B \leftrightarrow \angle E$, $\angle C \leftrightarrow \angle F$, $\overline{AB} \leftrightarrow \overline{DE}$, $\overline{BC} \leftrightarrow \overline{EF}$, $\overline{AC} \leftrightarrow \overline{DF}$

14. Find all 5 missing parts. (7.7)
$\angle B = 57°$, $\angle C = 40°$, $\angle D = 83°$, $x = 25$, $y = 48$

Construct triangle *XYZ* from measurements. (7.8)

Drawing

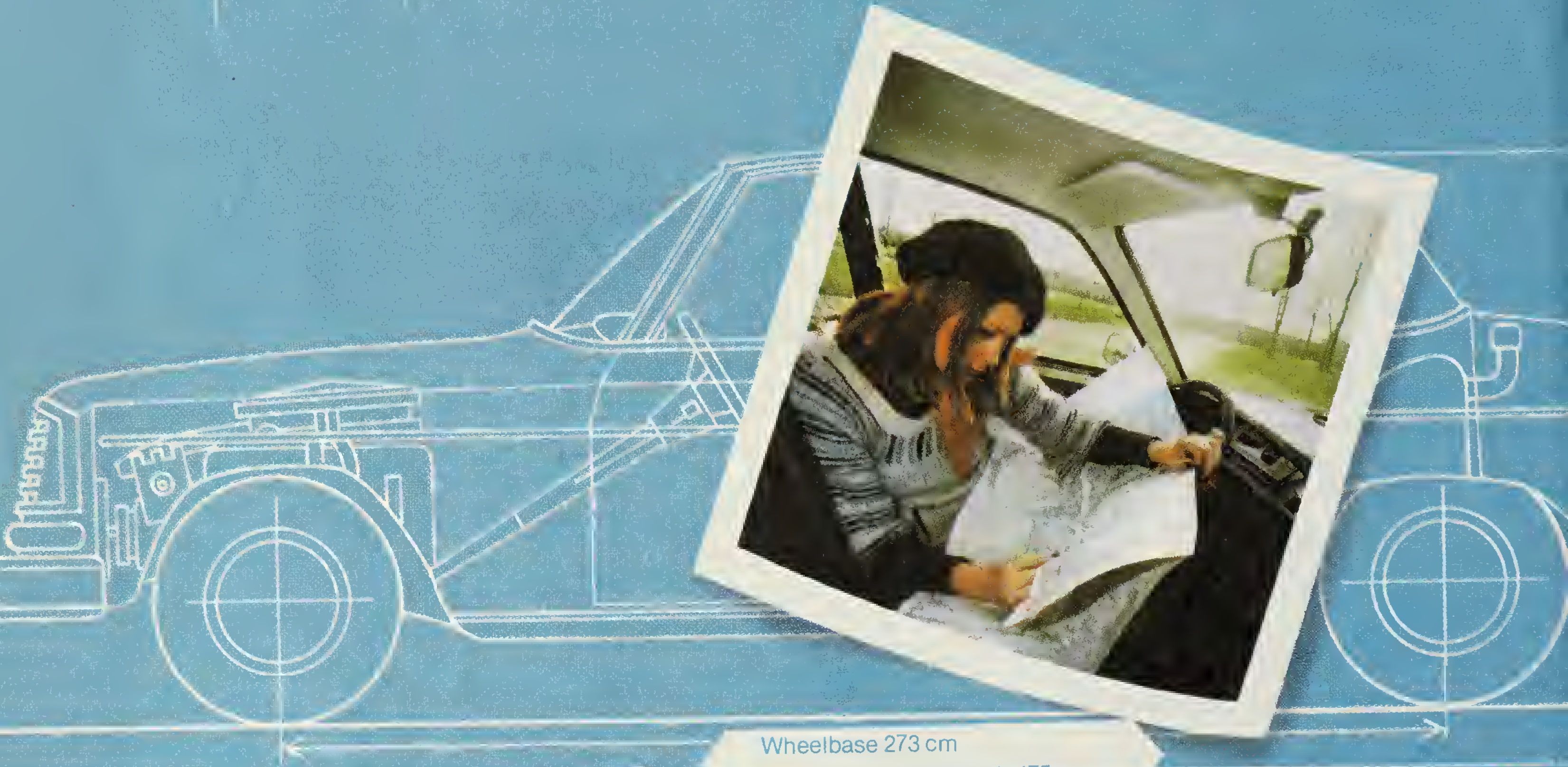
15. *XY* is 6 cm, $\angle X$ is 50°, and $\angle Y$ is 35°.



Make a scale drawing from the measurements at the right. Then measure, and use a proportion. (7.9)

16. Find the width of the river.
About 28 m

7 Scale Drawings


Wheelbase 273 cm
Overall length 475 cm

arage
Kitchen

PICTURE PREVIEW: Have you ever seen a floor plan? Have you ever used a map? These are just two examples of scale drawings. Drawings made to scale can give much information to someone who knows how to read them. The pictures here give some idea of the uses for scale drawings.

Fireplace
Family room
P

Height 154 cm
Fourth Ave

A transit is used to measure angles and distances. Its main parts are a telescope for accurate sighting and scales for precise measuring and reading of angles.

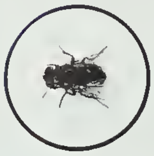
Photo credits clockwise
from top left
Zefa
Photri
Zefa
Brent Jones
Photri
Photri

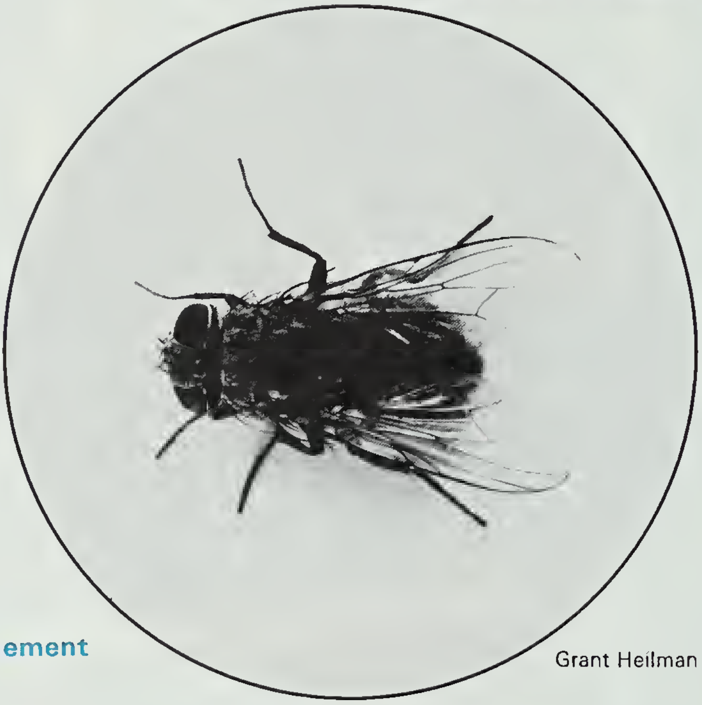
7.1 Enlargements and Reductions

Performance Objective: The student can determine the ratio of enlargement (or reduction) for a pair of figures and draw an enlargement (or a reduction) specified by a given ratio.

Monkmeyer Press Photo Service

If you looked at an insect through a microscope, you would see an *enlargement* of the insect.


original


enlargement

Grant Heilman

If you measured both pictures, you would find that each measurement in the enlargement is 5 times as large as the corresponding measurement in the original. We say:

The new picture is an enlargement by a factor of 5.

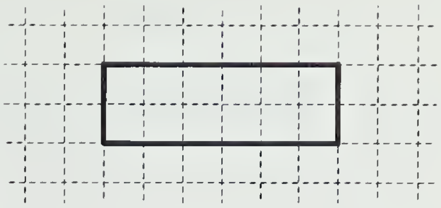
It is also convenient to use ratios:

The ratio of the enlargement to the original is

5 to 1 or 5:1 or $\frac{5}{1}$

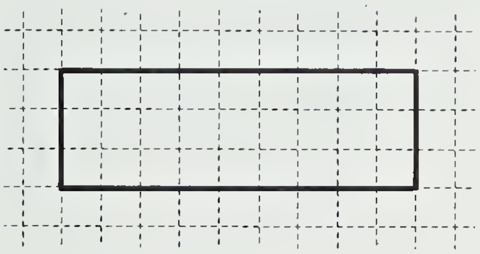
Note that in the ratio, the *new* figure always comes *first*, or is the numerator. The *original* figure always comes *second*, or is the denominator.

Example 1: Make a 3 to 2 enlargement of the rectangle.



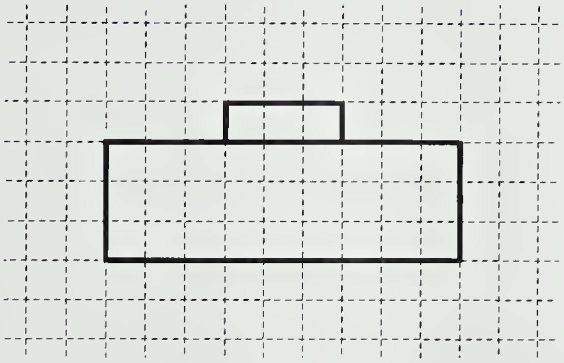
The ratio, new figure: original figure, is to be 3:2. (The new figure will be $\frac{3}{2}$, or $1\frac{1}{2}$ times, the size of the original figure.)

The new figure, the enlargement, is drawn with **3** units for every **2** units in the original.



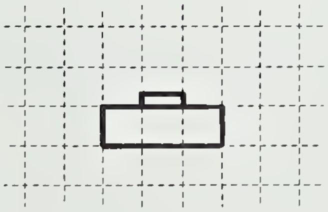
It is just as easy to make a *reduction*, a smaller new figure.

Example 2: Make a 1:3 reduction of the figure below.



The ratio, new figure: original figure, is to be 1:3. (The new figure will be $\frac{1}{3}$ the size of the original.)

The new figure is drawn with only **1** unit for every **3** units in the original.

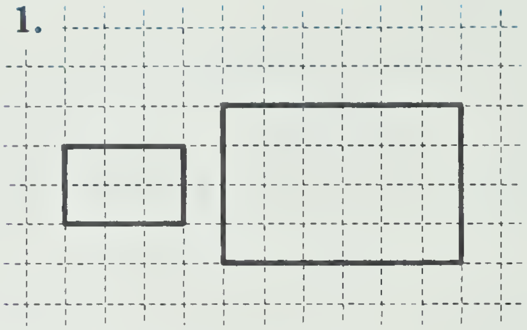


Notice that where the original figure has a length of 1 unit, the new figure has a length of $\frac{1}{3}$ unit.

Exercises

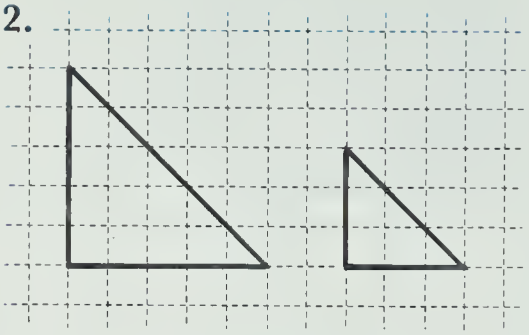
1–6. Tell whether each new figure is an enlargement or a reduction. Then give the ratio in lowest terms.

1.


original
new

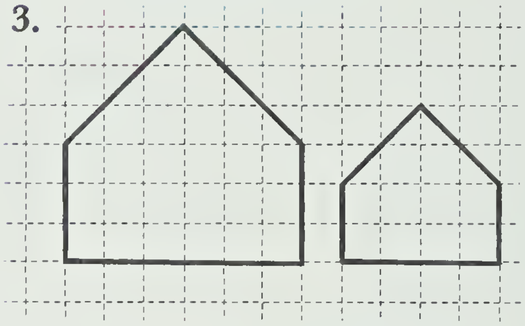
Enlargement, 2:1

2.


original
new

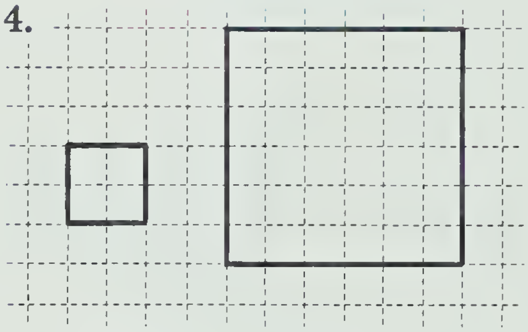
Reduction, 3:5

3.


original
new

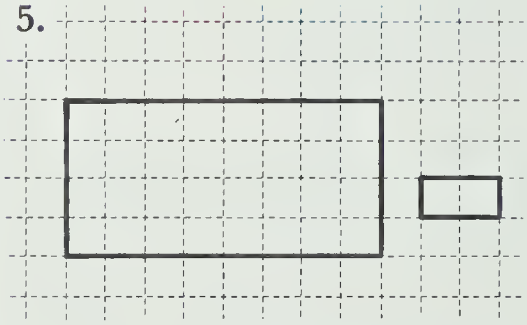
Reduction, 2:3

4.


original
new

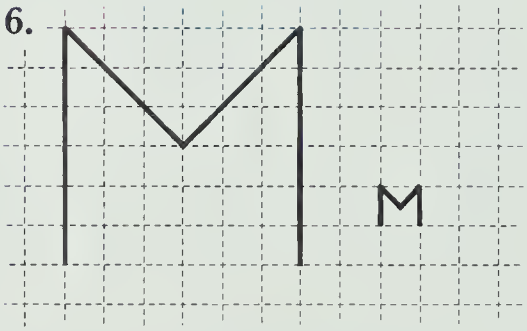
Enlargement, 3:1

5.


original
new

Reduction, 1:4

6.


original
new

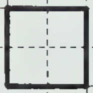
Reduction, 1:6

7–18. For each ratio, tell whether the new figure will be an enlargement, a reduction, or an identical copy.

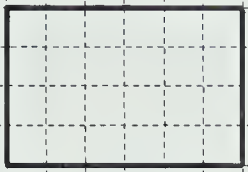
7. 2 to 5 Red.
8. 3 to 1 Enl.
9. 2:3 Red.
10. 4:4 Copy
11. $\frac{6}{2}$ Enl.
12. $\frac{1}{100}$ Red.
13. 7:8 Red.
14. $\frac{6}{10}$ Red.
15. $\frac{9}{5}$ Enl.
16. 3:2 Enl.
17. 5 to 5 Copy
18. 7 to 11 Red.

19–30. Assume that each figure is on paper with 4 squares to the centimetre. Use paper with 4 squares to the centimetre to draw each new figure.

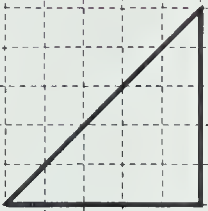
19. 5 to 2 enlargement



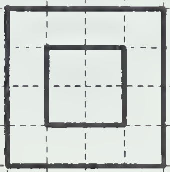
20. 1 to 2 reduction



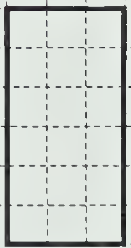
21. 2 : 5 reduction



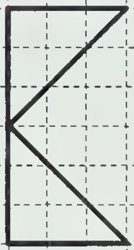
22. 5 : 2 enlargement



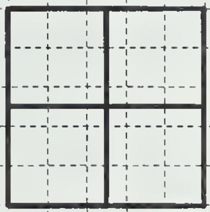
23. 5 : 3



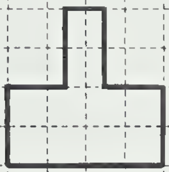
24. 8 : 3



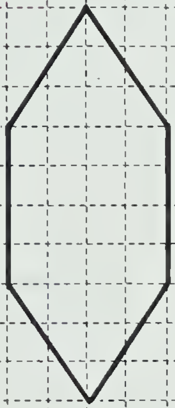
25. 4 : 5



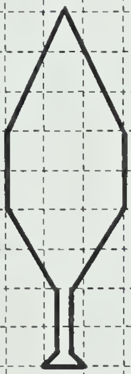
26. 4 : 2



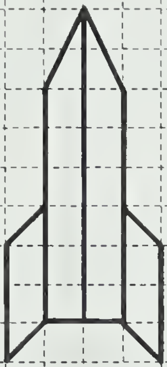
27. 2 : 1



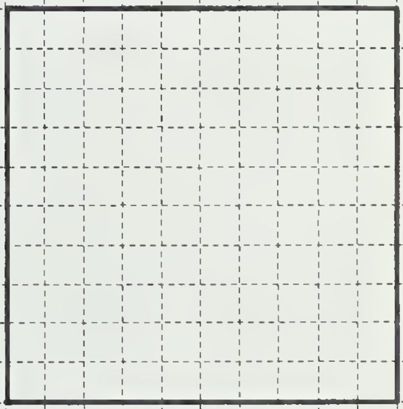
28. $\frac{3}{1}$



29. $\frac{3}{1}$



30. 1 : 20



Quick Quiz

Check this skill:

- *multiplying a decimal or a mixed number by a whole number*

1–8. Multiply.

1. 5×4.3 21.5

2. 8×3.5 28

3. 12×2.5 30

4. 20×4.2 84

5. $6 \times 3\frac{1}{2}$ 21

6. $8 \times 2\frac{3}{8}$ 19

7. $12 \times 2\frac{1}{4}$ 27

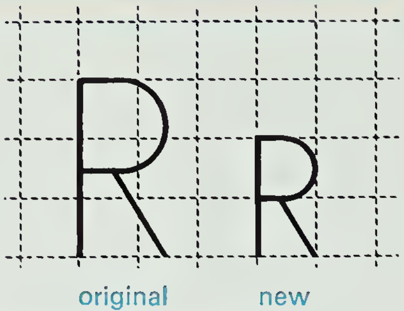
8. $24 \times 3\frac{5}{8}$ 87

7.2 Scale Drawings

The enlargement or reduction ratio is called the **scale.**

Performance Objective: The student can use the scale of a drawing to determine real measures represented in the drawing.

Example 1: Give the scale for the R's.



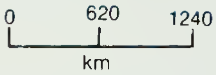
original
new

The new figure is a 2:3 reduction. Therefore, we say:

The scale is 2 units to 3 units.

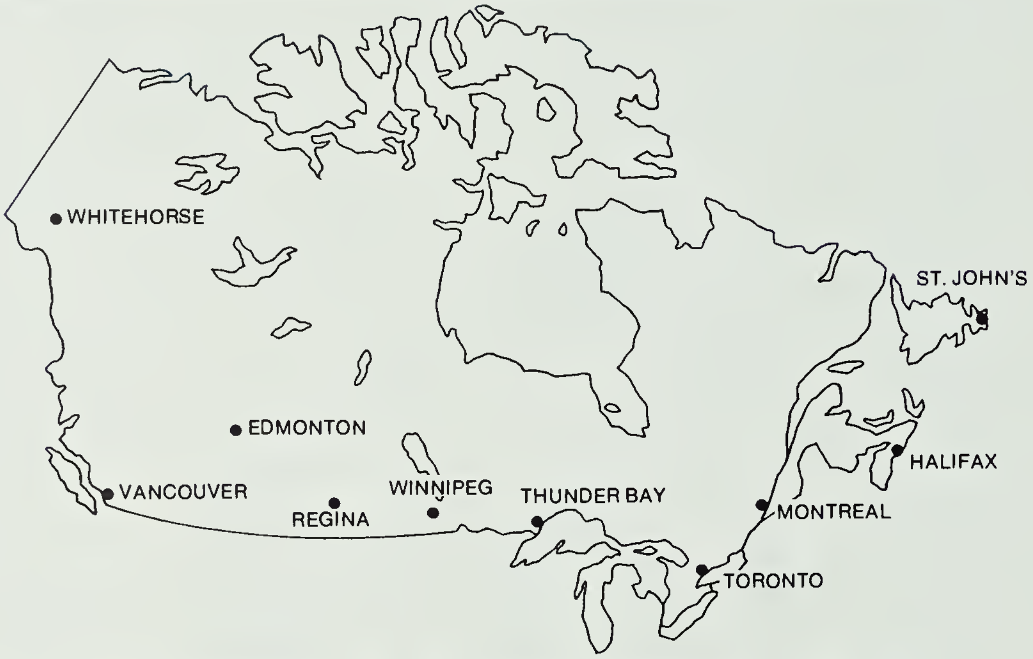
Example 2: Find the distance from Vancouver to Halifax.

Scale:



0
620
1240
km

1 cm to 620 km



WHITEHORSE
ST. JOHN'S
EDMONTON
HALIFAX
VANCOUVER
WINNIPEG
THUNDER BAY
REGINA
MONTREAL
TORONTO

For every centimetre on the map, there are 620 real kilometres.

Measure. ▶ From the Vancouver dot to the Halifax dot is 9 cm.

Multiply. ▶ 9 x 620 = 5580

The distance from Vancouver to Halifax is 5580 km.

Note

You could find the actual ratio, new figure: original figure. The new figure is the map. The original figure is, of course, the real Canada.

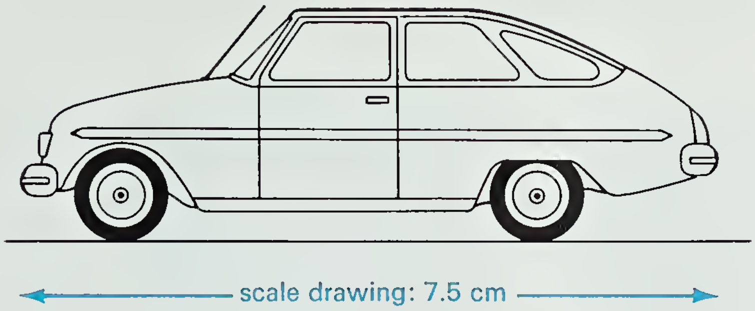
1 km = 1000 m
1 m = 100 cm

620 km x 1000 = 620 000 m
620 000 m x 100 = 62 000 000 cm

So the ratio is 1 cm to 62 000 000 cm.

The map is $\frac{1}{62\,000\,000}$ the size of the real Canada.

Example 3: Find the car's length from the scale drawing.


scale drawing: 7.5 cm

Scale: 1 cm = 0.6 m

This means that 1 cm on the drawing represents 0.6 m on the real car.

The drawing is 7.5 cm long. ◀ Measure.

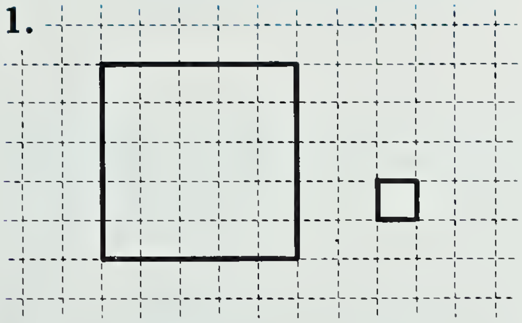
7.5 x 0.6 = 4.50 ◀ Multiply.

The real car is 4.5 m long.

Exercises

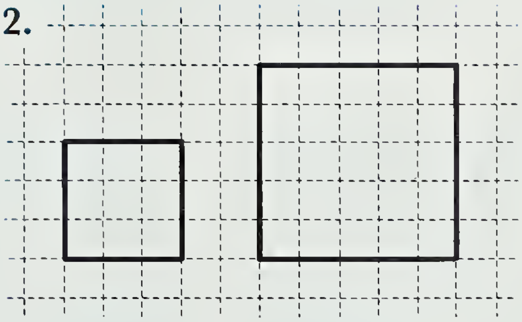
1–4. Give the scale for each pair of figures.

1.


original
new

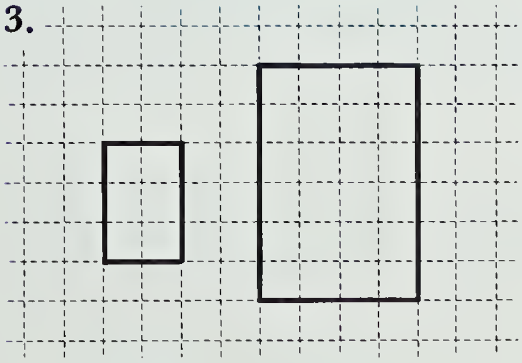
1 unit to 5 units

2.


original
new

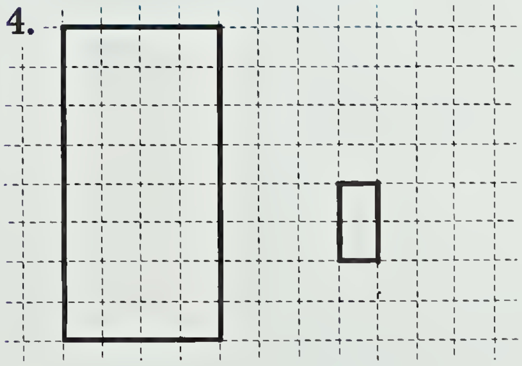
5 units to 3 units

3.


original
new

2 units to 1 unit

4.

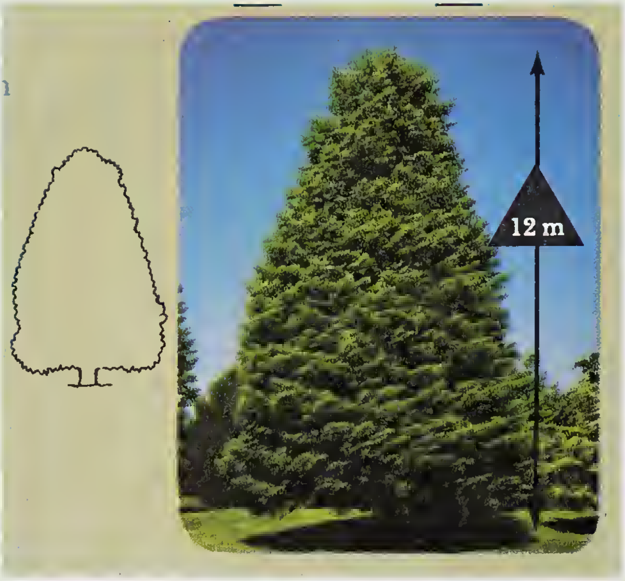

original
new

1 unit to 4 units

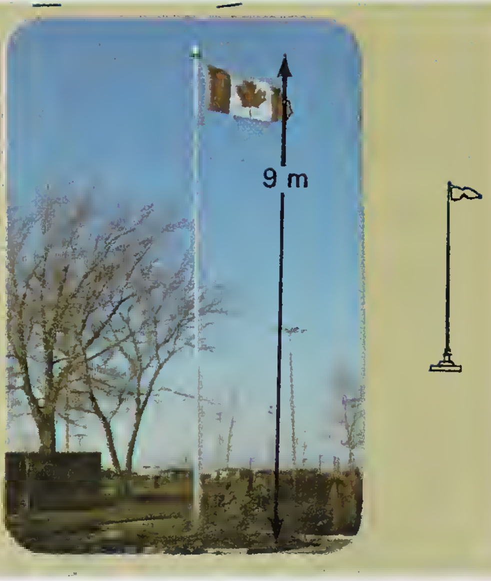
5–10. Carefully measure each drawing in centimetres (depending on how the measure of the real object is given). Then give the scale of the drawing.

7. 1 cm = 7 mm
8. 2.5 cm = 0.02 cm
9. 2.5 cm = 12 800 km
10. 1 cm = 6 m

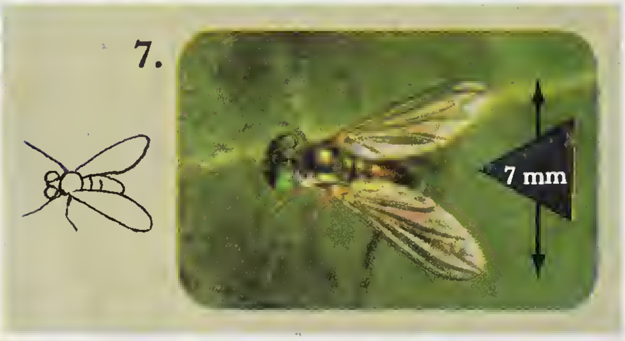
5. 2.5 cm = 12 m



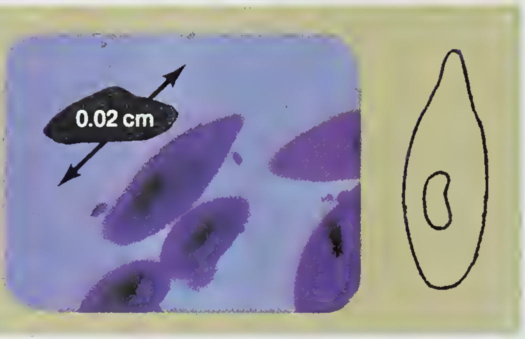
6. 2 cm = 9 m

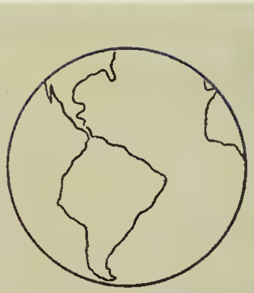


7.

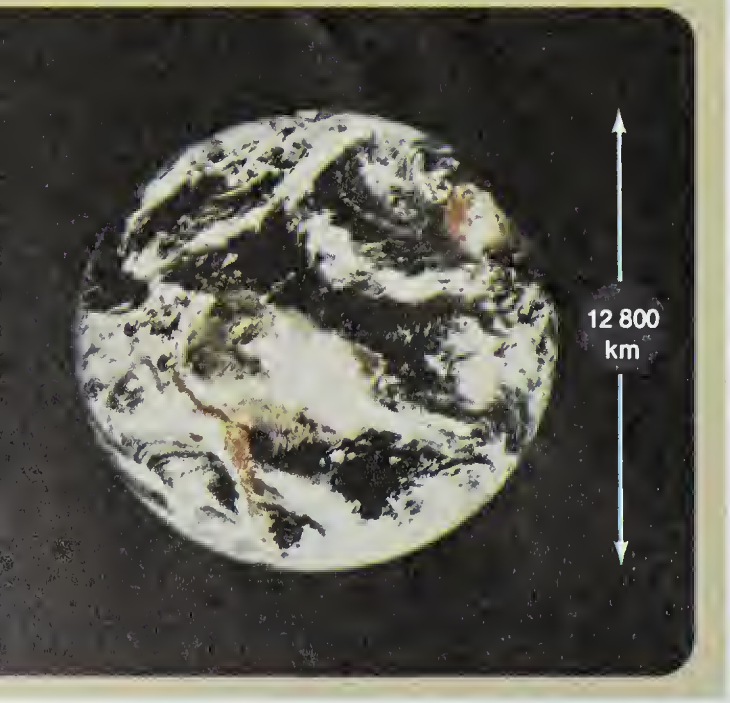


8.





9.



10.

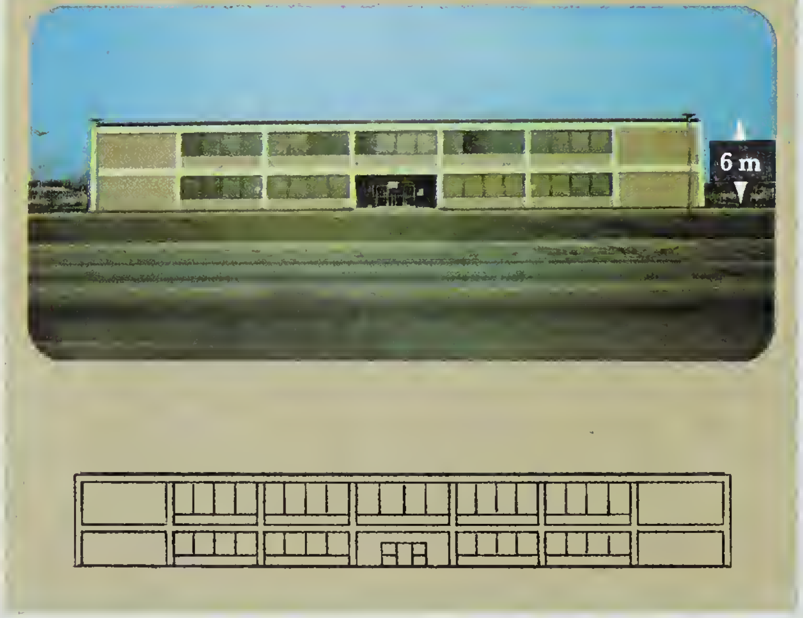


Photo credits: 5. Grant Heilmann, 6. Robert Brown, 7. Dr. E. R. Degginger, 8. Owczarak/Taurus Photos, 9. Photo Research International, 10. Frank Kuechmann.

Ex. 11–28: Answers are approximations.

11–16. Copy and complete the chart. Use the car on page 297.

	Find each measurement	Drawing to nearest 0.5 cm	Real car in metres
11.	Height (roof to ground)	2.5	1.5
12.	Wheelbase (front wheel center to rear wheel center)	4.5	2.7
13.	Outside diameter of tires	1	0.6
14.	Width of door	1.5	0.9
15.	Length of antenna	1	0.6
16.	Length of side molding	6.5	3.9

17–26. Copy and complete the chart. Use the map on page 296.

	Find the distance from:	Map distance to nearest millimetre	Real distance in kilometres
17.	Winnipeg to Edmonton	34	2108
18.	St. John's to Toronto	41	2542
19.	Whitehorse to Regina	43	2666
20.	Thunder Bay to Halifax	40	2480
21.	Vancouver to Montreal	71	4402
22.	Toronto to Vancouver	65	4030
23.	Edmonton to Halifax	72	4464
24.	Vancouver to St. John's	97	6014
25.	Regina to Montreal	46	2852
26.	Winnipeg to Thunder Bay	11	682

27–28. Use your answers to Exercises 5 and 10.

27. Measure the length of the body of the scale drawing of the fly in Exercise 5. How long is the real fly? 0.9 cm, 6.3 mm

28. Measure the width of the scale drawing of the building in Exercise 10. How wide is the real building? 7 cm, 42 m

CAREER CORNER

SCALE DRAWINGS AT WORK . . .

A. Devaney, Inc.

Scale drawings are used in industry as blueprints, plans, or charts. Such drawings can be used to check the accuracy of manufactured parts.

Scale drawings are also used in construction work as the floor plan for an apartment or a house. On the opposite page is the floor plan for an apartment. This plan might be used by several people during and after construction of the apartment.

Use the plan for the problems on this and the opposite page. Take measurements to the nearest millimetre.

CARPENTER:

How long is the row of cabinets (dashed line) above the sink in the kitchen? If 6 equal-size cabinet doors were made to fit the space exactly, how wide would each door be? 2.4 m, 0.4 m or 40 cm

GLAZIER:

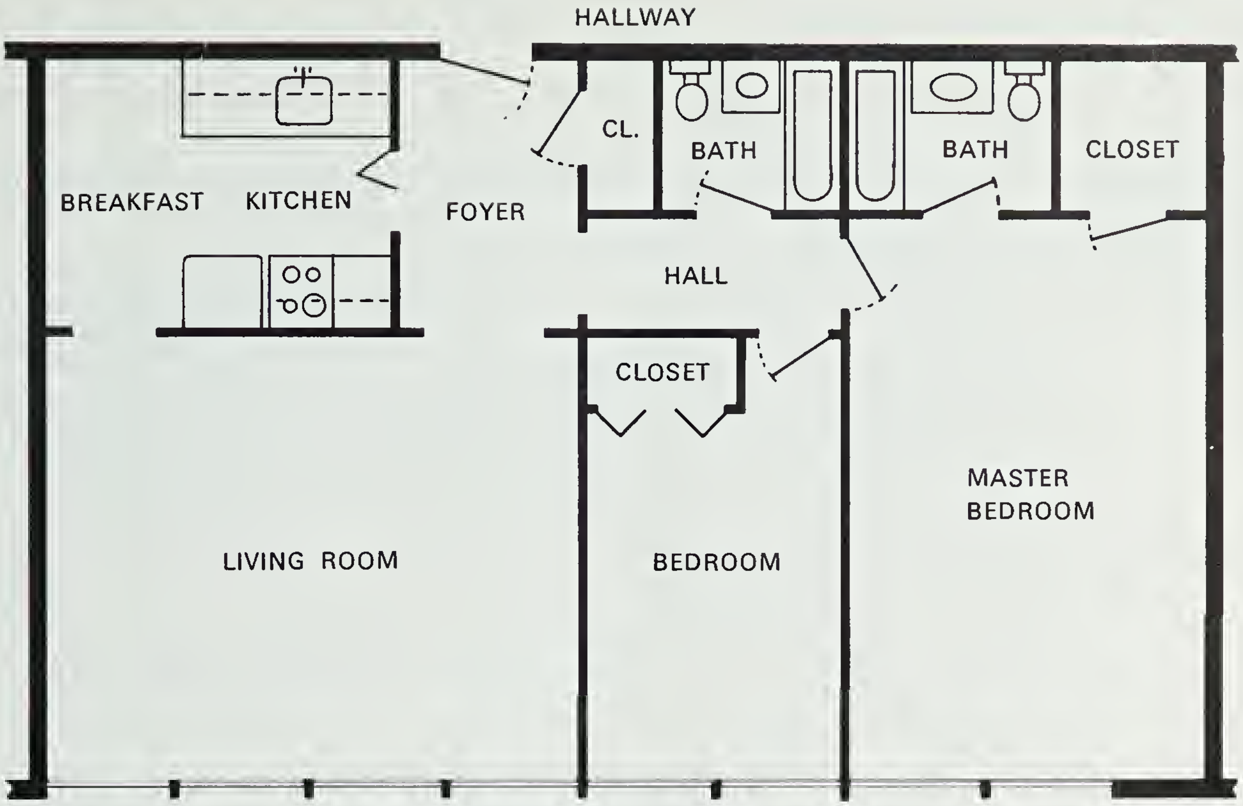
About how wide is each window? 1.5 m

How many windows are there in the whole apartment? If each window has two panes of glass, how many panes are needed? 8 windows; 16 panes

DECORATOR:

What are the length and the width of the living room? What are the dimensions of the master bedroom? 5.7 m long, 4.7 m wide; 7 m by 3.8 m

Copy the plan on a sheet of paper. Then draw in all the furniture you would use. Make sure to draw the furniture to the proper scale.


HALLWAY
CL.
BATH
BATH
CLOSET
BREAKFAST
KITCHEN
FOYER
HALL
CLOSET
MASTER BEDROOM
LIVING ROOM
BEDROOM

Floor plan scale: 1 cm = 1 m

. . . AND AT HOME

CONSUMER CORNER

Suppose you lived in an apartment. You could use the floor plan to find several different kinds of information.

CARPETS:

If you installed wall-to-wall carpeting in the living room, how many square metres would you need? 26.8 m^2

How many square metres would be needed for the master bedroom? 26.6 m^2

DRAPES:

If you hung draperies across the window wall in the living room, what total width would you need? (Drapery widths are given in centimetres.) 570 cm

TILE:

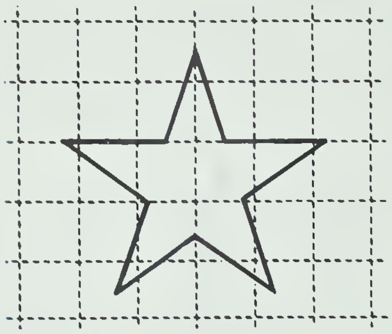
If you put down tile in the foyer, how many 1 dm by 1 dm tiles would you need? 600 tiles

How many tiles would be needed for the foyer if the tiles were 1.5 dm by 1.5 dm? (HINT: How many *tiles* long is the room? How many *tiles* wide?) 600 tiles (30 tiles by 20 tiles)

7.3 The Grid Method

A scale drawing can be made with a *grid* (ruled squares).

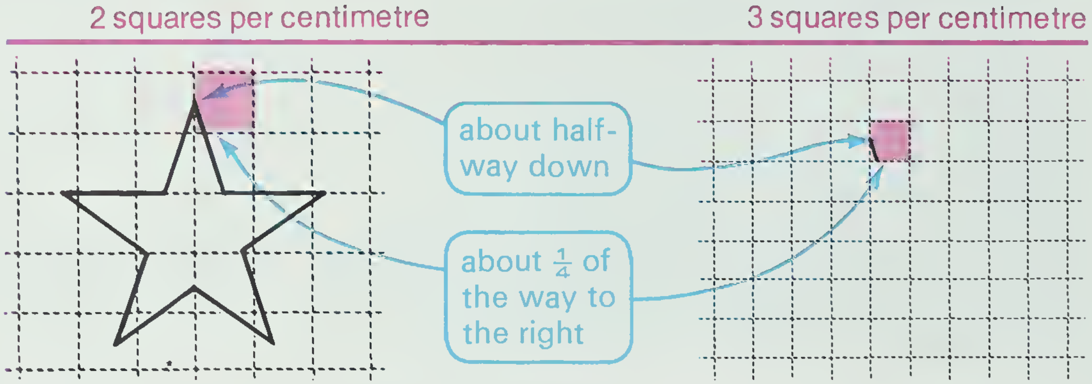
Performance Objective: The student can apply the grid method to make an enlargement or a reduction.



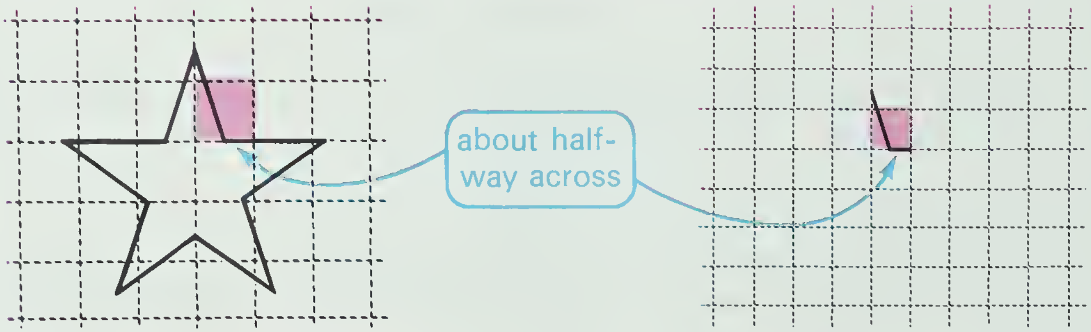
Example 1: Make a 2:3 reduction of the star.

To make a reduction, use a grid with smaller squares. Assume that the star is on a grid with 2 squares to the centimetre. For a ⅔ reduction, use a grid with squares ⅔ that size.

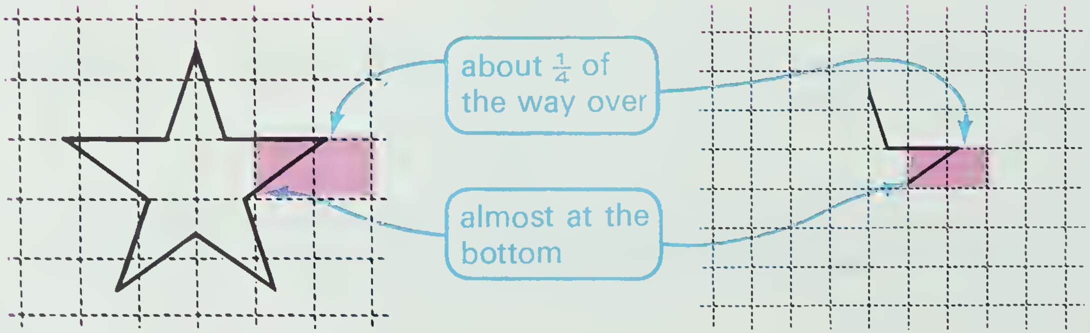
Study and redraw one grid square at a time.


2 squares per centimetre
3 squares per centimetre
about half-way down
about $\frac{1}{4}$ of the way to the right

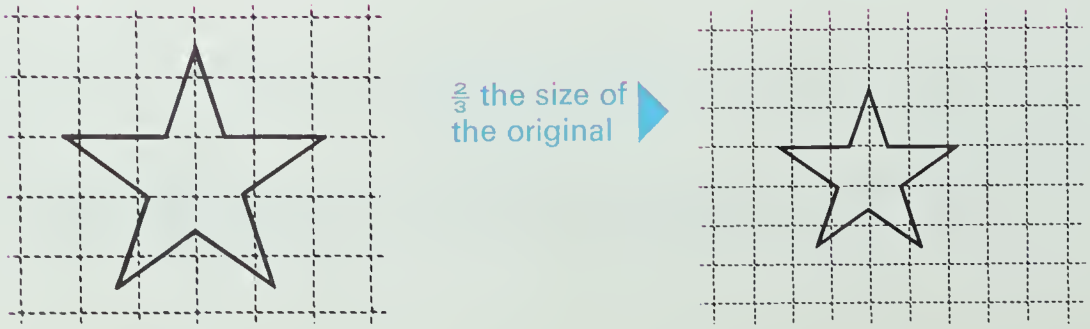
For each square, guess about where the figure meets the sides of the square.


about half-way across

Sometimes more than one square can be done at a time.

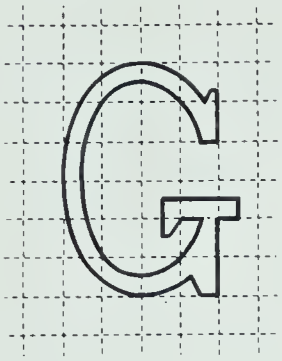

about $\frac{1}{4}$ of the way over
almost at the bottom

When all the squares are done, you will have a new figure in the proper size.


$\frac{2}{3}$ the size of the original

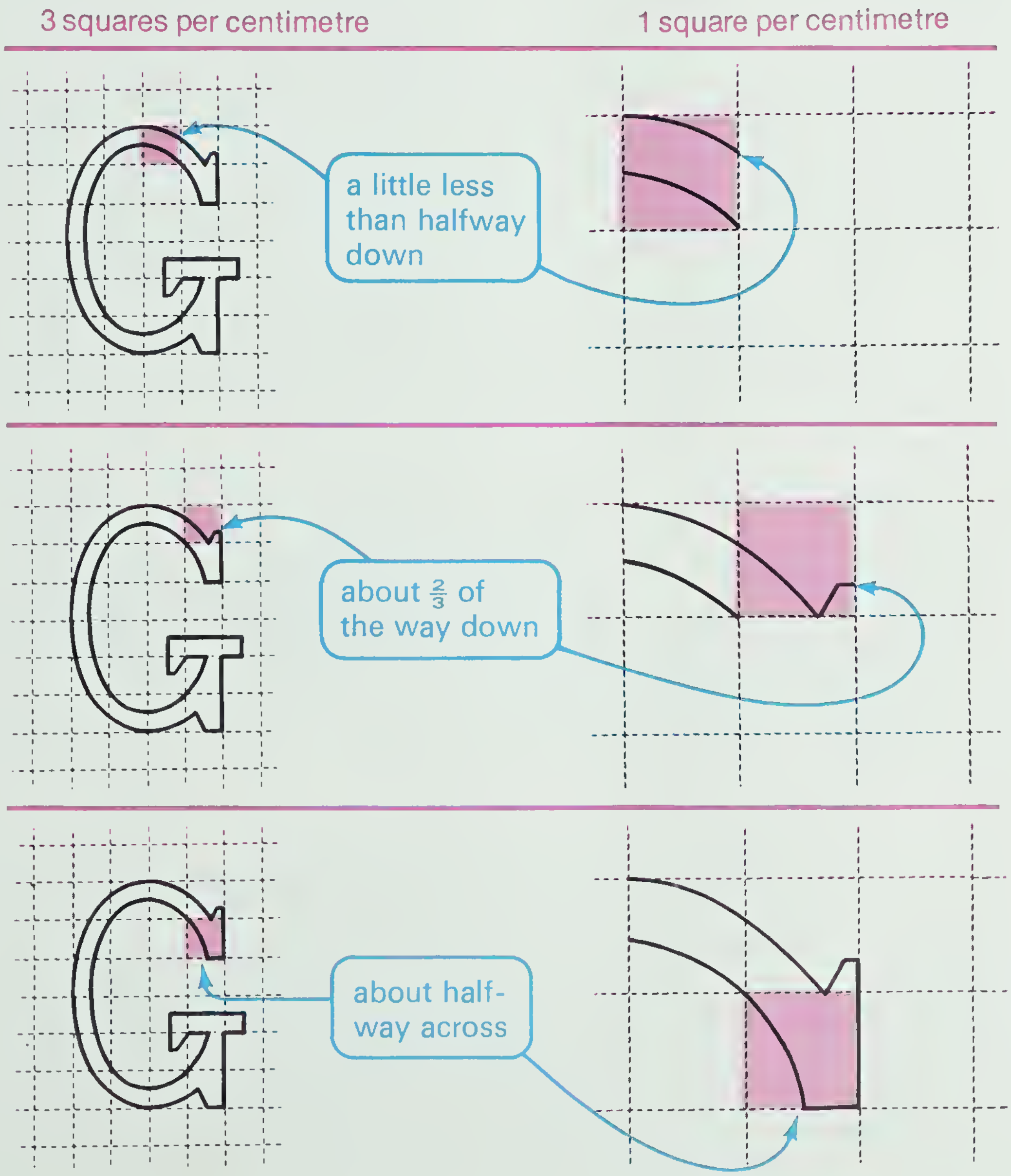
The new star is smaller than the original because a grid with smaller squares was used. The smaller the squares, the smaller the new figure will be.

To make an enlargement, use a grid with larger squares.



Example 2: Make a 3:1 enlargement of the letter G.

The new figure is to be 3 times larger, so use a grid with squares 3 times as large. Assume that the G is on a grid with 3 squares to the centimetre. Squares 3 times as large would be 1 centimetre.


3 squares per centimetre
1 square per centimetre
a little less than halfway down
about $\frac{2}{3}$ of the way down
about half-way across

For each square, guess about where the figure meets the square.

Draw curves in proportional size.

When completed the new figure will be 3 times as large.

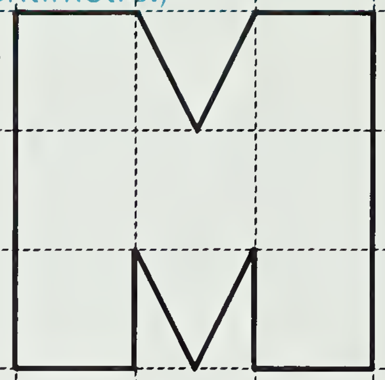
Exercises

1-16. *Drawings*

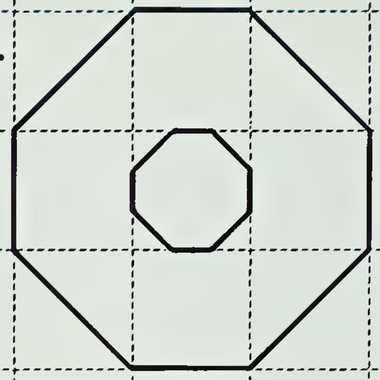
Assume that this grid has 1 square per centimetre.

1–2. Make a 1:2 reduction. (Use a grid with 2 squares per centimetre.)

1.

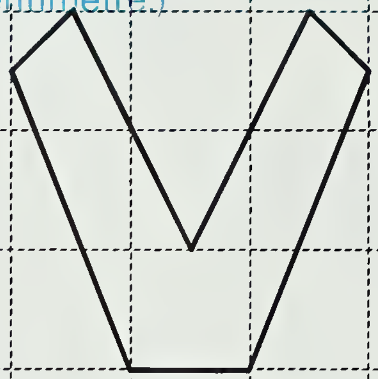


2.

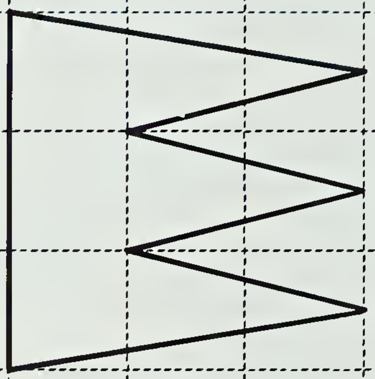


3–4. Make a 1:3 reduction. (Use a grid with 3 squares per centimetre.)

3.



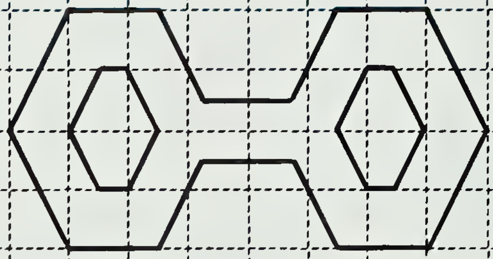
4.



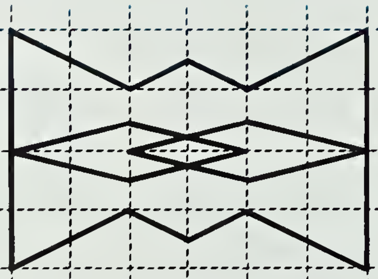
5–8. Make a 2:3 reduction. (Use a grid with 3 squares per centimetre.)

Assume that this grid has 2 squares per centimetre.

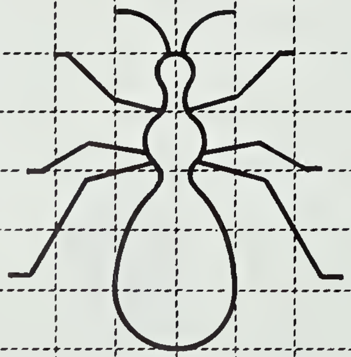
5.



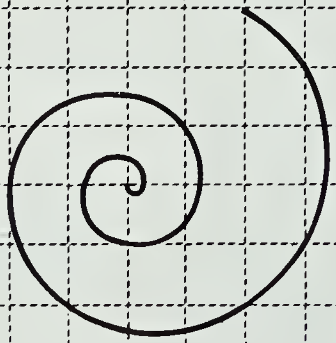
6.



7.



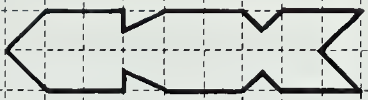
8.



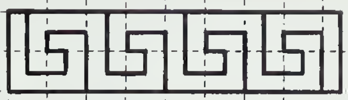
9–12. Make a 3:2 enlargement. (Use a grid with 4 squares per centimetre.)

Assume that this grid has 6 squares per centimetre.

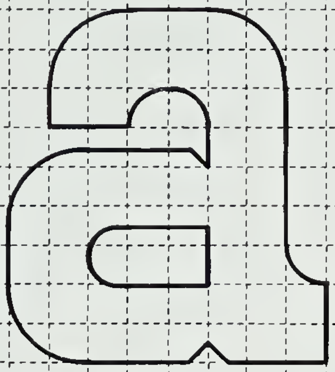
9.



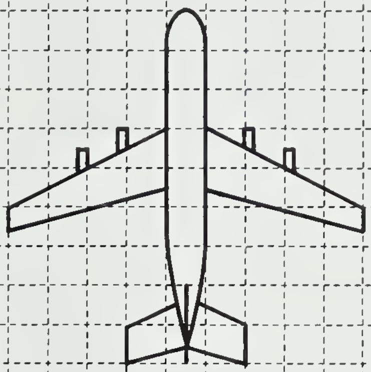
10.



11.

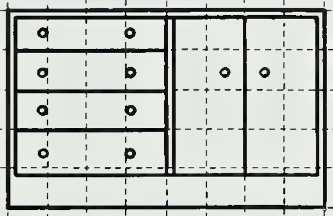


12.

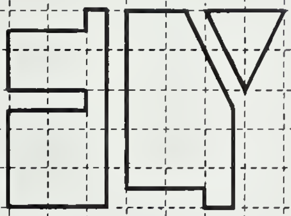


13–16. Make a 3:1 enlargement. (Use a grid with 2 squares per centimetre.)

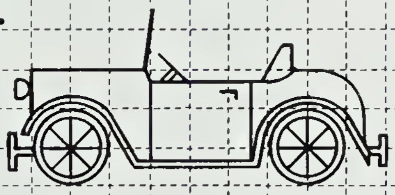
13.



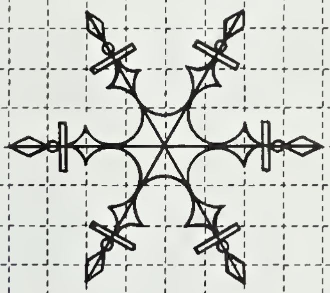
14.



15.



16.



17–20. Tell what size grid should be used for each.

17. 3:1 enlargement, original grid with 6 squares per centimetre
2 squares per centimetre

18. 4:1 enlargement, original grid with 8 squares per centimetre
2 squares per centimetre

19. 2:3 reduction, original grid with 4 squares per centimetre
6 squares per centimetre

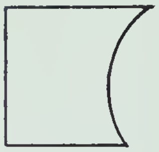
20. 1:3 reduction, original grid with 2 squares per centimetre
6 squares per centimetre

7.4 The Point Method

Performance Objective: The student can apply the point method to make an enlargement or a reduction.

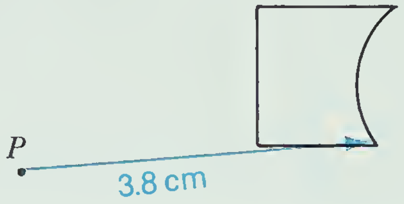
A second way to make a scale drawing is to use a *point* from which to measure distances.

Example 1: Make a 2:1 enlargement.



Begin by drawing a point *P* anywhere near the figure.

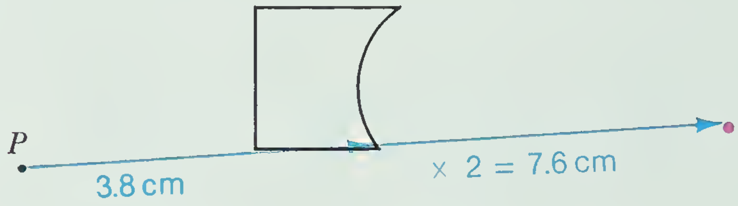
Measure the distance from *P* to a corner of the figure.



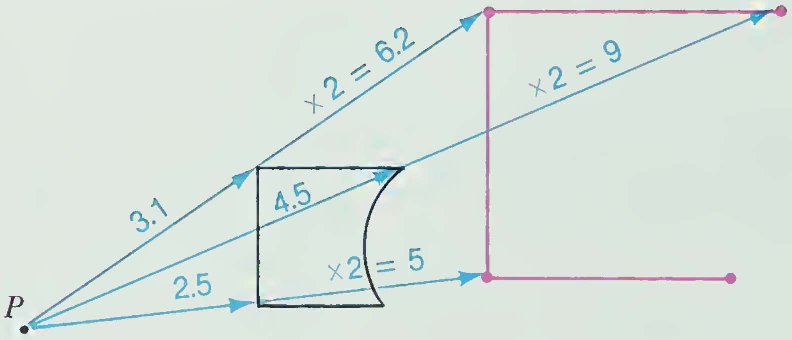
We want the new figure to be 2 times as large.
$3.8 \times 2 = 7.6$

Multiply the distance by the enlargement factor.

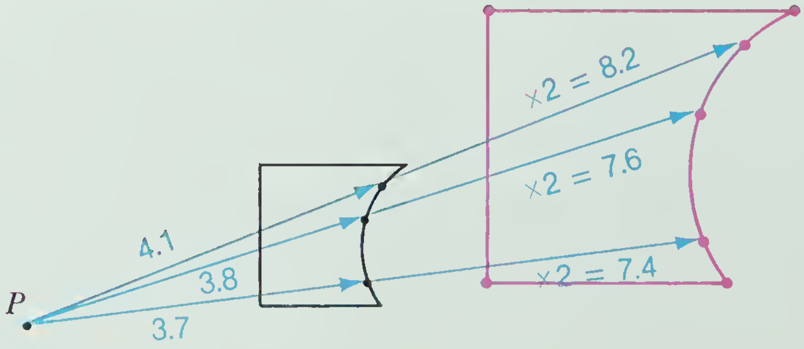
Make a dot the new distance from point *P* *along the same line.*



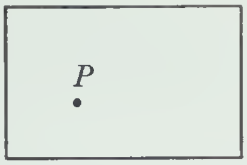
Do the same for the other corners, and draw in the sides.
All measurements are in centimetres.



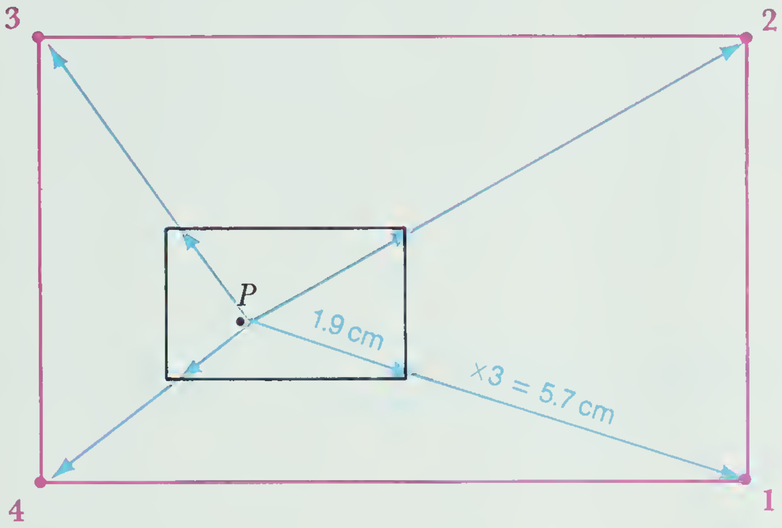
Curved lines require a few extra points. Use *any* points on the figure that are easy to measure. Then draw in the curve.



Example 2: Make a 3:1 enlargement of the rectangle, using point *P*.



For each corner, measure the distance from *P*. Multiply that distance by 3, and draw a new dot along the same line.



Here are the calculations:

1. $1.9 \times 3 = 5.7$
2. $2 \times 3 = 6$
3. $1.2 \times 3 = 3.6$
4. $1 \times 3 = 3$

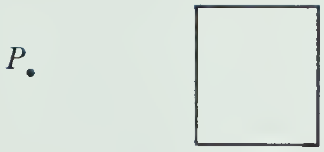
Measure the sides of both figures to see that the new figure really is a 3:1 enlargement.

Exercises

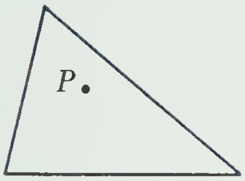
1–4. Trace each figure and point onto a sheet of paper. Then make a 2:1 enlargement using the point.

1–18. *Drawings*

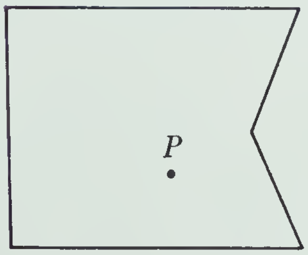
1.



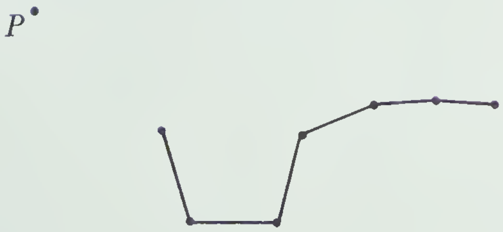
2.



3.

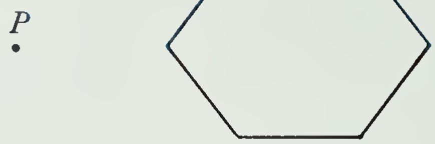


4.

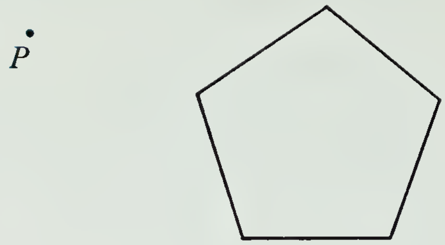


5–8. Trace and make a 3:1 enlargement.

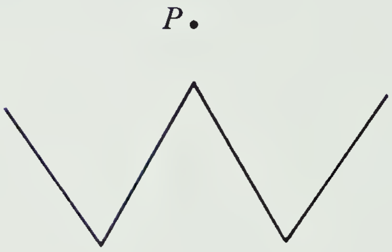
5.



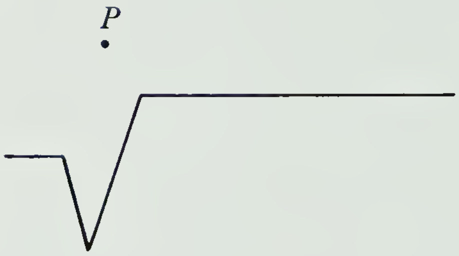
6.



7.

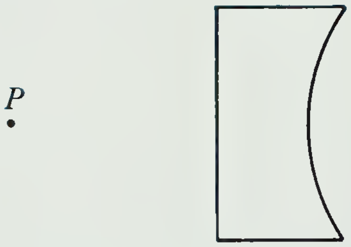


8.

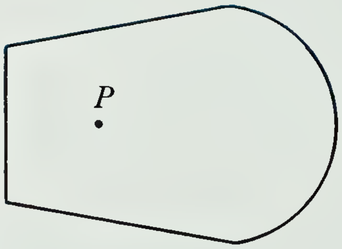


9–12. Trace and make a 2:1 enlargement. For curves, use as many points as you can accurately measure.

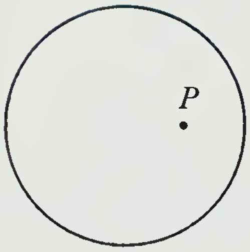
9.



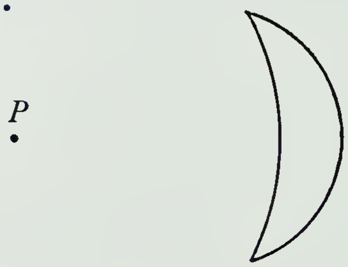
10.



11.

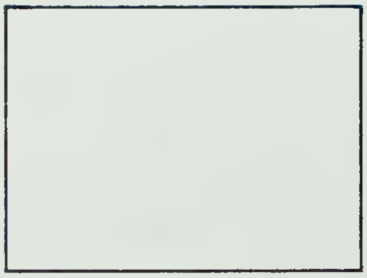


12.

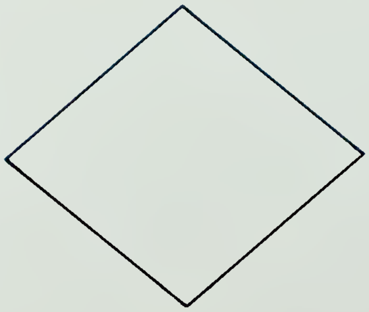


13–14. Trace each figure. Then draw in a point wherever you like and make a 3:1 enlargement.

13.

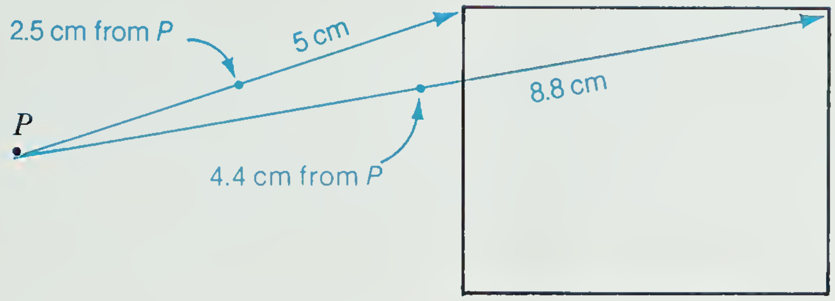


14.



15–18. Make a 1:2 reduction. Multiply each measurement by $\frac{1}{2}$, and make a dot that distance away from the point along the same line. (Exercise 15 has been started for you.)

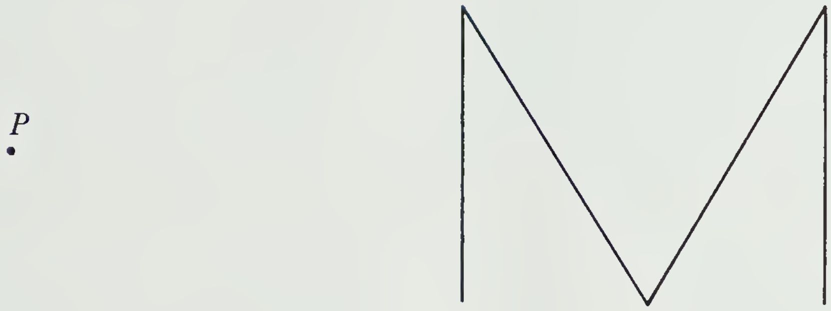
15.


2.5 cm from P
5 cm
P
8.8 cm
4.4 cm from P

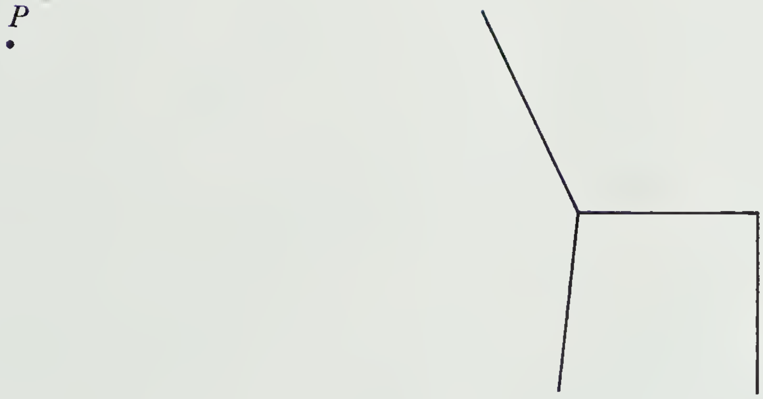
$5 \times \frac{1}{2} = 2.5$

$8.8 \times \frac{1}{2} = 4.4$

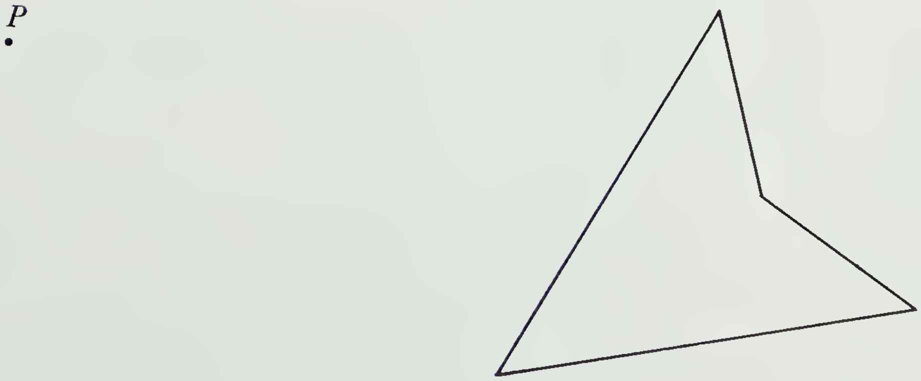
16.



17.



18.

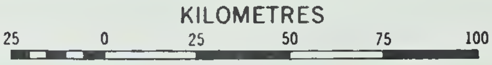


7.5 Using Maps

Performance Objective: The student can use the scale of a map to find real distances by measuring.

A map contains more information than the location of roads and cities. It also shows cities of various sizes, lakes, airports, parks, and in some cases, places of historical interest.

Every map has a *legend*, a section that gives information about the map. The legend below is for the map on page 311.



REFERENCE

(TOLL) ═══ ═══	Divided Highway
━━━	Principal Through Highway (Paved)
━ ━ ━	Principal Through Highway (Gravel)
───	Other Main Highways (Paved)
─ ─ ─	Other Main Highways (Gravel)
───	Other Roads
17	Trans-Canada Highway Marker
⑩ ㉙	Highway Markers Provincial
★ 25 ★	Kilometres between stars (Divided Highways)
★ 50 ★	Kilometres between stars
🟨	Park: National
🟩	Park: Provincial
- - - - - - -	Time Zone Boundary

POPULATED PLACES

OTTAWA	Nat. Capital
TORONTO	Prov. Capital
Montréal	over 200 000
Sarnia	50 000 to 200 000
Alma	10 000 to 50 000
Perth-Andover	2 000 to 10 000
Margaretsville	0 to 2000

Example: Find the shortest road distance and the straight-line distance from Saint John (B-4) to Chatham (B-2).

road ▶ 106 km (highway 7) + 177 km (highway 8) = 283 km

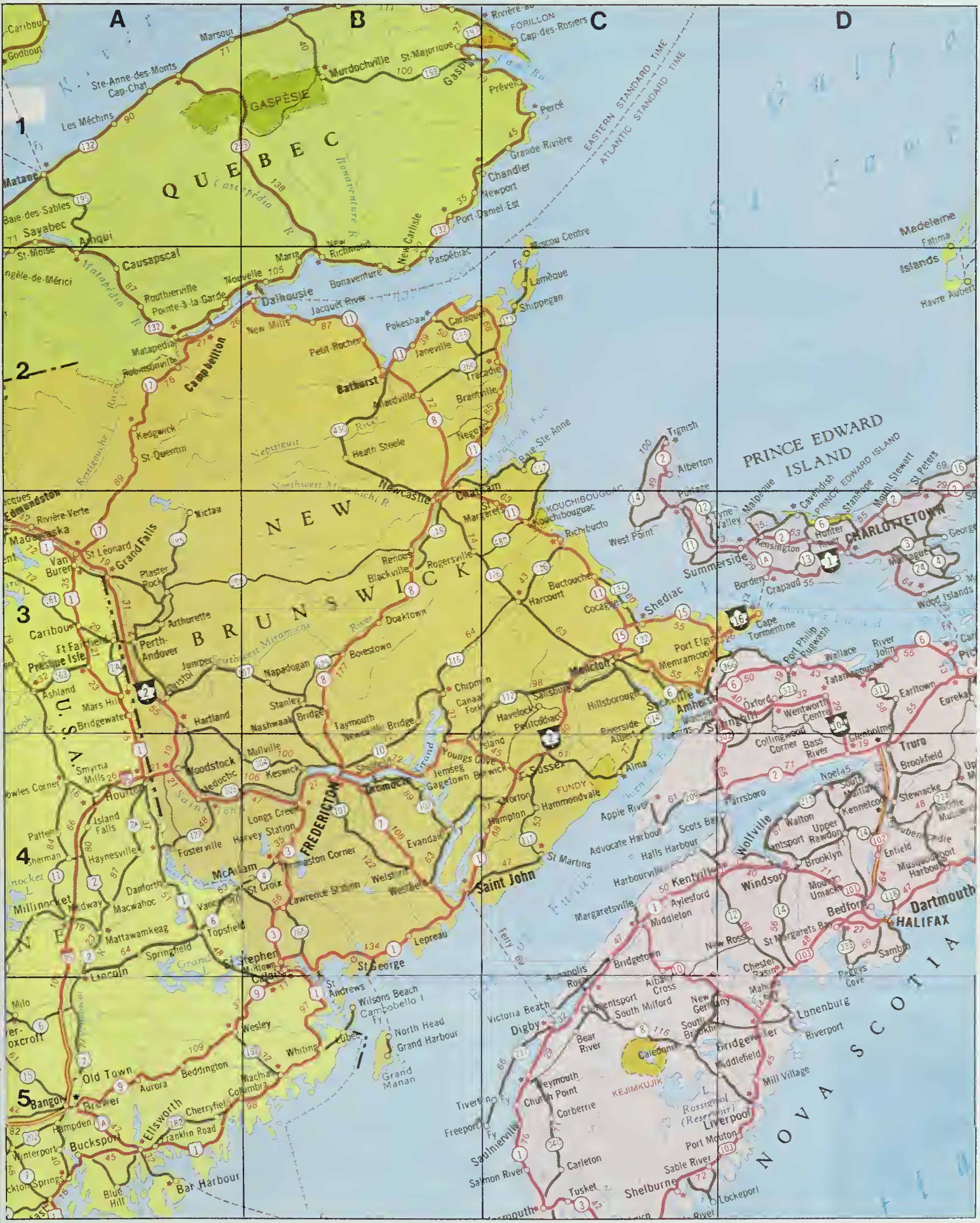
straight ▶ The map measurement is 7 cm and 1 cm = 28.5 km

$$7 \times 28.5 = 199.5 \text{ km}$$

Exercises

1–10. Use the map and the legend.

1. What is the scale of the map? 1 cm = 28.5 km
2. Name 2 provinces and 1 foreign country that border New Brunswick. Nova Scotia, Quebec, U.S.A.
3. Name a provincial park Gaspésie
4. Which water separates New Brunswick and Prince Edward Island? Northumberland Strait
5. Name the capital of Nova Scotia. Halifax
6. Name the capital of New Brunswick? Fredericton


A
B
C
D
1
2
3
4
5
QUEBEC
GASPÉSIE
Caribou
Godbout
Marsoui
Ste-Anne-des-Monts
Cap-Chat
Les Méchins
Matane
Baie-des-Sables
Sayabec
Amqui
Murdochville
St-Majorique
Gaspé
FORILLON
Cap-des-Rosiers
Rivière-au-
Prével
Percé
Grande Rivière
Chandler
Newport
Port-Daniel-Est
Paspébiac
New Carlisle
New Richmond
Bonaventure
Maria
Nouvelle
Causapscal
St-Moïse
Angèle-de-Mérici
Routhierville
Pointe-à-la-Garde
Matapedia
Robinsonville
Dalhousie
Jacquet River
New Mills
Campbellton
Kedgwick
St-Quentin
Petit-Rocher
Bathurst
Allardville
Heath Steele
Pokeshaw
Caraquet
Janeville
Tracadie
Brantville
Neguac
Miscou Centre
Lamèque
Shippegan
Baie-Ste-Anne
Newcastle
Chatham
St Margarets
Kouchibouguac
KOUCHIBOUGUAC
Richibucto
Rexton
Renous
Blackville
Rogersville
Doaktown
Boiestown
Napadogan
Harcourt
Buctouche
Cocagne
Shediac
Moncton
Hillsborough
Riverside-Albert
Alma
FUNDY
Hammondvale
Salisbury
Petitcodiac
Havelock
Canaan Forks
Chipman
Sussex
Norton
Hampton
St Martins
Saint John
Lepreau
St George
St Andrews
St Stephen
Milltown
Calais
McAdam
St Croix
Vanceboro
Topsfield
Fredericton
Oromocto
Gagetown
Jemseg
Youngs Cove
Welsford
Westfield
Lawrence Station
Harvey Station
Longs Creek
Keswick
Millville
Nashwaak Bridge
Taymouth
Stanley
Juniper
Bristol
Hartland
Woodstock
Meductic
Houlton
Perth-Andover
Plaster Rock
Arthurette
Grand Falls
St Léonard
Van Buren
Madawaska
Edmundston
Rivière-Verte
Nictau
Caribou
Fort Fairfield
Presque Isle
Ashland
Mars Hill
Bridgewater
Smyrna Mills
Island Falls
Haynesville
Danforth
Macwahoc
Mattawamkeag
Springfield
Lincoln
Millinocket
Medway
Sherman
Patten
Knowles Corner
Milo
Dover-Foxcroft
Old Town
Bangor
Brewer
Aurora
Beddington
Machias
Columbia
Cherryfield
Ellsworth
Franklin Road
Bar Harbour
Blue Hill
Bucksport
Hampden
Winterport
Bucksport Springs
Wesley
Whiting
Lubec
Wilsons Beach
Campobello I
North Head
Grand Harbour
Grand Manan
NEW BRUNSWICK
U.S.A.
NE
EASTERN STANDARD TIME
ATLANTIC STANDARD TIME
PRINCE EDWARD ISLAND
Tignish
Alberton
Portage
West Point
Tyne Valley
Malpeque
Cavendish
PRINCE EDWARD ISLAND
Stanhope
Mount Stewart
St Peters
Kensington
Hunter River
CHARLOTTETOWN
Summerside
Borden
Crapaud
Montague
Georgetown
Wood Islands
Cape Tormentine
Port Elgin
Memramcook
Sackville
Amherst
Springhill
Joggins
Oxford
Port Philip
Pugwash
Wallace
Tatamagouche
River John
Wentworth Centre
Collingwood Corner
Bass River
Earltown
Truro
Brookfield
Glenholme
Noel
South Maitland
Kennetcook
Stewiacke
Parrsboro
Advocate Harbour
Apple River
Scots Bay
Halls Harbour
Harbourville
Kentville
Wolfville
Walton
Upper Rawdon
Hantsport
Windsor
Brooklyn
Mount Uniacke
Enfield
Musquodoboit Harbour
Aylesford
Middleton
Margaretsville
New Ross
Bedford
Dartmouth
HALIFAX
St Margarets Bay
Chester Basin
Mahone Bay
Peggys Cove
Sambro
Lunenburg
Riverport
Bridgewater
Middlefield
Mill Village
Liverpool
Port Mouton
Sable River
Lockeport
Shelburne
Bridgetown
Annapolis Royal
Bridgeport
South Milford
Albany Cross
New Germany
South Brookfield
Caledonia
KEJIMKUJIK
Rossignol L (Reservoir)
Victoria Beach
Digby
Bear River
Weymouth
Church Point
Corberrie
Carleton
Tusket
Yarmouth
Saulnierville
Salmon River
Tiverton
Freeport
NOVA SCOTIA
Madeleine Islands
Fatima
Havre Aubert
Ferry
Fy
Eureka
Westville
Northumberland Strait
Bay of Fundy
Gulf of St Lawrence
Chaleur Bay
Miramichi Bay
Nepisiguit R
Northwest Miramichi R
Southwest Miramichi R
Restigouche R
Matapedia R
Cascapédia R
Bonaventure R
Saint John R
Grand L
Grand Falls
Cap-Chat
132
195
299
198
197
132
11
8
17
2
1
95
3
7
2
16
104
102
103
101

7. In which province will you find Grand Lake?
New Brunswick

8. In which province will you find Minas Basin?
Nova Scotia

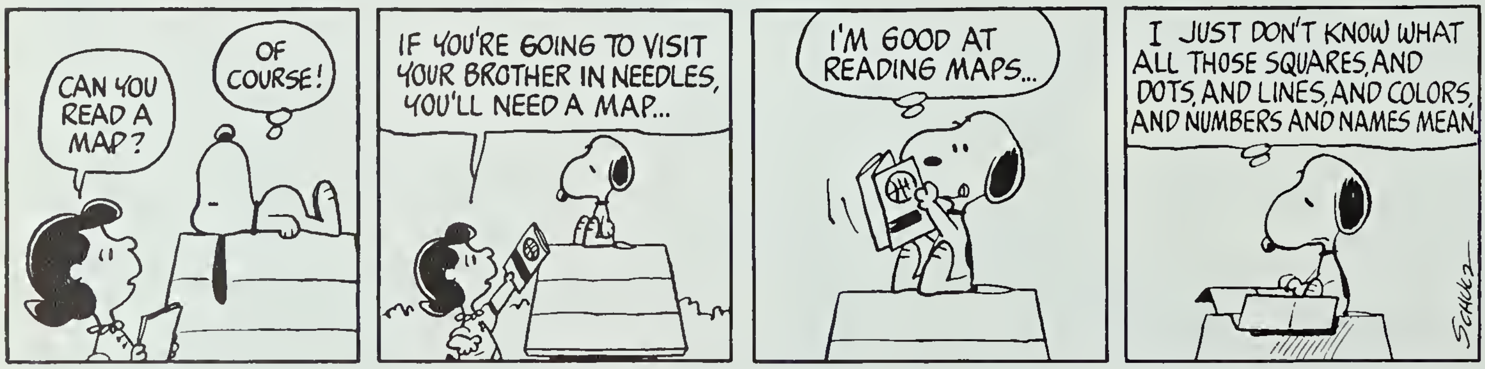
9. What divided highway is in Nova Scotia? 102

10. Name the largest lake in Nova Scotia.
Lake Rossignol

11—22. Copy and complete the chart. Use the map on page 311. Measure map distances to the nearest millimetre.

Ex. 11-22 *and* 27-34: *Answers are approximations (measured from center to center).*

	Straight line distance from	Map (cm)	Real (km)	Shortest Road distance (km)
	Example: Parrsboro (D-4) to Pugwash (D-3)	2.6	71	65 + 50 = 115
11.	Summerside (D-3) to Charlottetown (D-3)	1.8	51	63
12.	Newcastle (B-3) to Moncton (C-3)	4.1	117	137
13.	Fredericton (B-4) to St. Stephen (B-4)	3.3	94	128
14.	Bridgewater (D-5) to Shelburne (C-5)	3.2	91	117
15.	Digby (C-5) to Yarmouth (C-5)	3.2	91	105
16.	Moncton (C-3) to Truro (D-4)	5	143	175
17.	Bathurst (B-2) to Campbellton (A-2)	3.1	88	113
18.	Sussex (C-4) to Sackville (C-3)	3.2.	91	135
19.	Tignish (C-2 to Summerside (D-3)	2.2	63	92
20.	Halifax (D-4) to Kentville (C-4)	3	86	111
21.	Saint John (B-4) to Fredericton (B-4)	3	86	106
22.	Grand Falls (A-3) to Woodstock (A-4)	3.5	100	105


CAN YOU READ A MAP?
OF COURSE!
IF YOU'RE GOING TO VISIT YOUR BROTHER IN NEEDLES, YOU'LL NEED A MAP...
I'M GOOD AT READING MAPS...
I JUST DON'T KNOW WHAT ALL THOSE SQUARES, AND DOTS, AND LINES, AND COLORS, AND NUMBERS AND NAMES MEAN.
SCHULZ

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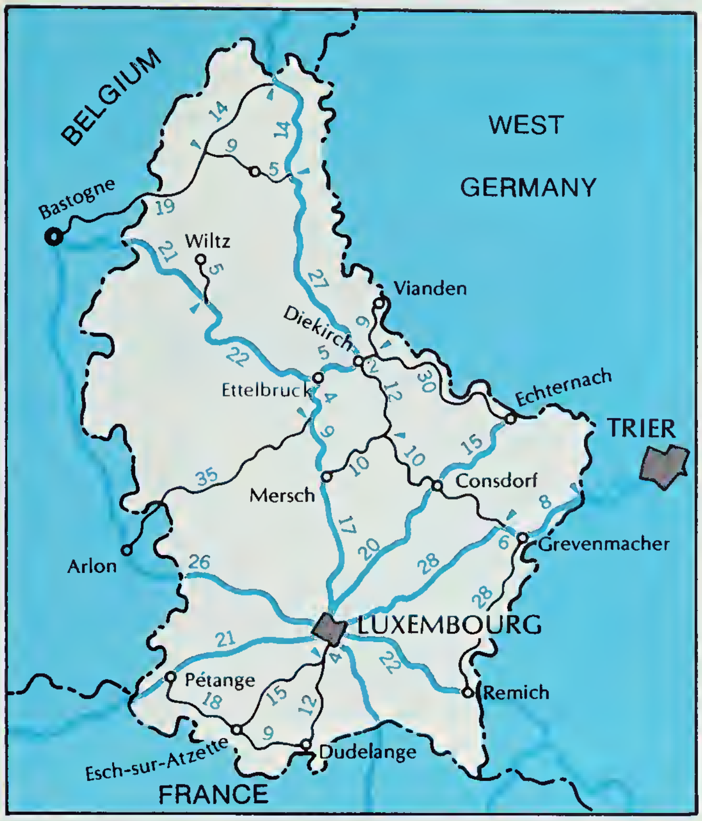
BELGIUM
WEST
GERMANY
Bastogne
Wiltz
Vianden
Diekirch
Ettelbruck
Echternach
TRIER
Mersch
Consdorf
Grevenmacher
Arlon
LUXEMBOURG
Pétange
Remich
Esch-sur-Atzette
Dudelange
FRANCE
14
9
14
5
19
21
5
27
9
5
22
4
12
30
9
15
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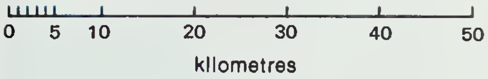
18
9

Scale:


0
5
10
20
30
40
50
kilometres

1 cm = 10 km

Luxembourg is a small country in Europe. Its capital city is also called Luxembourg. As in our country, road distances are measured in kilometres.

23—26. Use the map and the scale.

23. What is the scale of the map? 1 cm = 10 km

24. Name the three bordering countries. Belgium, France, West Germany

25. The distances given in blue are in what unit? kilometres

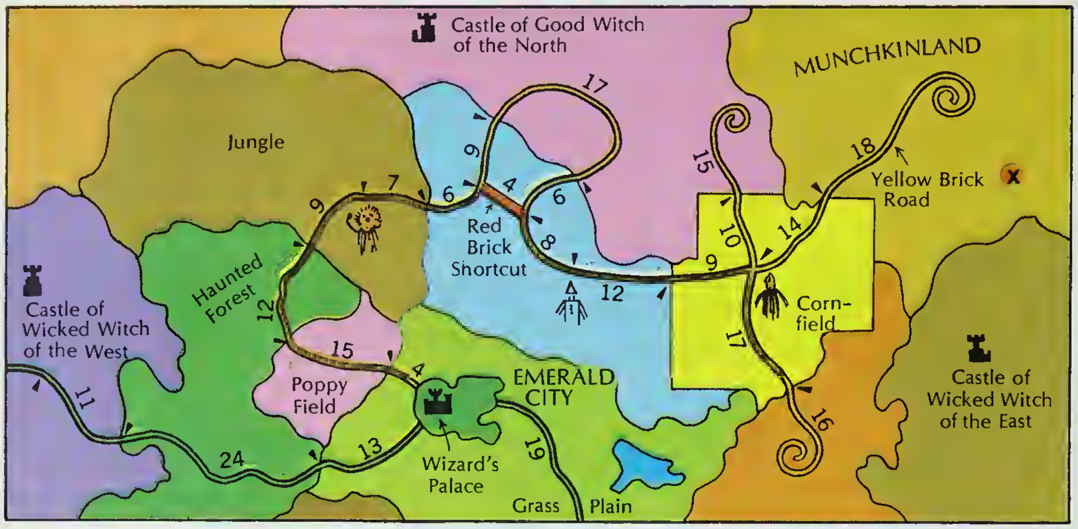
26. How many kilometres for a map distance of 3.4 cm? 34 km

27—34. Copy and complete the chart. Give map distances to the nearest 0.1 cm and real distances in kilometres.

Straight line distance from	Map	Real	Road distance	
Wiltz to Ettelbruck	1.8	18	5 + 22 = 27	◄ **Example**
27. Pétange to Dudelange	1.6	16	27	
28. Luxembourg to Grevenmacher	2.3	23	34	
29. Pétange to Echternach	4.6	46	56	
30. Mersch to Echternach	2.1	21	35	
31. Bastogne to Mersch	3.9	39	56	
32. Luxembourg to Trier	4	40	55	
33. Wiltz to Luxembourg	4.2	42	57	
34. Bastogne to Luxembourg	5.1	51	73	

FOLLOW THE YELLOW BRICK ROAD

There is an enchanted land that almost everyone has heard of but almost no one has visited. Distances in this land are measured in *gleebs*. You can walk a distance of 1 gleeb in about 5 minutes.



Castle of Good Witch of the North
MUNCHKINLAND
17
Jungle
15
18
Yellow Brick Road
X
9
7
6
4
6
Red Brick Shortcut
8
9
10
14
Haunted Forest
12
12
Castle of Wicked Witch of the West
Corn-field
15
4
EMERALD CITY
17
Poppy Field
Castle of Wicked Witch of the East
11
24
13
Wizard's Palace
19
16
Grass
Plain

Scale:
1 cm = 8 gleebs
X Place where Dorothy's house crashed
Place where Dorothy met:
the Scarecrow
theTinman
the Lion

1. The Munchkins walked Dorothy to the border of Munchkinland. How many gleebs was it from the border to the intersection where she met the scarecrow? 14 gleebs

2. How far along the yellow-brick road did Dorothy and the scarecrow travel before they met the tinman? 21 gleebs

3. How many gleebs would the red-brick shortcut have saved them? 28 gleebs

4. After they met the lion, how far was it to the Emerald City? 40 gleebs

5. What was the total distance Dorothy traveled on the yellow-brick road? If she had used the ruby slippers to fly straight from where her house crashed to the Wizard's palace, how far would she have gone? 146 gleebs; about 53 gleebs

YOUR PROVINCIAL MAP

Answers will vary.

Obtain a road map of the province in which you live. Study the map and the legend carefully.

1. What is the scale of the map?
2. Name the provinces that border your province.
3. Find the straight-line distance and the road distance between the two largest cities.
4. Name any rivers, mountains, lakes and highways.
5. If you were going to make a map of your province on a sheet of notebook paper, what would the scale of your map be?

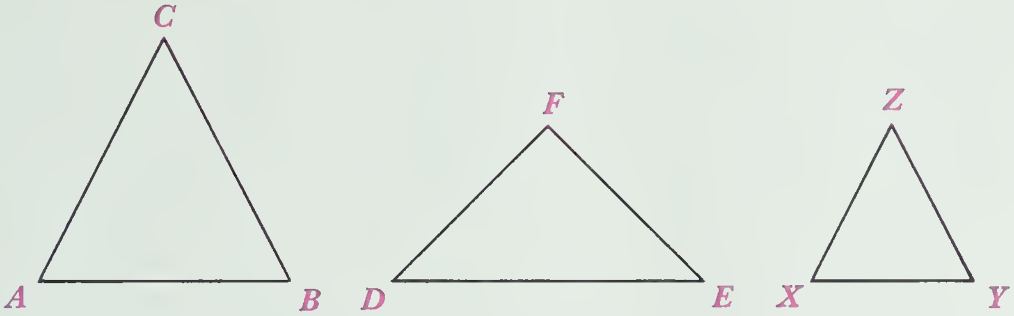
Similarity 7.6

Enlargements, reductions, and scale drawings are all examples of *similar* figures.

Performance Objective: The student can name all corresponding parts of two similar figures.

Two figures are **similar** if they have the same shape, but not necessarily the same size.

Triangles *ABC* and *XYZ* below are similar. If triangle *ABC* was reduced in size, it would look exactly like *XYZ*. But triangle *DEF* is *not* similar to the others. It has a different shape.



The similarity of two figures depends on the **correspondence**—the way the parts are matched up.

Example 1: Triangle *ABC* is similar to triangle *XYZ*. Give the corresponding parts.

Corresponding angles	Corresponding sides
angle $A \leftrightarrow$ angle X	$\overline{AB} \leftrightarrow \overline{XY}$
angle $B \leftrightarrow$ angle Y	$\overline{AC} \leftrightarrow \overline{XZ}$
angle $C \leftrightarrow$ angle Z	$\overline{BC} \leftrightarrow \overline{YZ}$

Since A corresponds to X and B corresponds to Y, side $\overline{AB}$ corresponds to $\overline{XY}$.

The corresponding parts are given by the *order* of the letters in the similarity statement.

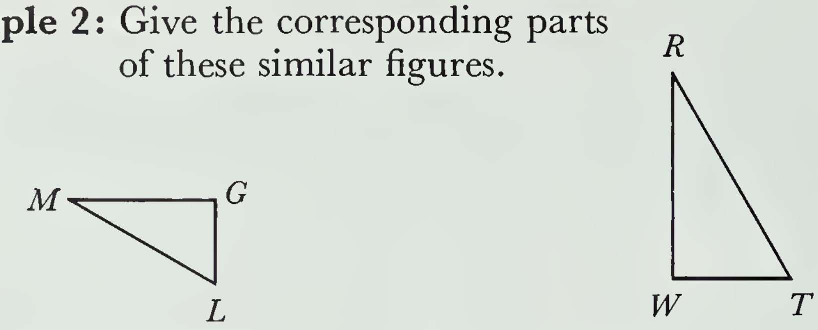
Triangle *ABC* is similar to triangle *XYZ*.

corresponding parts

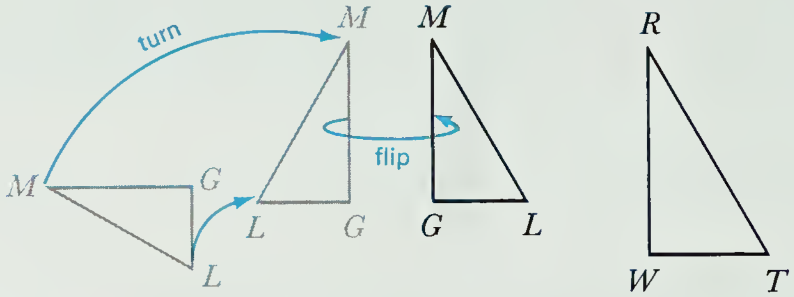
◀ Similarity statement

Sometimes you might have to imagine "moving" one of two similar figures to see the corresponding parts.

Example 2: Give the corresponding parts of these similar figures.



Give the smaller triangle first a "turn," then a "flip."



Now it is easy to write a similarity statement and give the corresponding parts.

The symbol ∠ means "angle."

Triangle *GLM* is similar to triangle *WTR*.

$\angle G \leftrightarrow \angle W$ $\overline{GL} \leftrightarrow \overline{WT}$

$\angle L \leftrightarrow \angle T$ $\overline{LM} \leftrightarrow \overline{TR}$

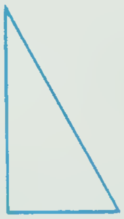
$\angle M \leftrightarrow \angle R$ $\overline{GM} \leftrightarrow \overline{WR}$

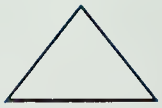
Can you see the corresponding parts in the original figures?

Exercises

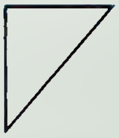
1–4. Give the letter of the one figure that looks similar to the blue figure.

1.

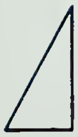




a.

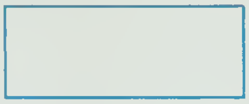


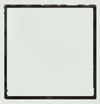
b.



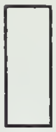
c.

2.

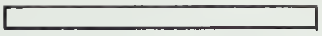




a.

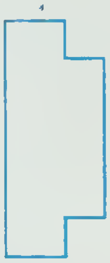


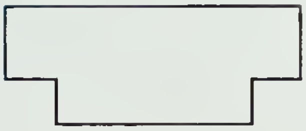
b.



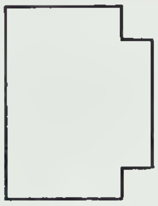
c.

3.

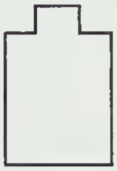




a.

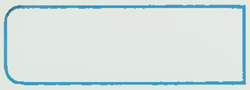


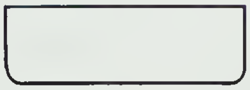
b.



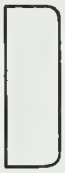
c.

4.

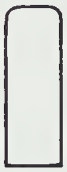




a.

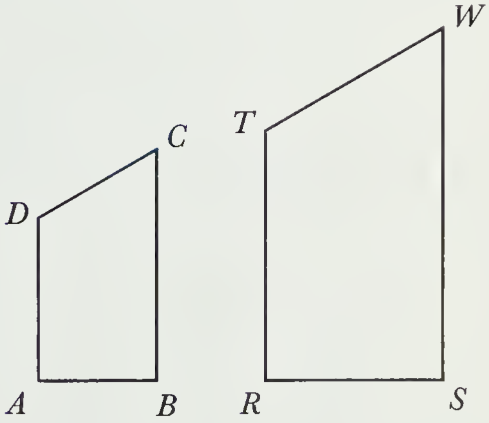


b.



c.

5–12. Give the corresponding angle or side. Figure *ABCD* is similar to figure *RSWT*.



Corresponding angles

5. $\angle A \leftrightarrow ?$ $\angle R$

6. $\angle B \leftrightarrow ?$ $\angle S$

7. $\angle D \leftrightarrow ?$ $\angle T$

8. $? \leftrightarrow \angle W$ $\angle C$

Corresponding sides

9. $\overline{AB} \leftrightarrow ?$ $\overline{RS}$

10. $\overline{AD} \leftrightarrow ?$ $\overline{RT}$

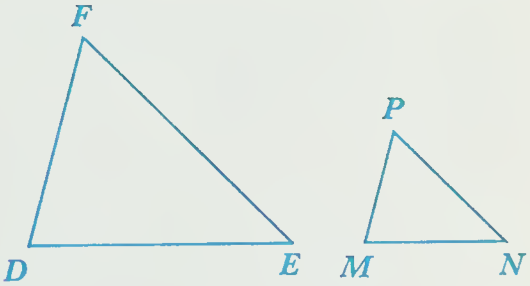
11. $? \leftrightarrow \overline{WT}$ $\overline{CD}$

12. $\overline{BC} \leftrightarrow ?$ $\overline{SW}$

13. Triangle *DEF* is similar to triangle *MNP*. Give the 3 pairs of corresponding angles and the 3 pairs of corresponding sides.

$\angle D \leftrightarrow \angle M$, $\angle E \leftrightarrow \angle N$, $\angle F \leftrightarrow \angle P$,

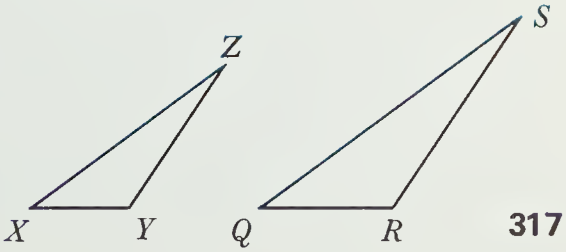
$\overline{DE} \leftrightarrow \overline{MN}$, $\overline{EF} \leftrightarrow \overline{NP}$, $\overline{DF} \leftrightarrow \overline{MP}$



14. Triangle *XYZ* is similar to triangle *QRS*. Give the 3 pairs of corresponding angles and the 3 pairs of corresponding sides.

$\angle X \leftrightarrow \angle Q$, $\angle Y \leftrightarrow \angle R$, $\angle Z \leftrightarrow \angle S$,

$\overline{XY} \leftrightarrow \overline{QR}$, $\overline{YZ} \leftrightarrow \overline{RS}$, $\overline{XZ} \leftrightarrow \overline{QS}$

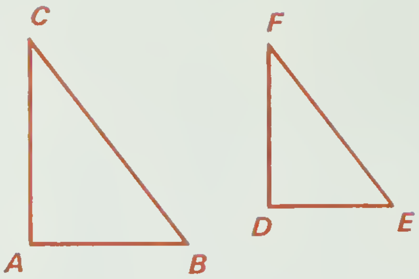


15–22. For each pair of similar figures, write a similarity statement and give *all* pairs of corresponding parts.

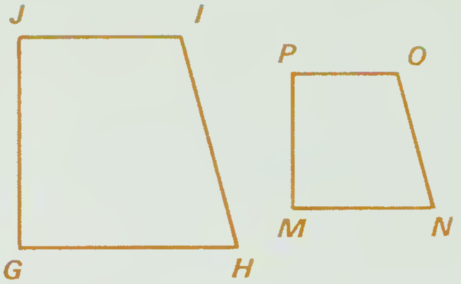
15. ABC is similar to DEF.
$\angle A \leftrightarrow \angle D$ $\overline{AB} \leftrightarrow \overline{DE}$
$\angle B \leftrightarrow \angle E$ $\overline{BC} \leftrightarrow \overline{EF}$
$\angle C \leftrightarrow \angle F$ $\overline{AC} \leftrightarrow \overline{DF}$

16. GHIJ is similar to MNOP.
17. RSTU is similar to YXZW.
18. CAT is similar to ODG.
19. OEA is similar to PMQ.
20. DFK is similar to YZX.

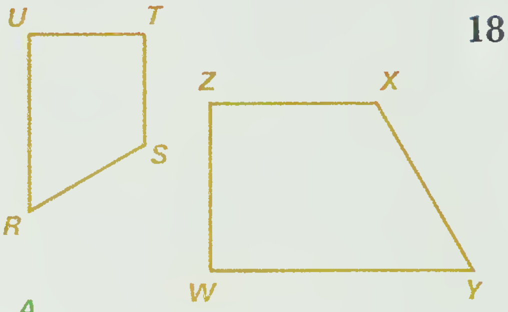
15.



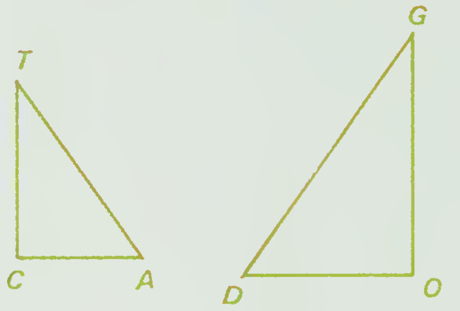
16.



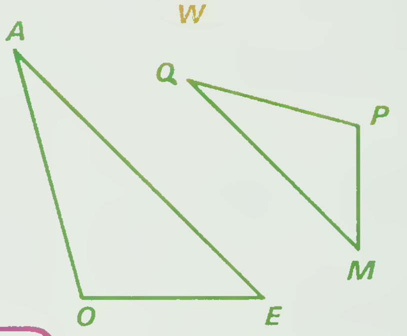
17.



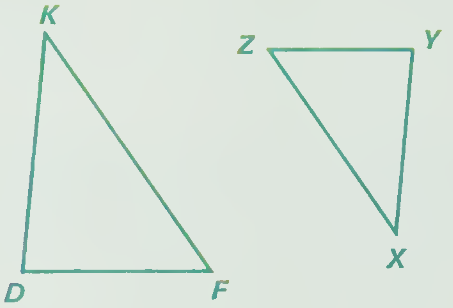
18.



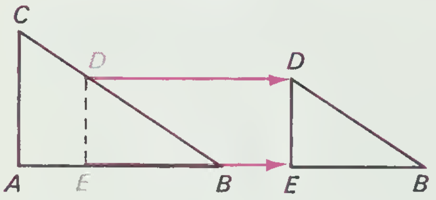
19.



20.

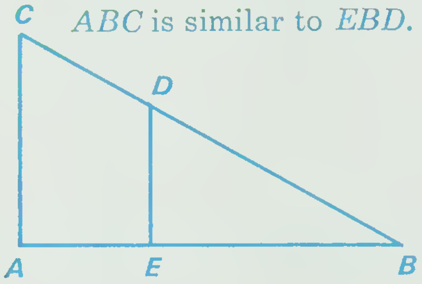


Hint for Exercises 21 and 22:



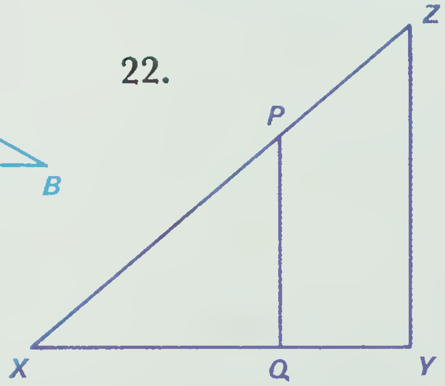
Imagine "sliding" the smaller triangle off the larger one. Notice that $\angle B$ is in *both* triangles. It corresponds to itself.

21. ABC is similar to EBD.



XYZ is similar to XQP.

22.



23–26. Give all pairs of corresponding angles and sides.

23. Triangle PQR is similar to triangle MTF.
$\angle P \leftrightarrow \angle M$, $\angle Q \leftrightarrow \angle T$, $\angle R \leftrightarrow \angle F$, $\overline{PQ} \leftrightarrow \overline{MT}$, $\overline{QR} \leftrightarrow \overline{TF}$, $\overline{PR} \leftrightarrow \overline{MF}$

24. Triangle YWZ is similar to triangle PEG.
$\angle Y \leftrightarrow \angle P$, $\angle W \leftrightarrow \angle E$, $\angle Z \leftrightarrow \angle G$, $\overline{YW} \leftrightarrow \overline{PE}$, $\overline{WZ} \leftrightarrow \overline{EG}$, $\overline{YZ} \leftrightarrow \overline{PG}$

25. Triangle DEF is similar to triangle KLO.
$\angle D \leftrightarrow \angle K$, $\angle E \leftrightarrow \angle L$, $\angle F \leftrightarrow \angle O$, $\overline{DE} \leftrightarrow \overline{KL}$, $\overline{EF} \leftrightarrow \overline{LO}$, $\overline{DF} \leftrightarrow \overline{KO}$

26. Triangle TUV is similar to triangle ABC.
$\angle T \leftrightarrow \angle A$, $\angle U \leftrightarrow \angle B$, $\angle V \leftrightarrow \angle C$, $\overline{TU} \leftrightarrow \overline{AB}$, $\overline{UV} \leftrightarrow \overline{BC}$, $\overline{TV} \leftrightarrow \overline{AC}$

Similar Figures

7.7

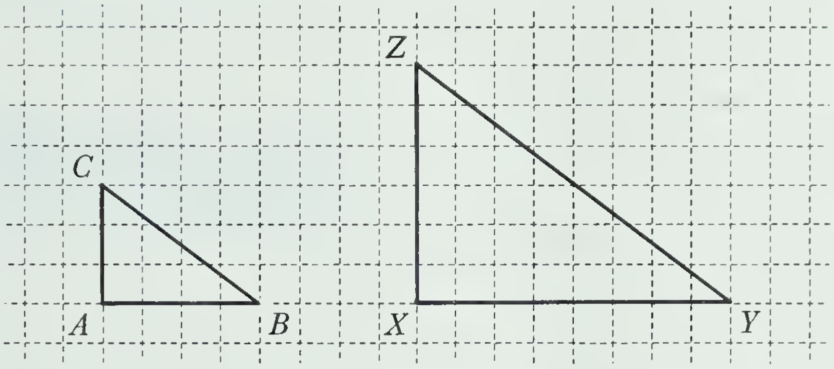
You know that similar figures are those that have the same shape, but here is a more precise description.

Similar figures are those in which:

1. Corresponding angles have the same measure.
2. Corresponding lengths are in the same ratio.

Performance Objective: The student can determine the missing sides (using proportions) and the missing angles of similar figures.

Example 1: Triangle *ABC* is similar to triangle *XYZ*.



By measuring the angles with a protractor, you can see that corresponding angles have the same measure.

Angle	Measure
$\angle A$	90°
$\angle B$	37°
$\angle C$	53°

Angle	Measure
$\angle X$	90°
$\angle Y$	37°
$\angle Z$	53°

By counting units of the grid, you can see that corresponding lengths are in the same ratio.

$$\frac{AB}{XY} = \frac{4}{8} = \frac{1}{2} \qquad \frac{AC}{XZ} = \frac{3}{6} = \frac{1}{2} \qquad \frac{BC}{YZ} = \frac{5}{10} = \frac{1}{2}$$

Each ratio: $\frac{\text{length from triangle } ABC}{\text{corresponding length of } XYZ} = \frac{1}{2}$

Note

To measure slanted sides, use a homemade ruler marked off in grid units.

We say the **ratio of similarity** of *ABC* to *XYZ* is $\frac{1}{2}$. (*ABC* is a $\frac{1}{2}$, or 1:2, reduction of *XYZ*)

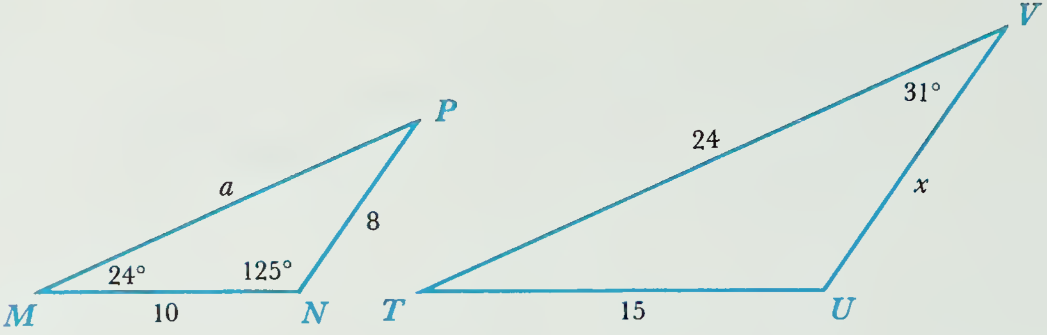
Note: The ratio of similarity of triangle *XYZ* to triangle *ABC* is $\frac{2}{1}$.

The properties in the box on the top of page 319 can be used to find missing parts of similar figures.

Example 2: Triangle MNP is similar to triangle TUV. Find all the missing parts.

a stands for the length of side $\overline{MP}$.

x stands for the length of side $\overline{UV}$.



There are 5 missing parts.

1. $\angle P$: $\angle P$ corresponds to $\angle V$, so $\angle P$ is 31°.
2. $\angle T$: $\angle M$ corresponds to $\angle T$, so $\angle T$ is 24°.
3. $\angle U$: $\angle N$ corresponds to $\angle U$, so $\angle U$ is 125°.
4. $\overline{MP}$: The ratio of corresponding lengths are the same, so a *proportion* can be set up.

Notice that corresponding letters are above and below one another.

$$\frac{MN}{TU} = \frac{MP}{TV}$$

Substitute the known lengths. $$\frac{10}{15} = \frac{a}{24}$$

Take the cross-products. $$10 \times 24 = 15 \times a$$

$$240 = 15a$$

Divide both sides by 15. $$\frac{240}{15} = \frac{15a}{15}$$

$$16 = a$$ So MP is 16.

5. $\overline{UV}$: Again use a proportion. $$\frac{MN}{TU} = \frac{NP}{UV}$$

Substitute. $$\frac{10}{15} = \frac{8}{x}$$

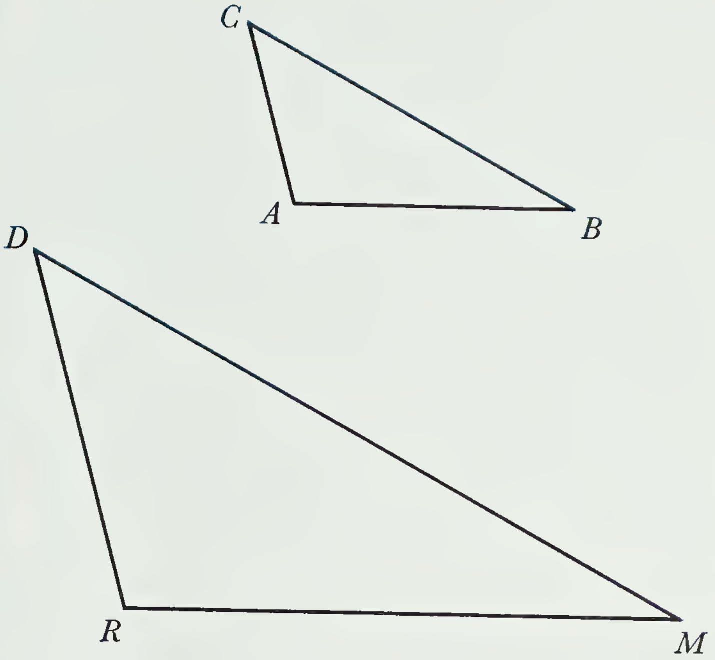
Take cross-products. $$10x = 120$$

Divide. $$x = 12$$ So UV is 12.

Exercises

1–8. Copy and complete each chart. Use triangles *ABC* and *RMD*. Measure angles in degrees and sides in centimetres.

	Angle	Measure
	∠*A*	105°
1.	∠*B*	29°
2.	∠*C*	46°
3.	∠*R*	105°
4.	∠*M*	29°
5.	∠*D*	46°



	Ratio of lengths	In lowest terms
6.	$\frac{AB}{RM} = \frac{3}{6}$	$= \frac{1}{2}$
7.	$\frac{BC}{MD} = \frac{4}{8}$	$= \frac{1}{2}$
8.	$\frac{AC}{RD} = \frac{2}{4}$	$= \frac{1}{2}$

9. Are the triangles similar? Yes

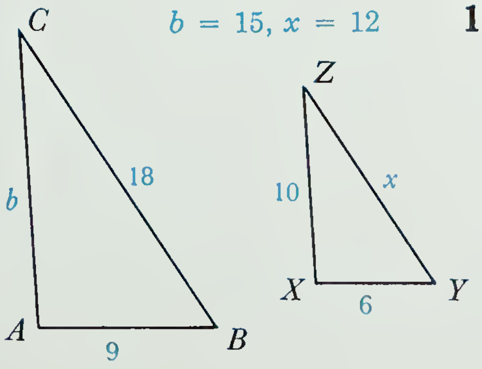
10. Write a similarity statement. Triangle *ABC* is similar to triangle *RMD*.

11. Give the ratio of similarity of triangle *ABC* to triangle *RMD*. $\frac{1}{2}$

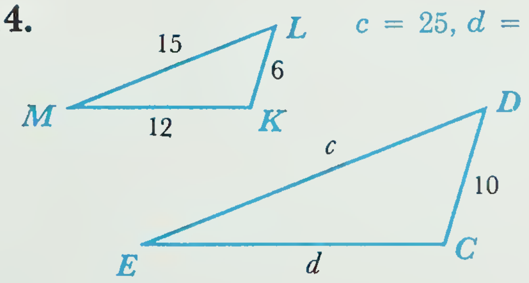
12. Give the ratio of similarity of triangle *RMD* to triangle *ABC*. $\frac{2}{1}$

13–16. The figures in each pair are similar. Use proportions to find the missing sides.

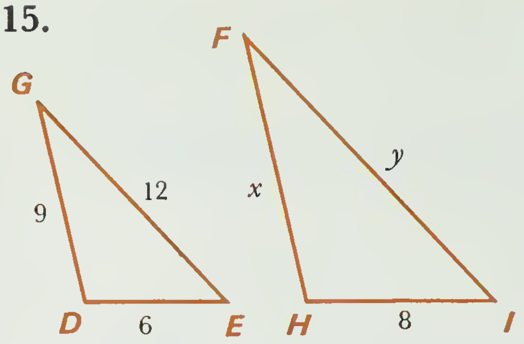
13. $b = 15, x = 12$



14. $c = 25, d = 20$



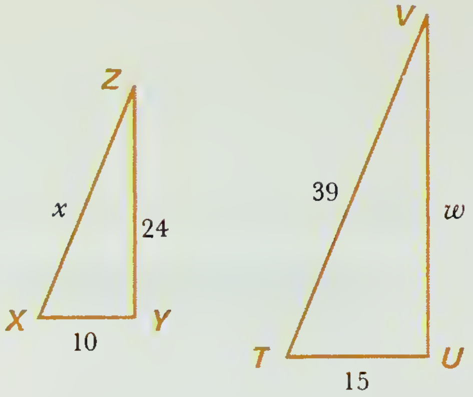
15.



G
F
12
y
9
x
D
6
E
H
8
I

$x = 12, y = 16$

16.



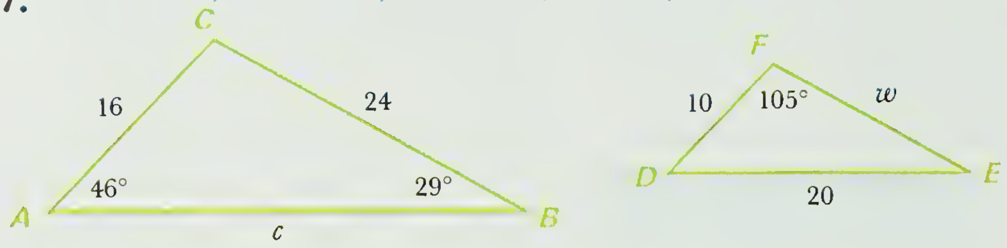
V
Z
39
w
x
24
X
10
Y
T
15
U

$x = 26, w = 36$

17–20. Find all the missing parts of the similar figures.

5 missing parts ▶

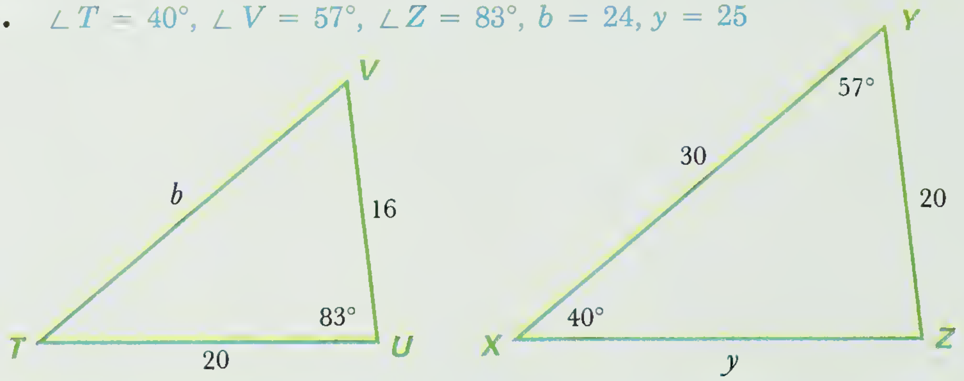
17. $\angle C = 105°, \angle D = 46°, \angle E = 29°, \ c = 32, w = 15$



C
16
24
46°
29°
A
c
B
F
10
105°
w
D
20
E

5 missing parts ▶

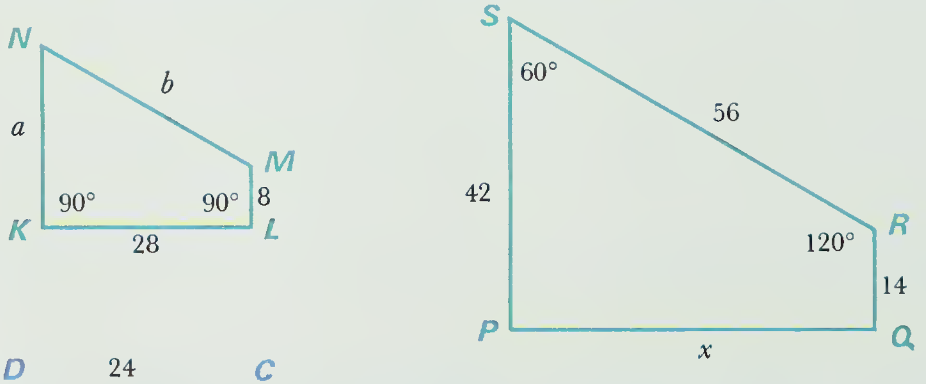
18. $\angle T = 40°, \angle V = 57°, \angle Z = 83°, b = 24, y = 25$



V
b
16
83°
T
20
U
Y
57°
30
20
40°
X
y
Z

7 missing parts ▶

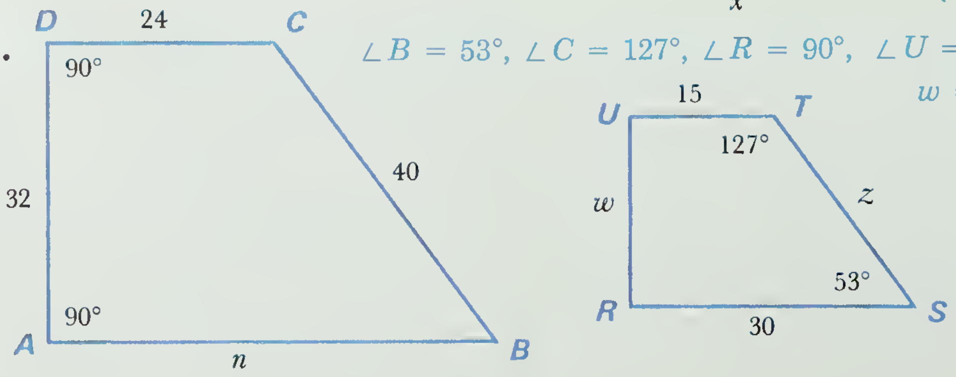
19. $\angle N = 60°, \angle M = 120°, \angle P = 90°, \ \angle Q = 90°, a = 24, b = 32, x = 49$



N
b
a
M
90°
90°
8
K
28
L
S
60°
56
42
120°
R
14
P
x
Q

7 missing parts ▶

20. $\angle B = 53°, \angle C = 127°, \angle R = 90°, \ \angle U = 90°, n = 48, w = 20, z = 25$



D
24
C
90°
40
32
90°
A
n
B
U
15
T
127°
w
z
53°
R
30
S

Constructing Triangles from Measurements

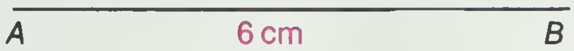
7.8

It is possible to construct a complete triangle when only some of the measurements are given.

Performance Objective:
The student can
- draw a triangle given essential measurements.
- draw a triangle similar to a given triangle and check lengths using proportions.

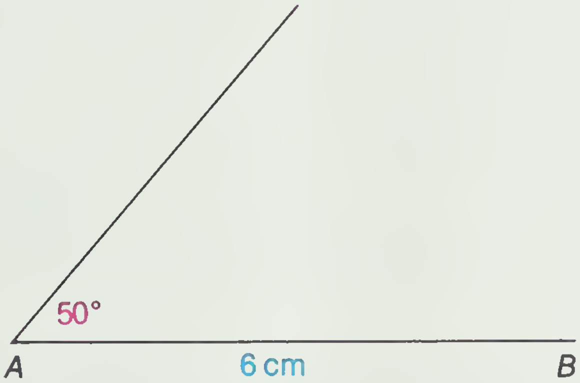
Example 1: Draw triangle ABC. AB is 6 cm, $\angle A$ is 50°, and $\angle B$ is 35°.

First draw the given side.

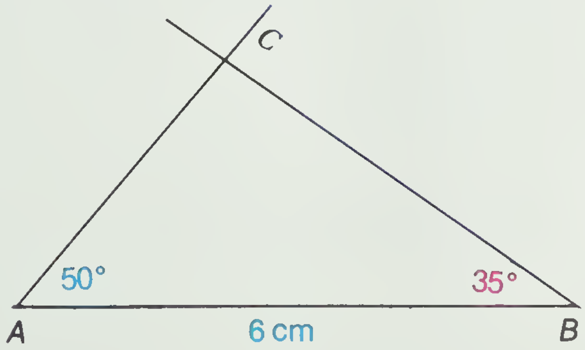


AB is 6 cm. Use a ruler to draw this side.

Then draw one of the given angles.



$\angle A$ is 50°. Use a protractor to draw a 50° angle at point A.

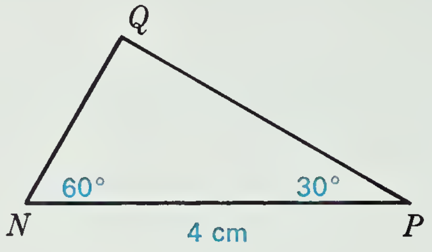


$\angle B$ is 35°. Use a protractor to draw a 35° angle at point B.

The point where the sides of $\angle A$ and $\angle B$ cross must be point C. The triangle is now complete.

In Example 1 a triangle was constructed from measurements. The same method can be used to draw a triangle similar to a given triangle.

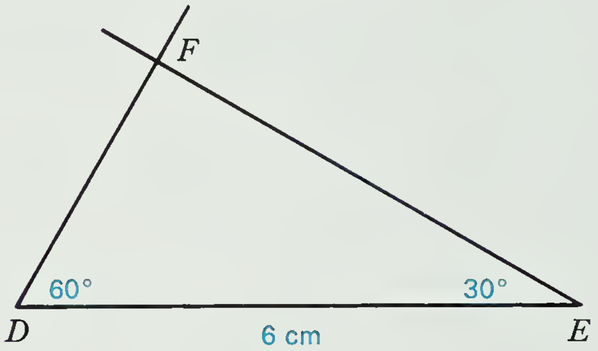
Example 2: Draw triangle DEF similar to triangle NPQ. Make $\overline{DE}$ 6 cm long.



The similarity statement shows the corresponding parts:

- $\overline{DE}$ corresponds to $\overline{NP}$, so draw $\overline{DE}$ in the same position as $\overline{NP}$, but 6 cm long.
- $\angle D$ corresponds to $\angle N$, so make $\angle D$ 60°.
- $\angle E$ corresponds to $\angle P$, so make $\angle E$ 30°.

The point where the sides of $\angle D$ and $\angle E$ cross is F. It corresponds to Q.



Example 3: Use proportions to show that sides $\overline{DF}$ and $\overline{EF}$ came out the correct size.

$\overline{DF}$: Measure its corresponding side $\overline{NQ}$. $\overline{NQ}$ is 2 cm long.

Ratios of corresponding lengths

$$\frac{DE}{NP} = \frac{DF}{NQ}$$

$$\frac{6}{4} = \frac{x}{2}$$

$$12 = 4x$$

$$3 = x$$

Now measure $\overline{DF}$ and $\overline{EF}$.

Is $\overline{DF}$ 3 cm long?

Yes!

$\overline{EF}$: Measure its corresponding side $\overline{PQ}$. $\overline{PQ}$ is 3.5 cm long.

$$\frac{DE}{NP} = \frac{EF}{PQ}$$

$$\frac{6}{4} = \frac{x}{3.5}$$

$$21.0 = 4x$$

$$5.25 = x$$

Is $\overline{EF}$ 5.25 cm long?

Close enough!

Exercises

1–6. Draw each triangle.

1–6. *Drawings*

1. Triangle ABC: AB is 90 mm, $\angle A$ is 90°, and $\angle B$ is 30°.

2. Triangle RST: RS is 5 cm, $\angle R$ is 80°, and $\angle S$ is 45°.

3. Triangle LMN: LM is 6 cm, $\angle L$ is 40°, and $\angle M$ is 65°.

4. Triangle GHJ: GH is 7.5 cm, $\angle G$ is 20°, and $\angle H$ is 40°.

5. Triangle XYZ: XY is 100 mm, $\angle X$ is 45°, and $\angle Y$ is 35°.

6. Triangle PQR: PQ is 4 cm, $\angle P$ is 70°, and $\angle Q$ is 55°.

7–10. Use your drawings for Exercises 1–4.

7. In Ex. 1, measure side $\overline{AC}$ to the nearest millimetre. 52 mm

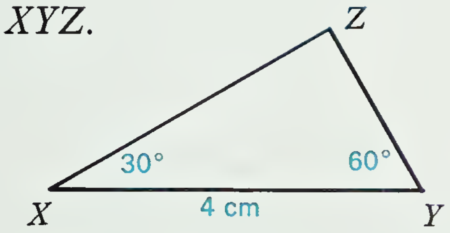
8. In Ex. 2, measure side $\overline{ST}$ to the nearest 0.5 cm. 6 cm

9. In Ex. 3, measure side $\overline{LN}$ to the nearest 0.5 cm. 5.5 cm

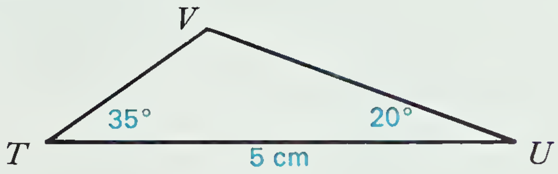
10. In Ex. 4, measure side $\overline{GJ}$ to the nearest millimetre. 56 mm

Ex. 11–14: Drawings omitted.

11. Draw triangle ABC similar to triangle XYZ. Make $\overline{AB}$ 8 cm long. Then follow Example **3** to check if $\overline{AC}$ and $\overline{BC}$ are the correct size.
BC is 4 cm; AC is about 7 cm.

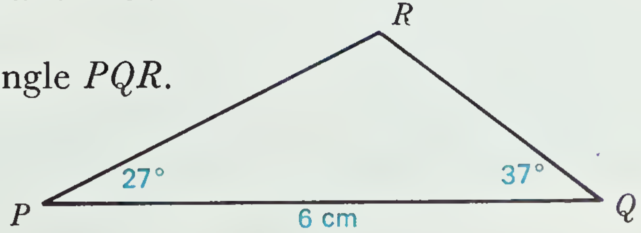


12. Draw triangle GHJ similar to triangle TUV. Make $\overline{GH}$ 10 cm long. Then follow Example **3** to check if $\overline{GJ}$ and $\overline{HJ}$ are the correct size.
GJ is about 4 cm; HJ is about 7 cm.



13. Draw triangle MNO similar to triangle PQR. Make $\overline{MN}$ 9 cm long. Check $\overline{MO}$ and $\overline{NO}$.
MO is about 6 cm; NO is about 4.5 cm.

14. Draw triangle DEF similar to triangle PQR. Make $\overline{DE}$ 8 cm long. Check $\overline{DF}$ and $\overline{EF}$.
DF is about 5.3 cm; EF is about 4 cm.

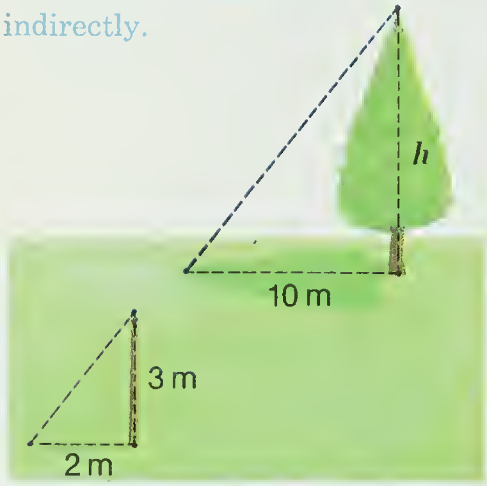


7.9 Indirect Measurement

Similar figures and proportions can be used to find measurements when it is difficult to measure directly.

Performance Objective: The student can use scale drawings and proportions to determine measures indirectly.

Example 1: A 3 m pole has a 2 m shadow. At the same time a tree has a 10 m shadow. How tall is the tree?



The sketch shows the two similar triangles. You can use a proportion.

$$\frac{2}{10} = \frac{3}{h}$$

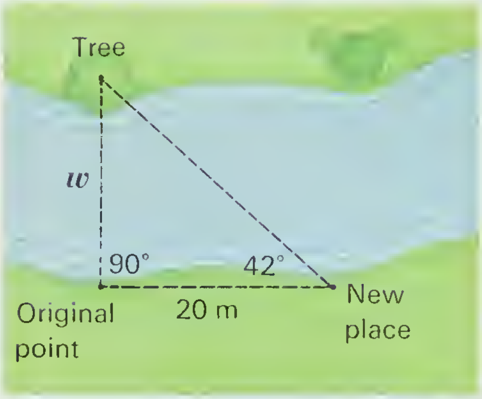
$$2h = 30$$ Take cross-products.

$$h = 15$$ Divide by 3.

The tree is 15 m tall.

The pole and its shadow were a ready-made similar figure. If there is no such figure, you can draw one from measurements. Straight distances can be measured with a metrestick. Angles can be measured with an instrument such as a transit, a device used by surveyors. (See page 291.)

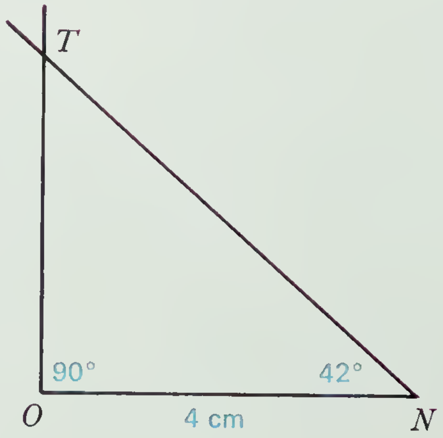
Example 2: From a point on the shore of a river directly across from a tree, you walk 20 m along the shore. The tree is now at a 42° angle with the shore. How wide is the river?



$\overline{ON}$ was drawn 4 cm long. Therefore the scale of the drawing is:

4 cm = 20 m

Make a scale drawing of any size from the given measurements.



Now that the scale drawing is finished, you can write a proportion. The distance to be found is the width w of the river. That corresponds to $\overline{OT}$ on the scale drawing.

Carefully measure $\overline{OT}$. It is 3.6 cm.

$$\frac{4}{20} = \frac{3.6}{w}$$

◀ Scale drawing measurements (cm)
◀ Corresponding real measurements (m)

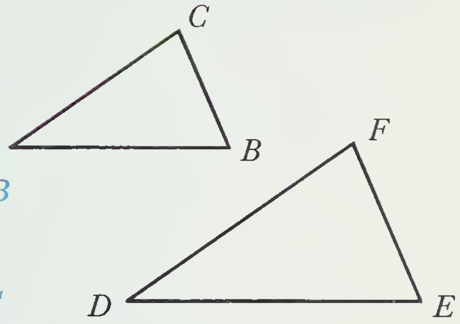
$$4w = 72.0$$

$$w = 18.0$$

The river is 18 m wide.

Exercises

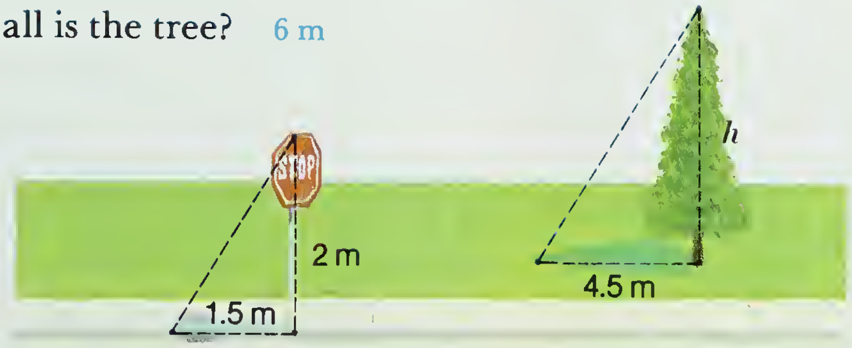
1–6. Triangle *ABC* is similar to triangle *DEF*. Copy and complete each proportion.



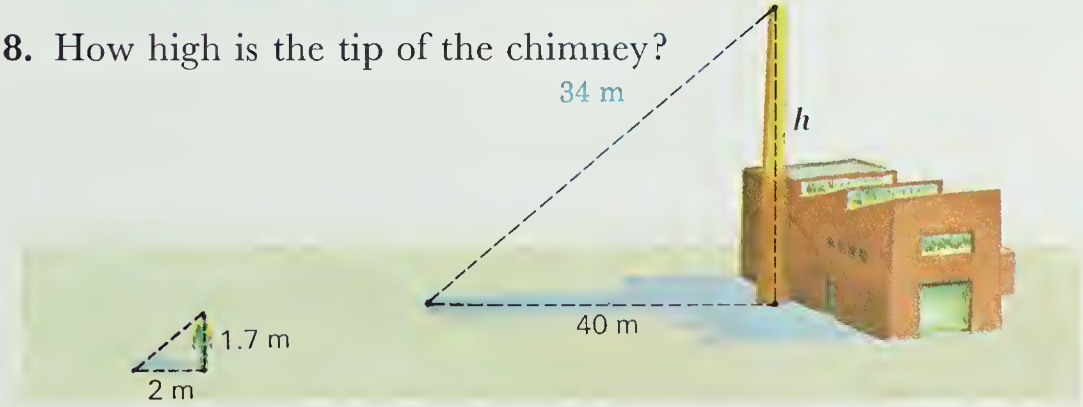
1. $\frac{AB}{DE} = \frac{BC}{?}$ *EF*
2. $\frac{BC}{EF} = \frac{AC}{?}$ *DF*
3. $\frac{BC}{EF} = \frac{?}{DE}$ *AB*
4. $\frac{AC}{DF} = \frac{?}{EF}$ *BC*
5. $\frac{DE}{AB} = \frac{EF}{?}$ *BC*
6. $\frac{DE}{AB} = \frac{DF}{?}$ *AC*

7–8. Find each measurement by using a proportion.

7. How tall is the tree? 6 m

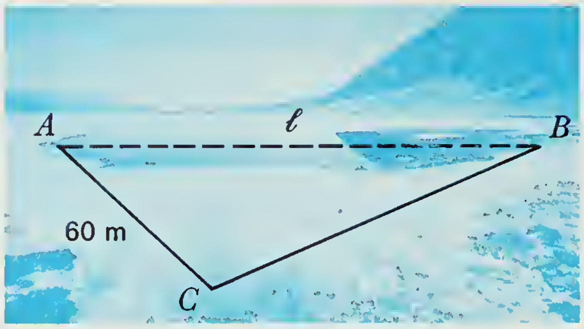


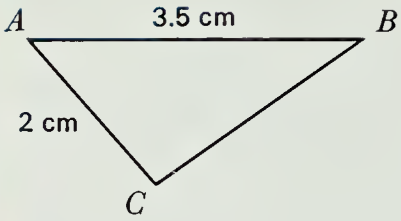
8. How high is the tip of the chimney? 34 m



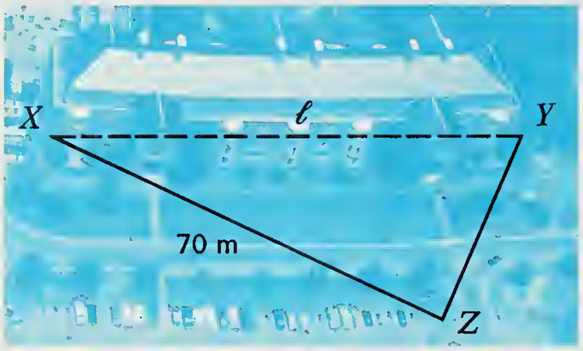
9–10. Use the scale drawing and a proportion.

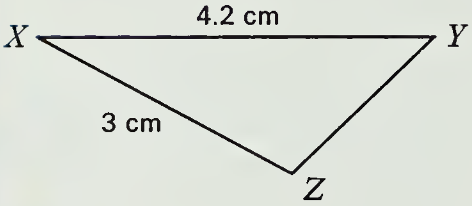
9. How long is the lake? 105 m




A
3.5 cm
B
2 cm
C

10. How long is the building? 98 m



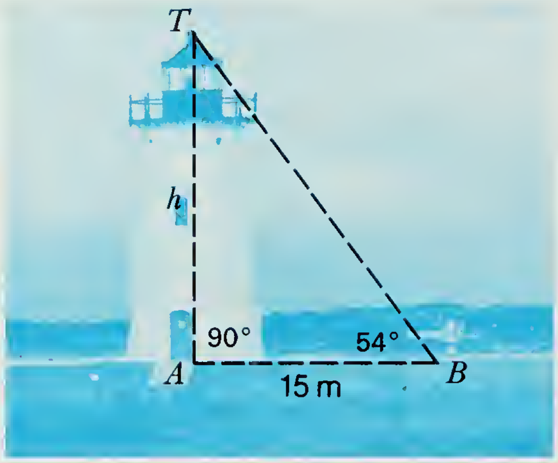

X
4.2 cm
Y
3 cm
Z

Ex. 11–20: Drawings omitted.

11–12. Make an accurate scale drawing from the given measurements. Then measure your drawing and use a proportion.

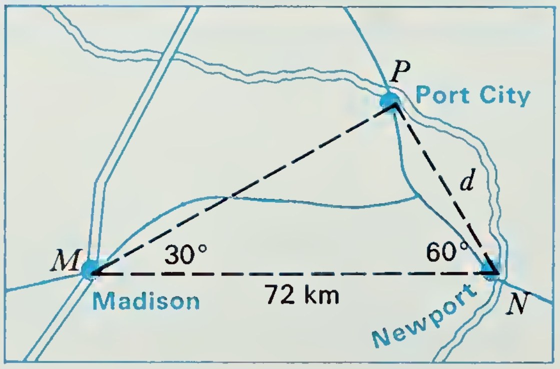
11. How tall is the lighthouse? About 21 m

Draw $\overline{AB}$ 5 cm long. Measure $\overline{AT}$ to the nearest millimetre.


T
h
90°
54°
A
15 m
B

12. How many kilometres is it from Newport to Port City? About 36 km

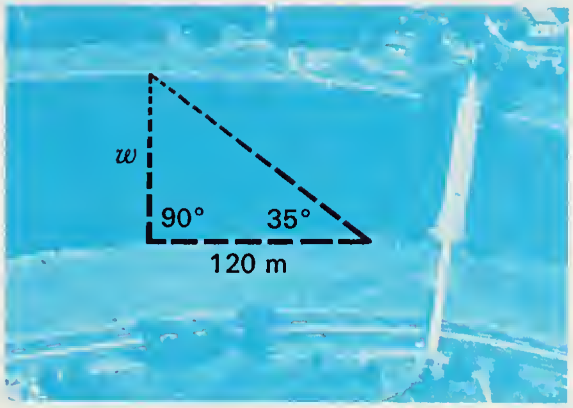
Draw $\overline{MN}$ 6 cm long. Measure $\overline{NP}$ to the nearest 0.5 cm.


P
Port City
d
M
30°
60°
Madison
72 km
Newport
N

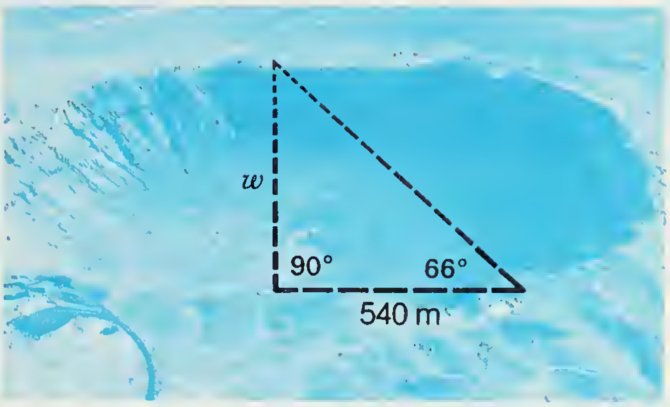
All Photos Courtesy American Airlines

13–18. Make a scale drawing of any size from the given measurements. Then measure your drawing and use a proportion.

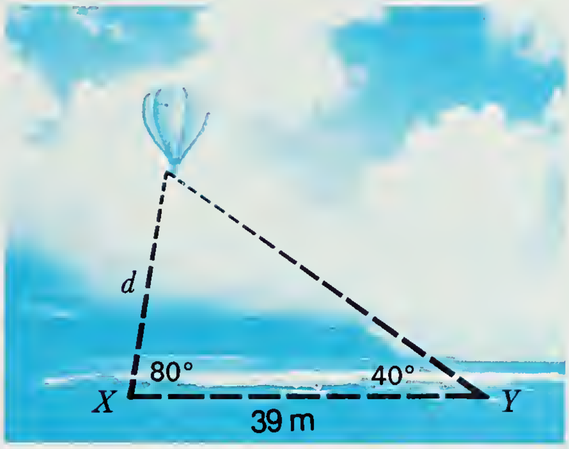
13. How wide is the river? About 84 m



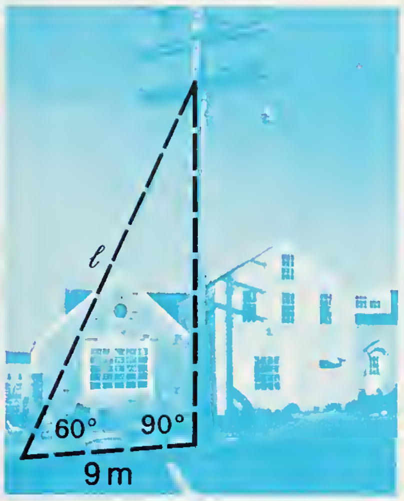
14. How wide is the crater? About 1200 m



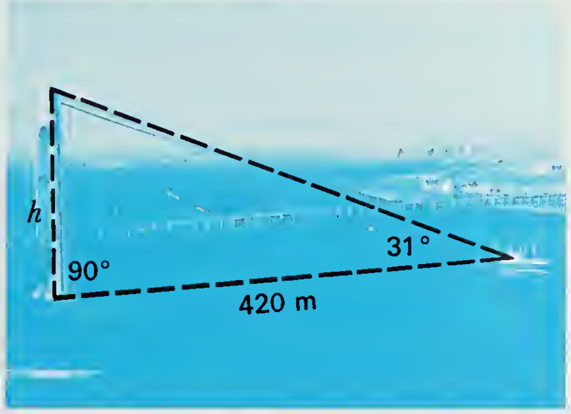
15. How far is the balloon from X? About 27 m



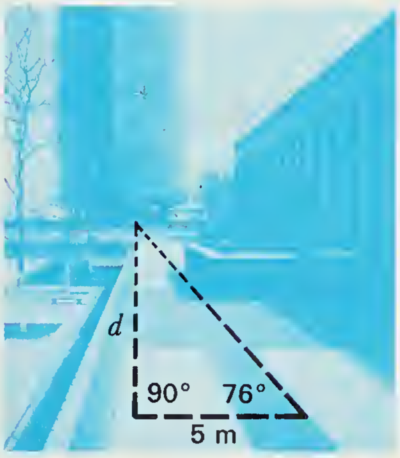
16. How long is the support wire? About 22 m



17. How tall is the bridge support? About 250 m



18. How far away is the building? About 20 m



19–20. Solve using a proportion. Make a sketch.

19. A person 180 cm tall has a shadow 120 cm long. How tall is a building that has a shadow 8 m long? 12 m

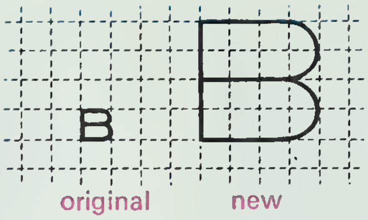
20. If a 1.5 m fence has a 1 m shadow, how tall is a flagpole with a 6 m shadow? 9 m

All Photos Courtesy American Airlines

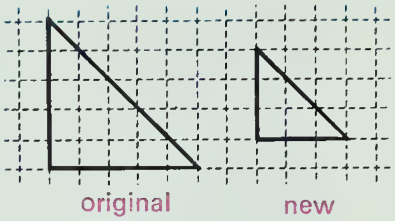
Test for Chapter 7

Give each ratio, new figure: original figure. (7.1)

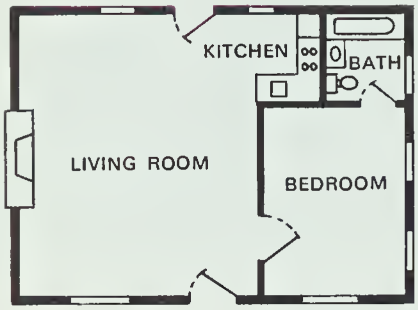
1. 4:1



2. 3:5



Floor plan scale:
1 cm = 2 m



From the floor plan find each measurement. (7.2)

3. length of living room 6 m

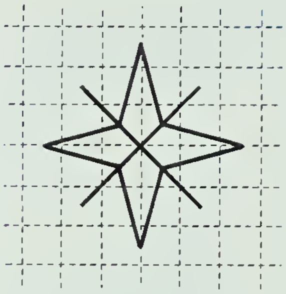
4. length of bedroom 4 m

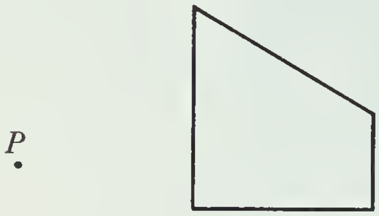
5. length of fireplace 2 m

6. width of bedroom 3 m

7–8. *Drawings*

7. Use the grid method to make a 3:2 enlargement of the figure at the right. (Use a grid with 2 squares per centimetre.) (7.3)





8. Use the point method to make a 3:1 enlargement of the figure at the left. (7.4)

A map has a scale of 1 cm = 16 km. Find the real distance for each map distance. (7.5)

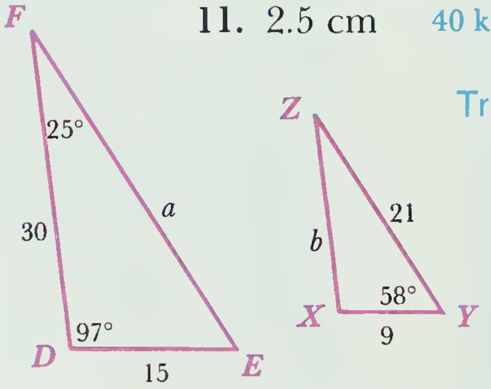
9. 2 cm 32 km

10. 1.75 cm 28 km

11. 2.5 cm 40 km

12. 3.125 cm 50 km

Triangle *DEF* is similar to triangle *XYZ*.



13. Give all 6 pairs of corresponding parts. (7.6)

14. Find all 5 missing parts. (7.7)

13.
$\angle D \leftrightarrow \angle X$, $\angle E \leftrightarrow \angle Y$. $\angle F \leftrightarrow \angle Z$,
$\overline{DE} \leftrightarrow \overline{XY}$, $\overline{EF} \leftrightarrow \overline{YZ}$, $\overline{DF} \leftrightarrow \overline{XZ}$

14.
$\angle E = 58°$, $\angle X = 97°$, $\angle Z = 25°$,
$a = 35$, $b = 18$

Construct triangle *MPQ* from measurements. (7.8)

15. *MP* is 5 cm, $\angle M$ is 50°, and $\angle P$ is 25°.
Drawing

Solve using a proportion. Make a sketch. (7.9)

16. A 1 m post has a 0.75 m shadow. How tall is a tree with a 9 m shadow? 12 m

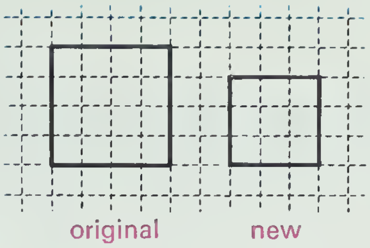
RATE YOURSELF

Number Correct	
16	Superior
14-15	Excellent
12-13	Good
9-11	Fair
0-8	Poor

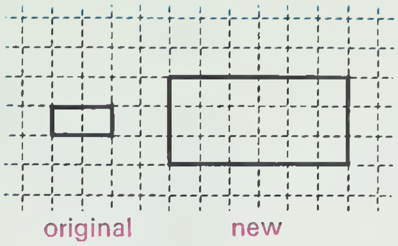
Chapter 7 Review

Give each ratio, new figure: original figure. (7.1)

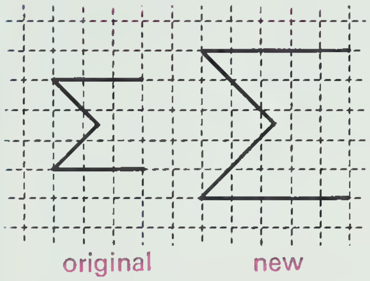
1. 3:4



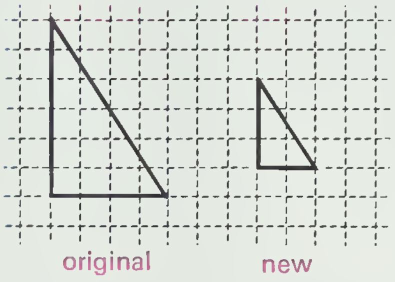
2. 3:1



3. 5:3



4. 1:2

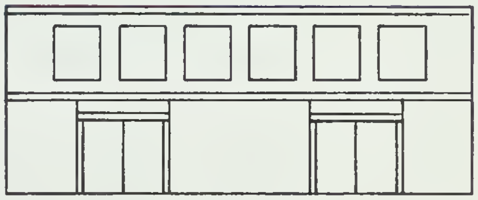


Copy and complete the chart. Use the scale drawing of the building at the right. (7.2)

Ex. 5–8 and 13–18: Answers are approximations.

	Measurement	Drawing (cm)	Real (m)
5.	Total length	5	15
6.	Total height	2	6
7.	Distance between doorways	1.5	4.5
8.	Width of each window	.5	1.5

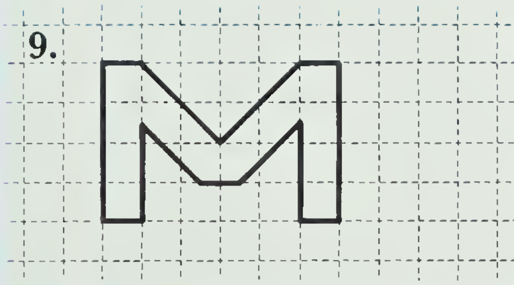
Scale:
1 cm = 3 m



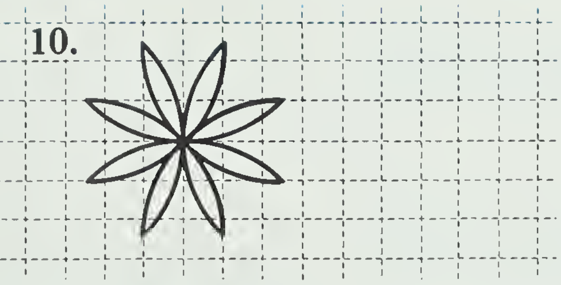
9–12. *Drawings*

Use the grid method to make a 3:2 enlargement. Use a grid with 2 squares to the centimetre. (7.3)

9.



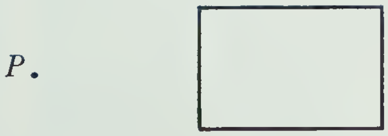
10.



Assume that this grid has 3 squares per centimetre.

Use the point method to make a 3:1 enlargement. (7.4)

11.



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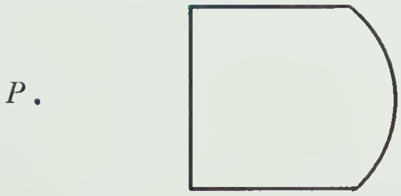
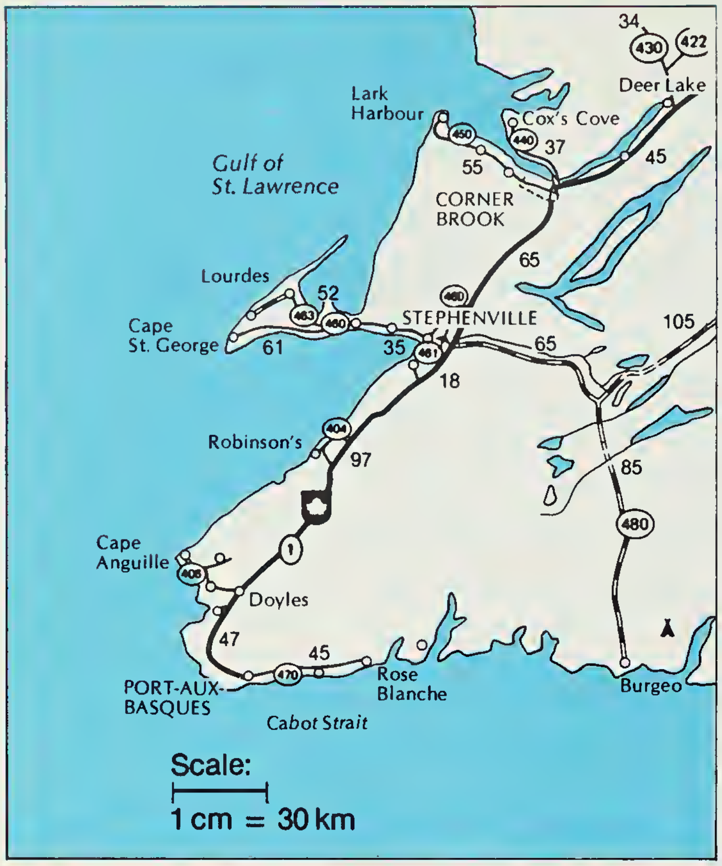




34
430
422
Deer Lake
Lark Harbour
450
Cox's Cove
440
37
55
45
Gulf of St. Lawrence
CORNER BROOK
65
Lourdes
52
460
463
460
STEPHENVILLE
105
Cape St. George
61
35
461
65
18
Robinson's
404
97
85
480
Cape Anguille
1
406
Doyles
47
45
470
Rose Blanche
PORT-AUX-BASQUES
Burgeo
Cabot Strait
Scale:
1 cm = 30 km

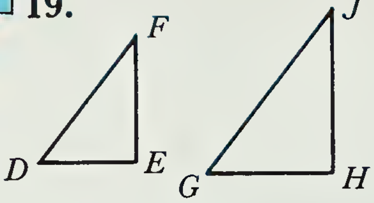
For each pair of places find the straight-line distance on the map to the nearest millimetre and the real distance in kilometres. Then find the shortest road distance. (7.5)

	From	To	Map	Real	Road
13.	Corner Brook	Deer Lake	1.5	45	45
14.	Stephenville	Port-aux-Basques	4.1	123	171
15.	Burgeo	Stephenville	4	120	150
16.	Deer Lake	Lark Harbour	2.5	75	92
17.	Lourdes	Doyles	2.9	87	219
18.	Port-aux-Basques	O'Regans	1.4	42	76

For each pair of similar triangles below write a similarity statement and give all 6 pairs of corresponding parts. *Answers at left.* (7.6)

19. *DEF* is similar to *GHJ*.
20. *KLM* is similar to *PQR*.

19.



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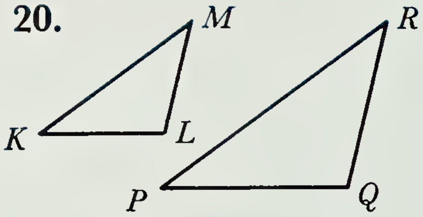
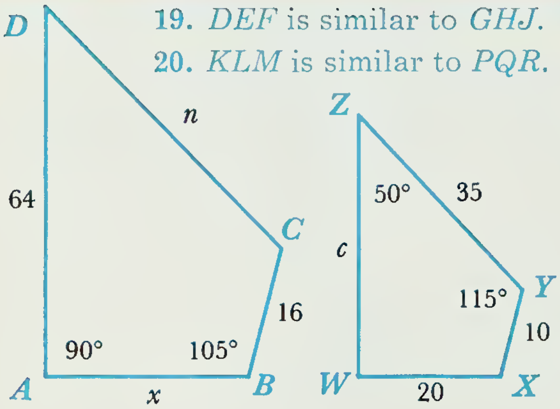


Figure *ABCD* is similar to figure *WXYZ*. (7.7)



D
n
64
C
16
90°
105°
A
x
B
Z
50°
35
c
115°
Y
10
W
20
X

21. Find the 4 missing angles.
$\angle C = 115°$, $\angle D = 50°$, $\angle W = 90°$, $\angle X = 105°$

22. Find the 3 missing sides.
$x = 32$, $n = 56$, $c = 40$

Construct each triangle from measurements. (7.8)

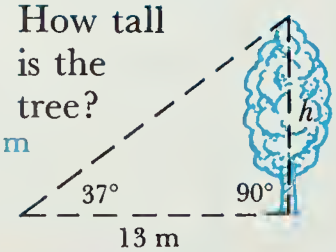
23–24. *Drawings*

23. Triangle *ABC*: *AB* is 6 cm, $\angle A$ is 40°, and $\angle B$ is 55°.

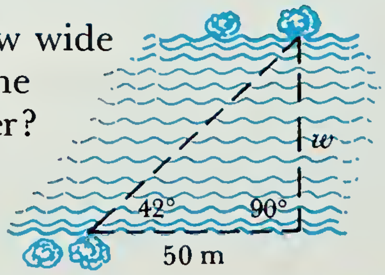
24. Triangle *TUV*: *TU* is 40 mm, $\angle T$ is 70°, and $\angle U$ is 45°.

Make a scale drawing, measure, and use a proportion. (7.9)

25. How tall is the tree?
About 7 m



26. How wide is the river?
About 45 m



Preview Test for Chapter 8

Find each cash-on-hand amount. (8.1)

Income	Expense	Cash on hand
		78.25
	22.65	**1.** 55.60
	9.42	**2.** 46.18
18.97		**3.** 65.15
	42.16	**4.** 22.99

Find each percent. (8.2)

Find each amount. (8.3)

Amount	Percent	Budget category	Percent	Amount
\$15	**5.** 25%	◀ food ▶	22%	**9.** \$55
\$12	**6.** 20%	◀ clothing ▶	16%	**10.** \$40
\$6	**7.** 10%	◀ savings ▶	8%	**11.** \$20
\$27	**8.** 45%	◀ other ▶	54%	**12.** 135
\$60	100%	◀ totals ▶	100%	\$250

Using unit costs, find the better buys. (8.4)

13. 50 g for 36¢ 0.72
125 g for 85¢ 0.68←

14. 225 g for 92¢ 0.41←
325 g for \$1.50 0.46

15. Make a cheque record (see form at right) and fill it in. Starting balance: \$126.84
Cheque No. 612: A & B Co., \$29.60, *groceries*
613: Tripp's, \$42.25, *clothing*
Deposit: \$82.14
614: D. Kohn, \$125.00, *rent* (8.5)

Cheque No	Description	Amount	Deposit	Bal
	To For			Bal 97.24
	To For			Bal 54.99
	To For			Bal 137.13
	To For			Bal 12.13

16. Make a form (see right) to reconcile the account. Bank balance: \$209.63; cheque record balance: \$151.55; outstanding cheques: \$84.28; outstanding deposit: \$25.00; service charge: \$1.20 (8.6)

	Bank balance		Cheque record balance
−	Outstanding cheques	−	Service charges
125.35		150.35	Corrected balance
+	Outstanding deposits		
150.35	Corrected balance		

17. Find C (credit cost) if n (number of payments) is 14, a (amount of each payment) is \$12.50, and b (amount borrowed) is \$160. \$15

$$C = na - b$$

(8.7)

18. Find R (annual percentage rate) if n (number of payments) is 14, b (amount borrowed) is \$400, and C (credit cost) is \$45. 18%

$$R = \frac{24C}{b(n+1)}$$

(8.8)

19. Find the tax on \$19,260 if the Tax Rate Schedule reads \$4510 + 36% of the amount over \$18,000. \$4963.60 (8.9)

Add or subtract the times. (8.10)

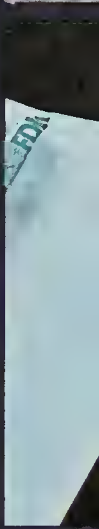
20. 4:28
+ 2:46
7:14

21. 10:26
− 2:47
7:39

22. 2:09 p.m.
− 9:42 a.m.
4:27 or 4 h 27 min

ne tax
rn

CANADA
CANADA
CANADA
CANADA



SUM
RIGHT
SOME OF THE B
UNDER T

8

Consumer $Skills

PICTURE PREVIEW: Everyone is a consumer, a user of goods and services. If you want to get the most for your money, you will want to find ways to watch your income and expenses. Also, you will want to be careful about purchases, especially those made on credit. The pictures on these pages show some of the areas covered in this chapter.

Photo credits clockwise from top left:
Johnson/De Wys, Inc., Brent Jones, Robert Brown, A. Devaney, Inc., William Parker, Richard Werle, William Means, William Parker, Robert Brown.

8.1 Managing Personal Finances

Performance Objective: The student can compute the cash on hand after each item in a record of expenses.

The first step toward managing your finances is to find how you spend your money. This requires keeping accurate records. One way is to use a chart such as the one below.

Example: High school student's record of spending for a week

Day	Item	Income	Expense	Cash on hand
	Left from last week			4.82
Sun.	Allowance	20.00		24.82
Mon.	Bus fare & lunch		1.65	23.17
	School supplies		.90	22.27
Tues.	Bus fare & lunch		1.85	20.42
Wed.	Bus fare & lunch		2.07	18.35
	Clothing		8.42	9.93
	Earnings	6.00		15.93
Thurs.	Bus fare & lunch		1.40	14.53
	Phonograph record		4.18	10.35
Fri.	Bus fare & lunch		1.72	8.63
	Entertainment & food		2.76	5.87
Sat.	Movie		2.50	3.37

Each income amount is added to the cash on hand. Each expense is subtracted. The cash-on-hand column tells you how much money you have at any given time.

Exercises

1. On the chart, Income is added to the cash on hand.
2. On the chart, Expenses are subtracted from the cash on hand.
3. What was the total income for the week? $26.00
4. How much was spent on clothing? $8.42

5–8. Find the amount of cash on hand for each letter **a** to **k**.

5. High school student—one week

Day	Item	Income	Expense	Cash on hand
	Left from last week			23.60
Mon.	Lunch		1.25	**a.** 22.35
	Radio repair		4.90	**b.** 17.45
Tues.	Lunch		.95	**c.** 16.50
	Gift		5.00	**d.** 11.50
Wed.	Lunch		1.27	**e.** 10.23
	Extra earnings	6.00		**f.** 16.23
Thurs.	Lunch		.98	**g.** 15.25
Fri.	Lunch		1.16	**h.** 14.09
	Paycheque	32.80		**i.** 46.89
Sat.	Deposit to savings		25.00	21.89
	Dentist bill		15.00	**j.** 6.89
	Movie & food		4.25	**k.** 2.64

◀ Checkpoint

If you get the number shown, your work up to this point is correct.

6. Single working person—one week

Day	Item	Income	Expense	Cash on hand
	Left from last week			11.40
Fri.	Lunch		2.56	**a.** 8.84
	Paycheque	187.50		**b.** 196.34
	Entertainment		9.47	**c.** 186.87
Sat.	Groceries		21.18	**d.** 165.69
	Food & entertainment		22.40	**e.** 143.29
Sun.	Gasoline		9.75	**f.** 133.54
Mon.	Lunch		2.27	**g.** 131.27
Tues.	Lunch		4.21	**h.** 127.06
	Deposit to savings		40.00	**i.** 87.06
Wed.	Lunch		4.82	82.24
Thurs.	Lunch		1.49	**j.** 80.75
	Telephone bill		12.47	**k.** 68.28

◀ Checkpoint

7. Family of 4—first two weeks of the month

Week	Item	Income	Expense	Cash on hand
	Previous month's balance			349.72
First week	Rent		275.00	**a.** 74.72
	Groceries		56.12	**b.** 18.60
	Gasoline & bus fares		12.90	**c.** 5.70
	Paycheque (two weeks)	427.80		**d.** 433.50
	Recreation		24.00	**e.** 409.50
Second week	Groceries		42.80	**f.** 366.70
	Clothing		45.18	**g.** 321.52
	Dentist bill		17.00	**h.** 304.52
	Gasoline & bus fares		9.75	**i.** 294.77
	Car payment		106.18	**j.** 188.59
	Personal needs		34.16	**k.** 154.43
	Recreation		18.40	136.03

Checkpoint ▶

8. Family of 4—second two weeks of the month

Week	Item	Income	Expense	Cash on hand
	Left from first two weeks			136.03
Third week	Groceries		41.16	**a.**
	Fuel, electric, telephone		34.98	**b.**
	Insurance premium		27.00	**c.**
	Paycheque (with overtime)	452.40		**d.**
	Car repairs		32.80	**e.**
	Savings		80.00	**f.**
	Gasoline & bus fares		14.40	**g.**
Fourth week	Groceries		49.60	**h.**
	Personal needs		43.64	**i.**
	Gasoline & bus fares		11.20	**j.**
	Clothing		21.50	**k.**
	Gifts & recreation		37.55	194.60

Checkpoint ▶

9–14. Use the charts in Exercises **7** and **8**.

9. What was the family's income for the whole month? $880.20

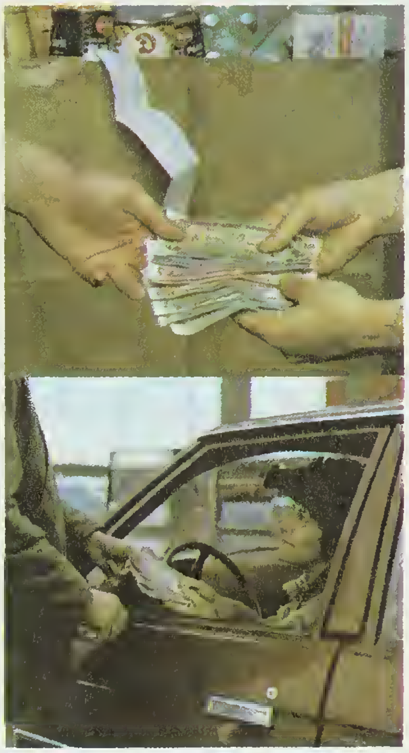
10. How much did the family spend on clothing for the entire month? $66.68

11. Find the month's total for groceries. $189.68

12. Find the total they spent on transportation (car payments, gasoline, repairs, bus fares). $187.23

13. How much money did they have at the beginning of the month (previous month's balance)? How much did they have at the end of the month? $249.72, $194.60

14. Did they spend more or less than their income during the month? How much more or less? $55.12



Robert Brown

TALK IT OVER

1. Some expenses are fixed, the same each month (such as rent). Other expenses are variable, change from month to month (such as recreation). Look over the expenses on the charts on page 338. Which expenses are fixed and which are variable?

Typical answer:

Fixed:	Variable:
Rent	Groceries
Car payment	Gasoline
Insurance	Recreation
	Clothing
	Medical
	Personal
	Utilities
	Repairs
	Savings
	Gifts

2. If income does not match expenses, something needs to be changed. How might income be changed? How can variable expenses be changed? How can fixed expenses be changed?
Typical answers: Income: work overtime, second job, etc.; variable: reduce unnecessary expenses, look for sales, etc.; fixed: get less expensive housing, smaller car, etc.

1–8. Reduce to lowest terms.

1. $\frac{3}{18}$ $\frac{1}{6}$

2. $\frac{4}{36}$ $\frac{1}{9}$

3. $\frac{4}{14}$ $\frac{2}{7}$

4. $\frac{6}{16}$ $\frac{3}{8}$

5. $\frac{8}{18}$ $\frac{4}{9}$

6. $\frac{6}{30}$ $\frac{1}{5}$

7. $\frac{3}{21}$ $\frac{1}{7}$

8. $\frac{10}{25}$ $\frac{2}{5}$

Quick Quiz

Check this skill:

- *reducing fractions to lowest terms*

8.2 How Do You Spend It?

After keeping records for a time, you will notice that most of your expenses fall into certain categories, or areas, such as the following:

Performance Objective: The student can compute the percent of income spent on each budget category, given the cash amount.

Food	groceries, beverages, meals out, etc.
Housing	rent, mortgage payments, property taxes, etc.
Clothing	clothes, cleaning, mending, etc.
Personal	allowances, medical & dental, education, etc.
Transportation	car payments, bus fares, parking, etc.
Recreation	movies, parties, vacations, etc.
Savings	savings accounts or other investments
Other	whatever is not included elsewhere

This is not the only way expenses can be divided. Different people will spend their money in different ways. But, no matter how you arrange your expenses, the next step is to find what part of your income *on the average* is spent on each area. This can be done with percents.

Example: Cathy has an average weekly income of $30. She usually spends her income as shown at the right. Find the percent of her income she spends on each category.

Food	$6
Clothing	$5
Recreation	$7
Personal	$3
Savings	$6
Other	$3
Total	$30

To find a percent, you can use a proportion.

$6 out of $30 is spent on food. That is the same as what out of 100?

$$\frac{6}{30} = \frac{x}{100}$$

$$600 = 30x$$ Take the cross-products.

$$20 = x$$ Divide both sides by 30.

20% of the total income is spent on food.

The following chart shows all the percents worked out:

Category	Average amount	Proportion used	Percent
Food	\$6	$\frac{6}{30} = \frac{x}{100}$	20%
Clothing	\$5	$\frac{5}{30} = \frac{x}{100}$	$16\frac{2}{3}\%$
Recreation	\$7	$\frac{7}{30} = \frac{x}{100}$	$23\frac{1}{3}\%$
Personal	\$3	$\frac{3}{30} = \frac{x}{100}$	10%
Savings	\$6	$\frac{6}{30} = \frac{x}{100}$	20%
Other	\$3	$\frac{3}{30} = \frac{x}{100}$	10%
Totals	\$30		100%

Notice that the percents add up to 100%. This is always true and allows you to check your work easily.

Shortcut for solving proportions

Sometimes, but not always, you can make a proportion easier to solve by *reducing* one of the sides.

$$\frac{6}{30} = \frac{x}{100}$$

$$\frac{1}{5} = \frac{x}{100} \qquad \text{Reduce } \tfrac{6}{30} \text{ to } \tfrac{1}{5}.$$

$$100 = 5x \qquad \text{Take cross-products.}$$

$$20 = x \qquad \text{Divide by 5.}$$

Compare this solution to the one on the opposite page.

Exercises

1–2. What proportion can be used to find each percent?

1. \$5 out of \$20 $\frac{5}{20} = \frac{x}{100}$

2. \$9 out of \$36 $\frac{9}{36} = \frac{x}{100}$

3–8. Find the percent spent out of each total income.

3. \$14 out of \$70 $\left(\text{Use } \frac{14}{70} = \frac{x}{100} \text{ and reduce } \frac{14}{70}.\right)$ 20%

4. \$15 out of \$60 $\left(\text{Use } \frac{15}{60} = \frac{x}{100} \text{ and reduce } \frac{15}{60}.\right)$ 25%

5. \$16 out of \$80 20%

6. \$7 out of \$70 10%

7. \$125 out of \$500 25%

8. \$120 out of \$600 20%

9–12. Copy and complete each chart. Where possible, reduce one side of the proportion to make it easier to solve.

Welch/Van Cleve Photography

9. High school student—average week

Category	Amount	Proportion	Percent
Food	\$6	$\frac{6}{25} = \frac{x}{100}$	24%
Clothing	\$5	$\frac{5}{25} = \frac{x}{100}$	20%
Personal	\$3	$\frac{3}{25} = \frac{x}{100}$	12%
Recreation	\$5	$\frac{5}{25} = \frac{x}{100}$	20%
Savings	\$4	$\frac{4}{25} = \frac{x}{100}$	16%
Other	\$2	$\frac{2}{25} = \frac{x}{100}$	8%
Totals	\$25		100%

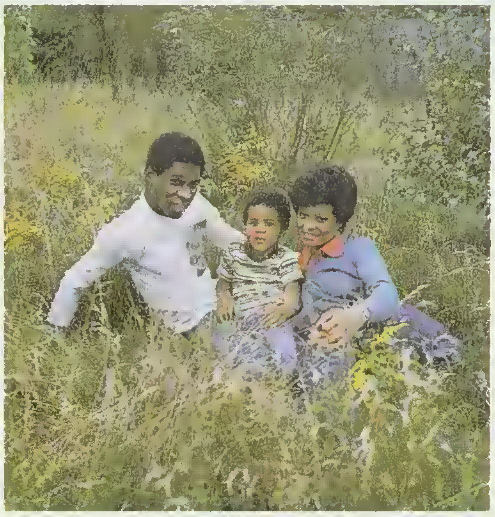
Walker/Taurus Photos

10. Single working person—average week

Category	Amount	Proportion	Percent
Rent	\$24	$\frac{24}{80} = \frac{x}{100}$	30%
Food	\$16	$\frac{16}{80} = \frac{x}{100}$	20%
Transportation	\$16	$\frac{16}{80} = \frac{x}{100}$	20%
Personal	\$4	$\frac{4}{80} = \frac{x}{100}$	5%
Recreation	\$8	$\frac{8}{80} = \frac{x}{100}$	10%
Other	\$12	$\frac{12}{80} = \frac{x}{100}$	15%
Totals	\$80		100%

11. Family of three—average month

Category	Amount	Proportion	Percent
Rent	$225	$\frac{225}{750} = \frac{x}{100}$	30%
Food	$210	$\frac{210}{750} = \frac{x}{100}$	28%
Clothing	$90	$\frac{90}{750} = \frac{x}{100}$	12%
Savings	$45	$\frac{45}{750} = \frac{x}{100}$	6%
Recreation	$75	$\frac{75}{750} = \frac{x}{100}$	10%
Personal	$30	$\frac{30}{750} = \frac{x}{100}$	4%
Other	$75	$\frac{75}{750} = \frac{x}{100}$	10%
Totals	$750		100%


Camerique

12. Family of five—average month

Category	Amount	Proportion	Percent
Housing	$336	$\frac{336}{960} = \frac{x}{100}$	35%
Food	$240	$\frac{240}{960} = \frac{x}{100}$	25%
Transportation	$96	$\frac{96}{960} = \frac{x}{100}$	10%
Savings	$48	$\frac{48}{960} = \frac{x}{100}$	5%
Clothing	$144	$\frac{144}{960} = \frac{x}{100}$	15%
Entertainment	$48	$\frac{48}{960} = \frac{x}{100}$	5%
Other	$96	$\frac{96}{960} = \frac{x}{100}$	10%
Totals	$960		100%

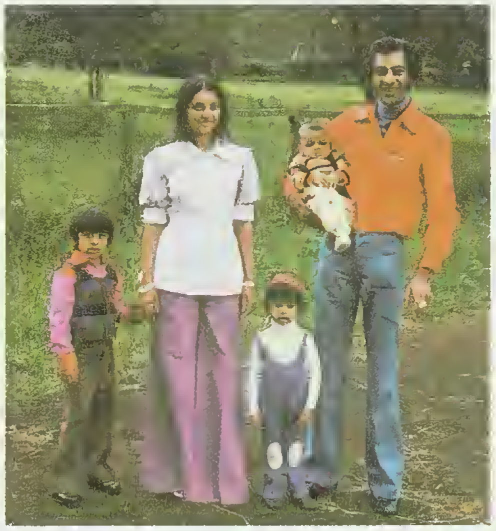


Photo Research International

TALK IT OVER

Ex. 1–4: Typical answers

1. Why must records be kept over a period of time to estimate your average expenses? Some expenses occur at irregular intervals, prices change, etc.

2. Do you suppose that the percent of income spent on each category would change with the amount of income? How? What other factors might determine the part of income that is spent on each category? Yes. % decreases for fixed expenses, increases for variable exp. Family size, age, etc.

3. Is a *written* plan necessary for good money management? Why? Yes, since it aids in controlling expenses and planning future income and expenditures.

4. Do you think it is a good idea to save a regular amount each week or month? Why? Yes, for future planned expenses, emergencies, security, interest income, etc.

THAT'LL BE 5 CLAMS

In primitive times a farmer who had extra grain might trade with a hunter who had extra meat. Then each of them could have some meat *and* some grain. The farmer might also trade with a potter, giving grain in exchange for clay pots. But what if the potter wanted meat instead of grain? Then the farmer would have to trade more grain with the hunter to get the meat to trade for the pots. Things could become even more complicated if the farmer also wanted goods such as clothing or tools.

To simplify things, some object that everyone valued was needed. The farmer and everyone else could trade one or more of these objects for whatever goods they wanted. *Money* is such an object.

Today we think of money as metal coins or paper bills or bank checks. But in different societies, "money" has meant objects such as cattle, hoes, rings, or fish. Here is a list of other moneys.

Col. Grover C. Criswell

Wampum, strings of beads made from polished shells, was used by North American Indians.

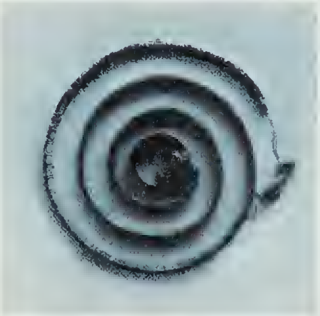
Whale teeth were used in the Fiji Islands.

Grant Heilman

Tool money was used in Ancient China.

Iron bars, often formed as spearheads or knives, were used in Africa.

Lumps of salt were used to pay Roman soldiers. A soldier who did not earn his pay was "not worth his salt."


Buffalo Museum of Science

Feathers of jungle birds were glued to a rope coil and used in the Santa Cruz Islands.

Coconuts were used in the Nicobar Islands.

Playing cards with the governor's signature on the back were used by early Canadian colonists.

Cowry shells were used in both Africa and the South Pacific.

The Chase Manhattan Bank

Japanese tree money had coins on "branches" that could be broken off to make change.

Typical answers

Below is a list of monetary units in use today. Find where each is used.

Cedi	Franc	Lira	Quetzal	Schilling
Dinar	Guilder	Mark	Rial	Yen
Dirham	Krone	Peso	Ruble	Yuan
Escudo	Kyat	Pound	Rupee	Zloty
Ghana	France	Italy	Guatemala	Austria
Middle East	Netherlands	Germany	Iran	Japan
Morocco	Denmark	Mexico	U.S.S.R.	China
Portugal	Burma	England	India	Poland

CAREER CORNER

MATH IN SUMMER JOBS

Most young people would like to work during summer vacation. Where can summer jobs be found? The best bets are places that are open only for the summer or are extra busy at that time—resorts, parks, playgrounds, plant nurseries, farms, and so on. Another possibility is replacing regular workers during their vacations.

Here is a list of some summer jobs. A few require experience. Most do not.

Baby-sitter
Bus girl or boy
Caddie
Camp counselor
Cannery worker
Cashier
Concession attendant
Delivery boy or girl
Elevator operator
Farmhand
Florist's aide
Food-store clerk
Gift wrapper
Greenhouse worker
Grounds keeper
Guide: museums, parks
Highway worker
Home repairer
Hospital worker
Ice-cream vendor
Laboratory assistant
Library aide
Lifeguard
Nursery-school worker
Nurse's aide
Office clerk
Playground attendant
Restaurant worker
Salesclerk
Telephone operator
Tennis-court attendant
Typist
Usher
Waiter, waitress
Window washer
Yard worker

Arejay

Here are a few problems that a summer worker may want to solve.

1. *Concession attendant:* This attendant gets 10¢ for each pennant sold and 25¢ for each baseball cap sold. How much would be earned on a day when 52 pennants and 29 caps are sold? $12.45

2. *Grounds keeper:* A certain grass seed should be used at a rate of 1.25 kg for each 100 m^2 of lawn. How much seed is needed for an area 25 m by 30 m? 6 kg

3. *Waiter, waitress:* The usual tip is 15% of the bill. How much of a tip should be expected on a bill of $25? $3.75

4. *Window washer:* The washing solution should be mixed at the rate of 30 g of cleaner for each litre of water. How much cleaner should be mixed with 6.5 L of water? 195 g

8.3 Budgets

Once you have found what part of your income *on the average* is spent on each category, you might decide something needs to be changed. Here are some questions you might ask.

Performance Objective: The student can compute the amount of income to be spent in each budget category, given the percent.

- Are there some areas where I should spend a smaller percent, such as clothing or recreation?
- Are there some areas where I should spend a larger percent, such as savings?
- Should I make allowance for expenses that come up less often than once a month, such as insurance payments or large purchases—stereos, furniture, etc.?
- Should I make allowance for unexpected expenses, such as repairs or medical expenses?

The answers to these questions may affect your *budget*, your plan for spending your income. The important thing is once you decide what part of your income to spend on each category, stick to it. Spend no more on any category than your budget allows.

Example: After studying the records of spending he had been keeping for a while, Bob decided on the percents shown at the right. If his weekly income is $40, how much can he spend on each category?

Food	20%
Clothing	18%
Recreation	25%
Personal	12%
Savings	10%
Other	15%
Total	100%

Solution for food—20% of 40

By proportion

$$\frac{x}{40} = \frac{20}{100}$$

$$\frac{x}{40} = \frac{1}{5}$$

$$5x = 40$$

$$x = 8$$

By multiplication

$$20\% = \frac{20}{100} = 0.20$$

$$40 \times 0.20 = 8.00$$

$8 can be spent on food.

The following chart shows all the amounts worked out:

Category	Budget percent	Amount to be spent
Food	20%	\$8.00
Clothing	18%	\$7.20
Recreation	25%	\$10.00
Personal	12%	\$4.80
Savings	10%	\$4.00
Other	15%	\$6.00
Totals	100%	\$40.00

Notice that the amounts add up to the total income. This is always true. You can check your work by adding the separate amounts to see if you get the total income.

Exercises

1. A plan for spending your income is called a Budget.

2. A budget shows what part of your income can be spent on each Category or area.

3–8. Use the chart above.

3. Show that the recreation amount is \$10.00 by solving a proportion. $\left(\text{Solve } \frac{x}{40} = \frac{25}{100}.\right)$

3. $\frac{x}{40} = \frac{1}{4}$
$4x = 40$
$x = 10$

4. Show that the recreation amount is \$10.00 by using multiplication. (Multiply \$40 by 0.25.) $40 \times 0.25 = 10.00$

5. Show that the clothing amount is \$7.20 by multiplying. $40 \times 0.18 = 7.20$

6. Show that the savings amount is \$4.00 by using a proportion.

6. $\frac{x}{40} = \frac{10}{100}$
$\frac{x}{40} = \frac{1}{10}$
$10x = 40$
$x = 4$

7. Show that the amount for personal needs is \$4.80 by using a proportion.

7. $\frac{x}{40} = \frac{12}{100}$
$\frac{x}{40} = \frac{3}{25}$
$25x = 120$
$x = 4.80$

8. Show that the amount for other expenses is \$6.00 by using multiplication. $40 \times 0.15 = 6.00$

9–12. Copy and complete each chart.

9. A high school student with a weekly income of $25.00

Category	Percent	Amount
Food	24%	$6.00
Clothing	20%	$5.00
Savings	8%	$2.00
Recreation	22%	$5.50
Personal	16%	$4.00
Other	10%	$2.50
Totals	100%	$25.00

10. A single working person with a weekly income of $120.00

Category	Percent	Amount
Housing	25%	$30.00
Food	20%	$24.00
Savings	10%	$12.00
Entertainment	18%	$21.60
Clothing	15%	$18.00
Other	12%	$14.40
Totals	100%	$120.00

11. A family of four with a monthly income of $800

Category	Percent	Amount
Rent	22%	$176
Food	28%	$224
Transportation	14%	$112
Savings	5%	$40
Entertainment	8%	$64
Personal	5%	$40
Clothing	14%	$112
Other	4%	$32
Totals	100%	$800

12. A family of four with a monthly income of $1500

Category	Percent	Amount
Housing	20%	$300
Food	22%	$330
Savings	10%	$150
Clothing	13%	$195
Transportation	10%	$150
Recreation	12%	$180
Personal	5%	$75
Other	8%	$120
Totals	100%	$1500

TALK IT OVER

Ex. 1–8: Typical answers.

1. Why is a budget necessary for good money management? It helps you to plan your expenses and to keep within your income.

2. How can budgeting help you to avoid getting into financial difficulties? It can help you to avoid overspending and to be sure of enough money for future expenses.

3. How might the amount of monthly income affect your budget? The amount of income affects the percents allotted to fixed and variable expenses.

4. Other than income, what might influence the way you budget your money? Age, occupation, area where you live, number and age of children, life-style, etc.

5. Do you think that following a workable budget will make you more likely to save money? Yes, if savings is included as a budget category.

6. What are some goals a young person might have that could be more easily achieved by using a budgeting plan? Education; large purchases such as a car, a stereo, clothing; etc.

7. What are some goals a young married couple might have to include in their budget? Saving for a house, children's education, etc.

8. What do you consider the most important rule for successful budgeting? Why? Making and following a reasonable budget plan.

8.4 Comparison Shopping

Performance Objective: The student can determine the best buy, using unit costs.

Photo Research International

One of the largest expenses in anyone's budget is food. It makes sense to buy the less expensive of two choices if price is the only difference.

How do you find the less expensive choice? You can use the *unit cost*, the cost of 1 mL or 1 g or 1 kg, etc.

Example 1: Which is the better buy?

150 g for 48¢
300 g for 75¢

a. Find the unit cost for the 150 g size.

150 g for 48¢ is the same as 1 g for how many cents?

$$\frac{150}{48} = \frac{1}{p}$$

$$150 \times p = 48 \times 1$$ Take the cross-products.

$$150p = 48$$

$$\frac{150p}{150} = \frac{48}{150}$$ Divide by 150.

$$p = 0.32$$

The unit cost is 0.32¢

Notice that when you find the unit cost, the proportion always has a 1 in it. When you take the cross-products and multiply by the 1, you get the same number again. Therefore, you can skip the proportion and go straight to the division.

Shortcut

150 g for 48¢

$$p = \frac{48}{150} \begin{matrix} \leftarrow \text{cost} \\ \leftarrow \text{amount} \end{matrix}$$

$$p = 0.32 \leftarrow \text{unit cost}$$

b. Find the unit cost of the 300 g size using division.

$\frac{\text{cost}}{\text{amount}} = \text{unit cost}$ ▶ $\frac{75}{300} = 0.25$

The 300 g size cost 0.25¢ per gram.

The 150 g size costs 0.32¢ per gram, so the 300 g size is the better buy.

Example 2: Find the better buy: 250 mL for 82¢, 325 mL for $1.13.

$\frac{\text{cost}}{\text{amount}} = \text{unit cost}$ ▶ $\frac{82}{250} = 0.33$ $\frac{113}{325} = 0.35$

0.33¢ per gram 0.35 ¢ per gram

lower unit cost—better buy

Note that both divisions are done with cents.

Always use the same units.

NOTE: Quotients have been rounded to the nearest hundredth.

Exercises

1. To find the unit cost, divide the Cost by the Amount.
2. If there are no other differences, the better buy is the choice with the Lowest cost per gram.

3—12. Find the unit cost of each.

3. 150 g for 65¢ 0.43¢
4. 225 g for 56¢ 0.25¢
5. 400 g for 84¢ 0.21¢
6. 325 g for 96¢ 0.3¢
7. 500 mL for $1.62 0.32¢
8. 425 mL for $1.35 0.32¢
9. 1.5 kg for 69¢ 46¢
10. 2 kg for 56¢ 28¢
11. 5 kg for $2.90 58¢
12. 3 kg for $1.74 58¢

13—30. For each exercise, find the better buy by finding the unit cost of each choice. Where necessary, round quotients to the nearest hundredth.

13. 200 g for 35¢ 0.17
325 g for 48¢ 0.15 ←

14. 150 g for 54¢ 0.36
275 g for 80¢ 0.29 ←

15. 250 g for 36¢ 0.14
450 g for 56¢ 0.12 ←

16. 200 g for 63¢ 0.32 ←
275 g for 95¢ 0.35

17. 2 kg for 42¢ 21
5 kg for 99¢ 19.8 ←

18. 3 kg for 66¢ 22 ←
4 kg for 92¢ 23

19. 150 g for 45¢ 0.3
225 g for 55¢ 0.24 ←
300 g for 77¢ 0.26

20. 150 g for 36¢ 0.24
225 g for 60¢ 0.27
350 g for 81¢ 0.23 ←

21. 275 mL for 72¢ 0.26 ←
450 mL for $1.20 0.27

22. 150 mL for 60¢ 0.4 ←
400 mL for $1.75¢ 0.44

23. 200 g for 28¢ 0.14
700 g for 91¢ 0.13 ←

24. 300 g for 24¢ 0.08
500 g for 39¢ 0.078 ←

25. 150 g for 72¢ 0.48
275 g for $1.12 0.41 ←
400 g for $1.73 0.43

26. 200 g for 84¢ 0.42
300 g for $1.19¢ 0.4
450 g for $1.49¢ 0.33 ←

27. 700 mL for 54¢ 0.08
1.1 L for 72¢ 0.07 ←

28. 600 mL for 36¢ 0.06 ←
1.2 L for 76¢ 0.063

29. 675 g for 86¢ 0.127 ←
1.2 kg for $1.54 0.128

30. 750 g for 91¢ 0.12
1.1 kg for $1.07¢ 0.1 ←

31–34. A savings of ½ ¢ per gram may not seem like much money. But the amount a family saves in a full year by getting the best buys can be a large sum. Copy and complete the chart.

	Choices	Unit Cost		Average week's supply	Cost of average week's supply		Amount saved by getting the better buy	
		of a.	of b.		for a.	for b.	in 1 week	in 1 year
	Example: Cereal a. 225 g for 48¢ b. 325 g for 66¢	0.21	0.2	1000 g	1000 ×0.21 210 or $2.10	1000 ×0.2 200 or $2.00	2.10 −2.00 $0.10	0.10 × 52 $5.20
31.	Peanut butter a. 325 g for 78¢ b. 500 g for 99¢	0.24¢	0.2¢	450 g	$1.08	$0.90	$0.18	$9.36
32.	Jam a. 225 g for 76¢ b. 275 g for 85¢	0.34¢	0.31¢	325 g	$1.11	$1.01	$0.10	$5.20
33.	Bread a. 450 g for 56¢ b. 575 g for 64¢	0.12¢	0.11¢	1150 g	$1.38	$1.27	$0.11	$5.72
34.	Cola a. 300 mL can, 30¢ b. 1L btl, 65¢	0.1¢	0.07¢	2250 mL	$2.25	$1.58	$0.67	$34.84

TALK IT OVER

1. Add up the dollar amounts in the last column of the chart above. How much money can a family save in one year just on these four items? $60.32

2. How much do you think could be saved in a year by getting the best buy on every item purchased? *Answers will vary depending on family size.*

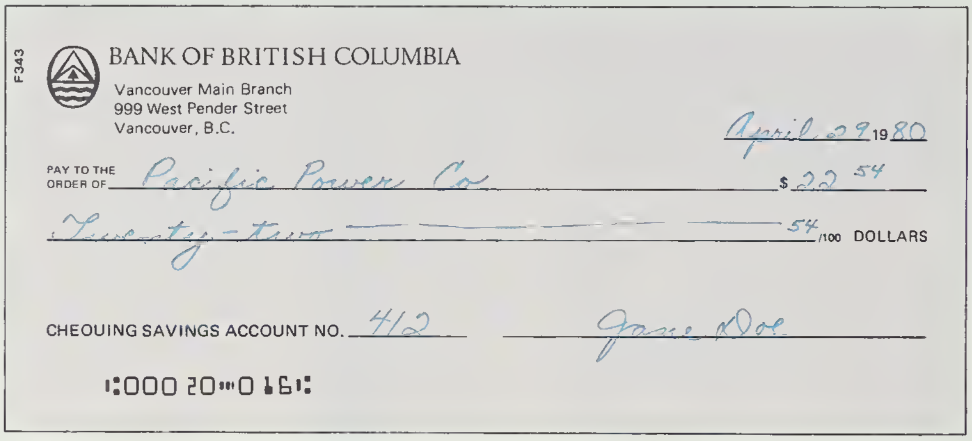
3. Do you suppose the best buy is always the least expensive? What, other than price, might be considered for different kinds of items? *Typical answer:* No, you should also consider the amount you need or can use, the quality, special features, personal taste, etc.

8.5 Chequing Accounts

Performance Objective: The student can write cheques and maintain a cheque record.

A cheque is an order to your bank to pay from your account the amount on the cheque. When you open a chequing account, you are provided with both cheques and a cheque record. The cheque record is used to keep track of the balance in your account. It works much like the records of expenses in Section 8.1.

Example:



F343

BANK OF BRITISH COLUMBIA
Vancouver Main Branch
999 West Pender Street
Vancouver, B.C.

April 29 19 80

PAY TO THE ORDER OF Pacific Power Co. $ 22 54

Twenty-two —— 54/100 DOLLARS

CHEQUING SAVINGS ACCOUNT NO. 412 Jane Doe

⑆000 20⑈0 16⑆

Courtesy of Bank of British Columbia

The Does wrote the cheque above to pay their electric bill for $22.54. The cheque is recorded as the last entry in their cheque record for the last part of April.

Cheque No.	Date	Description	Amount (—)		Deposit (+)		Bal	340	35
408	4-18	To D+K Management Co. For Rent	175	00			 Bal	-175 165	00 35
409	4-23	To Berg's Dept. Store For Clothing	56	20			 Bal	-56 109	20 15
410	4-23	To Provincial Bank For Car payment	98	27			 Bal	-98 10	27 88
	4-26	To For			150	00	 Bal	+150 160	00 88
411	4-27	To Hansen's Market For Groceries	31	18			 Bal	-31 129	18 70
412	4-29	To Pacific Power Co. For Electric bill	22	54			 Bal	-22 107	54 16
		To For					Bal		

Notice that the amount of each cheque is *subtracted from* the balance. Each deposit is *added to* the balance.

Exercises

1. In a cheque record, the amount of a cheque is______from the balance. Subtracted

2. In a cheque record, a deposit is Added to the balance.

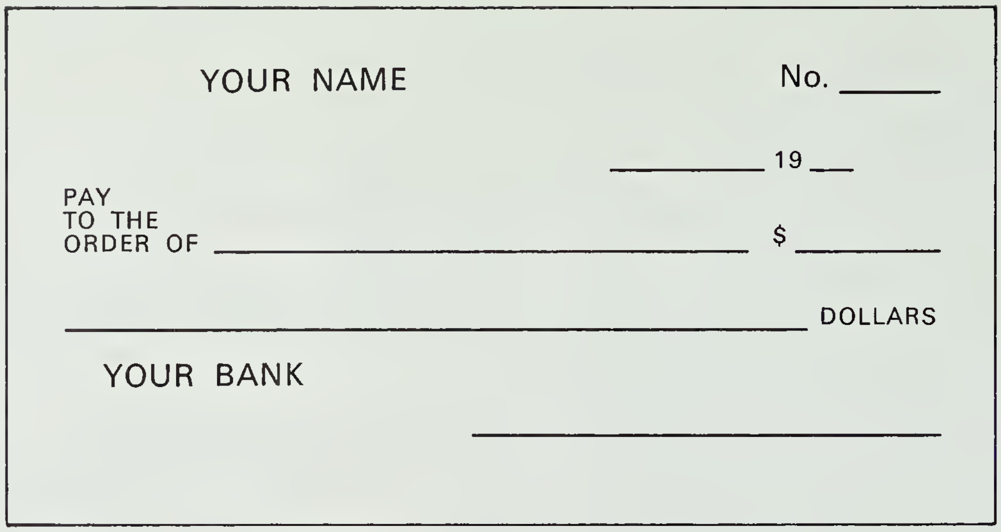
3–4. Complete each section of a cheque record by finding the balance for each letter **a** to **f**.

3.

Cheque No.	Date	Description	Amount (—)		Deposit (+)		Bal 40	62	
161	9-22	To Bell Telephone Co. For Telephone bill	18	46			−18 Bal a.	46	22.16
	9-23	To For			227	00	+227 Bal b.	00	249.16
162	9-25	To A + B Garage For Auto repair	29	00			−29 Bal c.	00	220.16
163	9-25	To Safety Insurance Co. For Insurance premium	78	21			−78 Bal d.	21	141.95
164	9-26	To Cash For	50	00			−50 Bal e.	00	91.95
165	9-30	To Olson's Market For Groceries	26	45			−26 Bal f.	45	65.50
166	9-30	To History Magazine For 1-year subscription	10	00			−10 Bal 55	00 50	

4.

Cheque No.	Date	Description	Amount (—)		Deposit (+)		Bal 104	50	
237	3-19	To Power Commission For Electric bill	36	17			−36 Bal a.	17	68.33
238	3-20	To B + G Market For Groceries	42	58			−42 Bal b.	58	25.75
239	3-20	To Peoples Gas Co. For Gas bill	24	80			−24 Bal c.	80	0.95
	3-22	To For			356	16	+356 Bal d.	16	357.11
240	3-24	To Receiver General For Taxes	58	00			−58 Bal e.	00	299.11
241	3-28	To E. Whitehouse For Rent	225	00			−225 Bal f.	00	74.11
242	3-30	To Sherman St. Garage For Monthly parking	40	00			−40 Bal 34	00 11	


YOUR NAME No. ______

______ 19 __

PAY TO THE ORDER OF ______________________ $ ______

______________________ DOLLARS

YOUR BANK

____

5–6. For each exercise make 3 copies of the cheque above and 1 copy of a cheque record like the ones on the previous page. Then fill in the cheques to pay the bills. Use today's date and your own signature. Finally, fill in the cheque record, and find the final balance.

5. Starting balance: $326.50
Cheque No. 511: Light and Power Corp., $32.84, *electric bill* 293.66
512: R. Young & Co., $26.18, *shoes* 267.48
513: Elkhorn Realty Co., $168.00, *rent* 99.48

6. Starting balance: $207.18
Cheque No. 601: Shell Oil, $38.54, *gasoline credit card* 168.64
602: Bell Telephone Co., $11.56, *telephone bill* 157.08
603: Richard J. Wren, D.D.S., $75.00, *dentist* 82.08

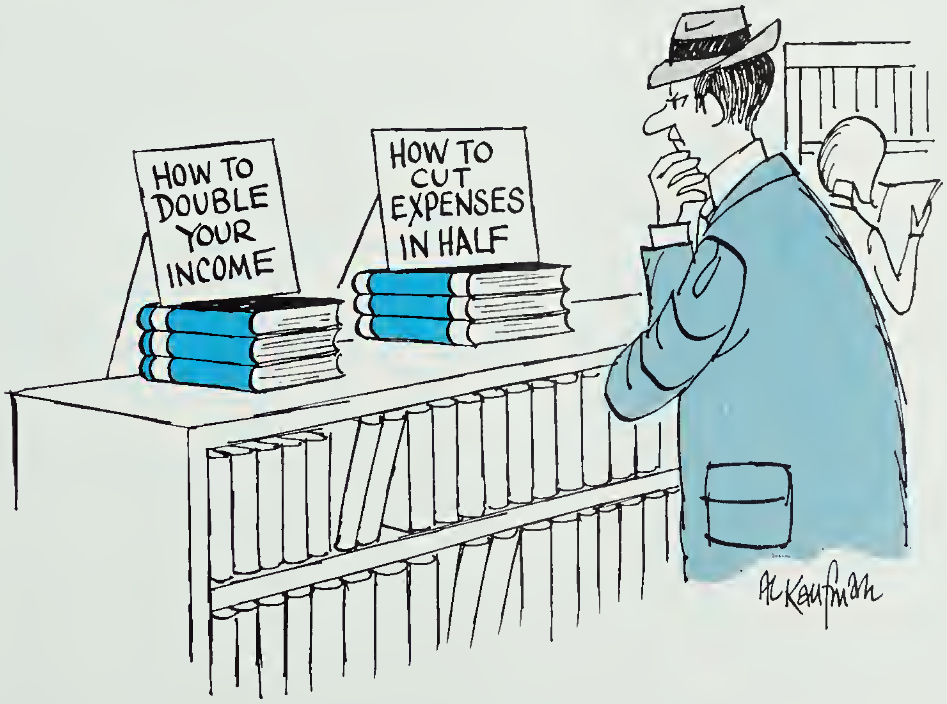
7–10. For each exercise make a copy of a cheque record and fill it in. Find each final balance.

7. Starting balance: $326.42
Cheque 739: Kobelt Travel Agency, $295.37, *airline tickets* 31.05
740: North Shore Ski Club, $15.00, *club dues* 16.05
741: Wilson's Gift Shop, $8.76, *stationery* 7.29
Deposit paycheque: $487.15 494.44
742: Bartok's Appliance Store, $368.14, *refrigerator* 126.30

8. Starting balance: $246.50
Cheque 819: Book-of-The-Month Club, $11.28, *books* 235.22
820: Bockner's Pharmacy, $5.85, *drugs* 229.37
Deposit: $164.18 393.55
821: Provincial Bank, $150.00, *deposit to savings* 243.55
822: Clausen's Market, $46.62, *groceries* 196.93

9. Starting balance: $189.47
Cheque 946: Robert R. Young, D.D.S., $25.00, *dentist* 164.47
947: Neary's Florist Shop, $16.82, *plants* 147.65
948: Bell Telephone Co., $9.24, *telephone bill* 138.41
949: March of Dimes, $25.00, *donation* 113.41
Deposit: $212.60 326.01
950: Eastern Trust, $100.00, *savings* 226.01

10. Starting balance: $386.12
Cheque 156: Marino's Camera Store, $16.95, *developing pictures* 369.17
157: Arkin's Garage, $24.00, *car repair* 345.17
158: T. Pace & Co., $206.88, *clothing* 138.29
Deposit: $347.90 486.19
159: Maslak Carpet Co., $168.00, *rug* 318.19
160: Smith Realty Co., $220.00, *rent* 98.19


HOW TO DOUBLE YOUR INCOME
HOW TO CUT EXPENSES IN HALF
Al Kaufman

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8.6 Bank Statements

Once a month the bank sends you a statement of your chequing account. This statement shows the amount of each deposit and the amount paid out of your account for each cheque. The bank also returns the paid cheques, called *canceled cheques*.

Performance Objective: The student can reconcile a chequing account, using information from a bank statement and a cheque record.

Each month it is necessary to *reconcile* the account. That is, you must account for any difference between the balance on the bank statement and the balance in your cheque record.

How can these two balances be different?

The bank statement may have items *not* in your cheque record, such as service charges or charges for an overdrawn account.

Such items should be included in your cheque record.

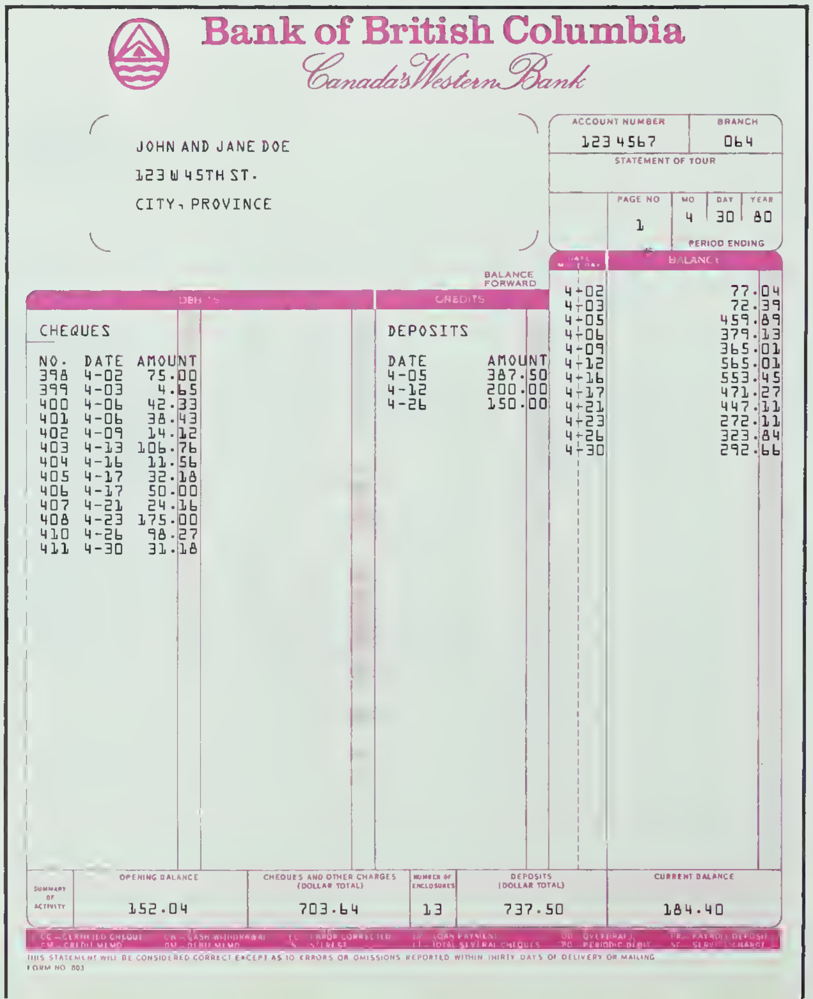
Your cheque record may have items *not* on the bank statement, such as cheques you have written but which have not yet reached the bank and are therefore not yet subtracted from your balance.

Example: Here is the Does' bank statement for April.

Compare this bank statement to the part of the cheque record on page 354.

Look at the dates on the cheque record and then at the dates for the same cheques on the bank statement. It takes time for a cheque you write to reach the bank.

Notice also that two cheques (409 and 412) did not get to the bank at all in April.



Bank of British Columbia

Canada's Western Bank

JOHN AND JANE DOE
123 W 45TH ST.
CITY, PROVINCE

ACCOUNT NUMBER	BRANCH
123 4567	064

STATEMENT OF YOUR

PAGE NO	MO	DAY	YEAR
1	4	30	80

PERIOD ENDING

BALANCE FORWARD

DEBITS — CHEQUES

NO.	DATE	AMOUNT
398	4-02	75.00
399	4-03	4.65
400	4-06	42.33
401	4-06	38.43
402	4-09	14.12
403	4-13	106.76
404	4-16	11.56
405	4-17	32.18
406	4-17	50.00
407	4-21	24.16
408	4-23	175.00
410	4-26	98.27
411	4-30	31.18

CREDITS — DEPOSITS

DATE	AMOUNT
4-05	387.50
4-12	200.00
4-26	150.00

DATE	BALANCE
4-02	77.04
4-03	72.39
4-05	459.89
4-06	379.13
4-09	365.01
4-12	565.01
4-16	553.45
4-17	471.27
4-21	447.11
4-23	272.11
4-26	323.84
4-30	292.66

SUMMARY OF ACTIVITY	OPENING BALANCE	CHEQUES AND OTHER CHARGES (DOLLAR TOTAL)	NUMBER OF ENCLOSURES	DEPOSITS (DOLLAR TOTAL)	CURRENT BALANCE
	152.04	703.64	13	737.50	184.40

CC – CERTIFIED CHEQUE
CM – CREDIT MEMO
CW – CASH WITHDRAWAL
DM – DEBIT MEMO
EC – ERROR CORRECTED
IN – INTEREST
LP – LOAN PAYMENT
TL – TOTAL SEVERAL CHEQUES
OD – OVERDRAFT
PD – PERIODIC DEBIT
PR – PAYROLL DEPOSIT
SC – SERVICE CHARGE

THIS STATEMENT WILL BE CONSIDERED CORRECT EXCEPT AS TO ERRORS OR OMISSIONS REPORTED WITHIN THIRTY DAYS OF DELIVERY OR MAILING
FORM NO. 803

Courtesy of Bank of British Columbia

When you compare the bank statement with the cheque record on page 354, you will see that two cheques written toward the end of the month have not yet been paid by the bank. Such cheques are called *outstanding cheques.*

Find outstanding cheques from the cheque record.

No. 409	56.20
No. 412	22.54
	78.74

◀ Total of outstanding cheques

To the bank statement

include whatever is in the cheque record that is not on the bank statement.

To the cheque record

include whatever is on the bank statement that is not in the cheque record.

184.40	Bank balance
−78.74	Subtract total of outstanding cheques.
105.66	
+ 0	Add total of outstanding deposits.
105.66	Corrected balance

107.16	Cheque record balance
− 1.50	Subtract any service charges.
105.66	Corrected balance

A form like this is usually provided on the back of the bank statement.

Make sure you know where all the information written on it can be found.

The two corrected balances should be the same. If they are not, then either you or the bank has made an error. Report errors made by the bank as soon as possible. Always adjust your cheque record to show the corrected balance. The Does should subtract 1.50 from the balance in their cheque record.

Exercises

1. Cheques paid by the bank from your account and returned to you are called______cheques. Canceled
2. Cheques you have written but that do not show as paid on the bank statement are called______cheques. Outstanding
3. The total of outstanding cheques should be______from the balance shown on the______. Subtracted, bank statement
4. Service charges should be______from the balance shown in your______. Subtracted, cheque record

5–6. Copy and complete each form to reconcile the account.

5.

264.12	Bank balance		
− 98.60	Subtract total of outstanding cheques.	166.72	Cheque record balance
165.52		− 1.20	Subtract any service charges.
+ 0	Add total of outstanding deposits.	165.52	Corrected balance
165.52	Corrected balance		

6.

174.20	Bank balance		
− 56.35	Subtract total of outstanding cheques.	170.35	Cheque record balance
117.85		− 2.50	Subtract any service charges.
+ 50.00	Add total of outstanding deposits.	167.85	Corrected balance
167.85	Corrected balance		

7–10. For each exercise make a form like the ones above and use it to reconcile the account.

7. Bank balance: $328.57; cheque record balance: $207.94; total of outstanding cheques: $147.43; service charge: $1.80; total of outstanding deposits: $25.00

7. 328.57
− 147.43 207.94
181.14 − 1.80
+ 25.00 206.14
206.14

8. Bank balance: $216.90; cheque record balance: $235.21; total of outstanding cheques: $58.19; service charge: $1.50; total of outstanding deposits: $75.00

8. 216.90
− 58.19 235.21
158.71 − 1.50
+ 75.00 233.71
233.71

9. Bank balance: $278.25; cheque record balance: $383.66; outstanding cheques: $35.00, $14.23, $22.56; service charge: $2.20; outstanding deposit: $175.00

9. 278.25
− 71.79 383.66
206.46 − 2.20
+ 175.00 381.46
381.46

10. Bank balance: $309.78; cheque record balance: $287.76; outstanding cheques: $14.95, $46.27, $12.50; service charge: $1.70; outstanding deposit: $50.00

10. 309.78
− 73.72 287.76
236.06 − 1.70
+ 50.00 286.06
286.06

11–12. For each exercise make a form like those on the opposite page. Then use the given bank statement and the correct cheque record on page 355 to find all the information needed to reconcile the account.

11. This bank statement goes with the cheque record in Exercise 3 on page 355.

168.96
−114.66 55.50
54.30 − 1.20
+ 0 54.30
54.30

BANK STATEMENT

35.46	Beginning balance
370.44	Total cheques
505.14	Total deposits
1.20	Service charge
168.96	Ending balance

CHEQUES PAID

No.	Date	Amount
155	9–3	8.20
156	9–6	14.91
157	9–7	190.00
158	9–7	3.12
159	9–16	49.60
160	9–22	7.15
161	9–26	18.46
162	9–29	29.00
164	9–30	50.00

DEPOSITS

Date	Amount
9–5	200.00
9–7	78.14
9–23	227.00

12. This bank statement goes with the cheque record in Exercise 4 on page 355.

154.51
−122.80 34.11
31.71 − 2.40
+ 0 31.71
31.71

BANK STATEMENT

119.48	Beginning balance
574.89	Total cheques
612.32	Total deposits
2.40	Service charge
154.51	Ending balance

CHEQUES PAID

No.	Date	Amount
232	3–4	34.20
233	3–5	6.47
234	3–11	108.00
235	3–14	4.25
236	3–15	118.22
237	3–26	36.17
238	3–27	42.58
241	3–31	225.00

DEPOSITS

Date	Amount
3–8	256.16
3–22	356.16

8.7 The Cost of Credit

Performance Objective: The student can determine the dollar cost of credit for a credit purchase.

More and more often people are making use of credit, a way to "buy now and pay later." You may already have used credit—borrowing money from a friend is a form of credit. Credit is often used through charge accounts or installment purchases.

But one thing should be remembered. Credit costs money. It costs money to borrow money.

Example 1: Suppose you buy a $100 TV on credit. You agree to pay $9 a month for 12 months. What is the cost of this credit?

Heron/Monkmeyer Press Photo Service

First find the total of the monthly payments, the total amount you must pay to borrow the $100.

$$\underbrace{12}_{\text{number of payments}} \times \underbrace{\$9}_{\text{amount of each}}$$

Total of payments ▶ $108

Then find the credit cost.

$$\underbrace{\$108}_{\text{total of payments}} - \underbrace{\$100}_{\text{amount borrowed}}$$

Cost of credit ▶ $8

The work above can be simplified by the use of a formula.

Formula for the dollar cost of credit

$$C = na - b$$

C: cost of credit (finance charge)
n: number of payments
a: amount of each payment
b: amount borrowed

Example 2: Solve the problem in Example **1** with the formula.

$$\begin{aligned} C &= na - b && \text{Write formula.}\\ C &= 12(9) - 100 && \text{Substitute.}\\ C &= 108 - 100 && \text{Multiply.}\\ C &= 8 && \text{Subtract.} \end{aligned}$$

The credit cost is \$8. You must spend \$8 more to buy the TV on credit than if you paid cash.

Often a credit purchase requires a down payment.

Example 3: Suppose you buy the \$100 TV under these conditions: a down payment of \$25 and 12 monthly payments of \$6.75. What is the credit cost?

First find the amount you must borrow.

$$\overset{\text{purchase price}}{\$100} \overset{\text{minus}}{-} \overset{\text{down payment}}{\$25}$$

$$\$75 \quad \blacktriangleleft \text{ Amount borrowed}$$

Now apply the formula.

$$\begin{aligned} C &= na - b\\ C &= 12(6.75) - 75\\ C &= 81.00 - 75\\ C &= 6.00 \end{aligned}$$

The credit cost is \$6.

Notice the credit cost is now lower. This is to be expected since less money is borrowed.

In general, the more money you borrow, the more the credit will cost. Also, the longer you take to pay, the more the credit will cost.

NOTE: By law, the dollar cost of credit, the finance charge, must be stated on all installment contracts.

Exercises

1. To find the credit cost, multiply the Number of payments times the Amount of each payment and then subtract the amount Borrowed

2. Give two common ways in which credit is used.
   Charge accounts and installment purchases
3. Tell what each letter stands for: $C = na - b$
   See page 362.
4. If you make a down payment on an installment purchase, the amount borrowed is the ______ minus the ______.
   Cash or purchase price, down payment

5–10. Find the credit cost for each amount borrowed.

	b Amount borrowed	n Number of payments	a Monthly payment	$C = na - b$ Credit cost
5.	\$80	12	\$7	\$4
6.	\$50	6	\$9	\$4
7.	\$90	8	\$12	\$6
8.	\$180	12	\$16	\$12
9.	\$400	14	\$31	\$34
10.	\$500	20	\$27	\$40

11–16. First find the amount borrowed, then the credit cost.

	p Cash price	d Down payment	$b = p - d$ Amount borrowed	n Number of payments	a Monthly payment	$C = na - b$ Credit cost
11.	\$100	\$10	\$90	12	\$8	\$6
12.	\$150	\$40	\$110	12	\$10	\$10
13.	\$375	\$55	\$320	14	\$25	\$30
14.	\$375	\$85	\$290	16	\$20	\$30
15.	\$800	\$150	\$650	20	\$36.25	\$75
16.	\$800	\$100	\$700	18	\$42.50	\$65

17–22. Solve in the same way as Ex. **11–16.**

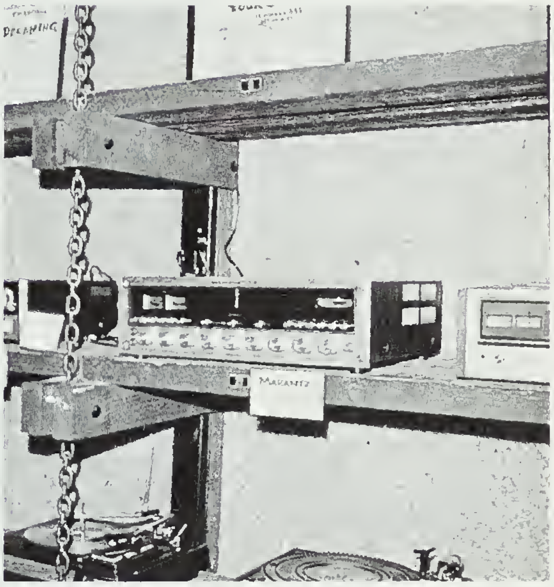
17. A $160 TV is purchased with a down payment of $45 and 12 monthly payments of $10.50. What is the credit cost? $11

18. A bicycle with a cash price of $120 is bought on credit. There is a down payment of $35 and 12 monthly payments of $7.75. Find the credit cost. $8

19. The easy-payment plan for a $349 stereo requires a down payment of $28 and 20 payments of $17.85. Find the credit cost. $36

20. A $329 sewing machine is bought on the installment plan. There is a down payment of $65 and 18 payments of $16.50. Find the credit cost. $33

21. The down payment on a refrigerator selling for $520 is $70. Then there are 24 payments of $22.15. What is the credit cost? $81.60


William Means

22. A $680 motorcycle is going for $120 down and 24 monthly installments of $26.33. Find the credit cost. $71.92

Quick Quiz

Check these skills:

- *dividing whole numbers*
- *reducing fractions*

1–4. Divide. Give answers as decimals.

1. $\frac{12}{200}$ 0.06 **2.** $\frac{36}{300}$ 0.12 **3.** $\frac{96}{800}$ 0.12 **4.** $\frac{91}{1300}$ 0.07

5–8. Reduce to lowest terms. Then multiply.

5. $\frac{4(6)}{9(16)}$ $\frac{1}{6}$ **6.** $\frac{10(12)}{16(15)}$ $\frac{1}{2}$ **7.** $\frac{24(12)}{80(15)}$ $\frac{6}{25}$ **8.** $\frac{24(26)}{200(13)}$ $\frac{6}{25}$

8.8 The Annual Percentage Rate

The dollar cost of credit is a figure you should know for any installment purchase. Another important number is the *annual percentage rate*, the percent of yearly interest you must pay for credit.

Performance Objective:
The student can determine the percentage cost of credit for a credit purchase.

What percent interest would you pay for the \$100 TV in Example 1, page 362? Payments were \$9 each for 12 months (1 year). The credit cost was found to be \$8.

\$8 out of \$100 is $\frac{8}{100}$, or 8% a year. Right?

Not really. If you borrow \$100 and make a single \$108 payment at the end of a year, you do pay 8% interest. But on this credit purchase you would not have the \$100 for a year. When you make your first \$9 payment after one month, you have the use of only \$91. After two months you owe only \$82, and so on. The true interest rate is therefore higher than 8%.

There is a formula by which you can find the true interest rate for any monthly plan.

Formula for the annual percentage rate

$$R = \frac{24C}{b(n+1)}$$

R: annual percentage rate (in decimal form)
C: credit cost
b: amount borrowed
n: number of payments

Example 1: Find the true annual percentage rate for the TV.

$$R = \frac{24C}{b(n+1)}$$

$$R = \frac{24(8)}{100(12+1)}$$ Substitute.

C is \$8.
b is \$100.
n is 12.

$$R = \frac{24(8)}{100(13)}$$ Add inside parentheses.

$$R = \frac{192}{1300}$$ Multiply.

$$R = 0.148$$ Divide; round to nearest thousandth.

The rate 0.148 can be changed to a percent.

$$0.148 = \frac{0.148}{1} \times \frac{100}{100} = \frac{14.8}{100} = 14.8\%$$

To change a decimal to a percent, move the decimal point two places to the right.

You would actually pay 14.8% yearly interest on the $100, not 8%.

Example 2: A $400 stereo is bought on credit. There are 18 monthly payments. The credit cost is $57. Find the annual percentage rate.

$$R = \frac{24C}{b(n+1)}$$

$$R = \frac{24(57)}{400(18+1)}$$ Substitute.

$$R = \frac{24(57)}{400(19)}$$ Add inside parentheses.

$$R = \frac{1368}{7600}$$ Multiply.

$$R = 0.18$$ Divide.

The annual percentage rate is 18%.

The difficult division $\frac{1368}{7600}$ can be avoided by *reducing* before doing the multiplication.

$$R = \frac{24(57)}{400(19)}$$

$$R = \frac{\overset{6}{\cancel{24}}(\overset{3}{\cancel{57}})}{\underset{100}{\cancel{400}}(\underset{1}{\cancel{19}})}$$ Divide 4 into 24 and 400. Divide 19 into 57 and 19.

$$R = \frac{18}{100}, \text{ or } 18\%$$

Why is 4 divided into 24 and 400, when it is possible to use a larger divisor?

NOTE: Like the dollar cost of credit, the true annual percentage rate must by law be stated on all installment contracts.

Exercises

1. The annual percentage rate is the Percent of yearly interest you must pay for credit.

2. Explain how to change a decimal to a percent.
Move decimal point 2 places to right.

3. Tell what each letter stands for: $R = \frac{24C}{b(n + 1)}$.
See page 366.

4. In Section 8.7 you found the cost of credit as an amount of money. In this section you find the cost of credit as a Percent.

5–10. Find each annual percentage rate. Reduce wherever possible before multiplying.

	b Amount borrowed	n Number of payments	C Credit cost	$R = \frac{24C}{b(n + 1)}$ Rate
5.	\$200	14	\$25	20%
6.	\$300	12	\$26	16%
7.	\$400	12	\$39	18%
8.	\$500	15	\$60	18%
9.	\$600	20	\$84	16%
10.	\$600	24	\$100	16%

11–16. First find the dollar cost of credit as you did in Section 8.7. Then find the annual percentage rate.

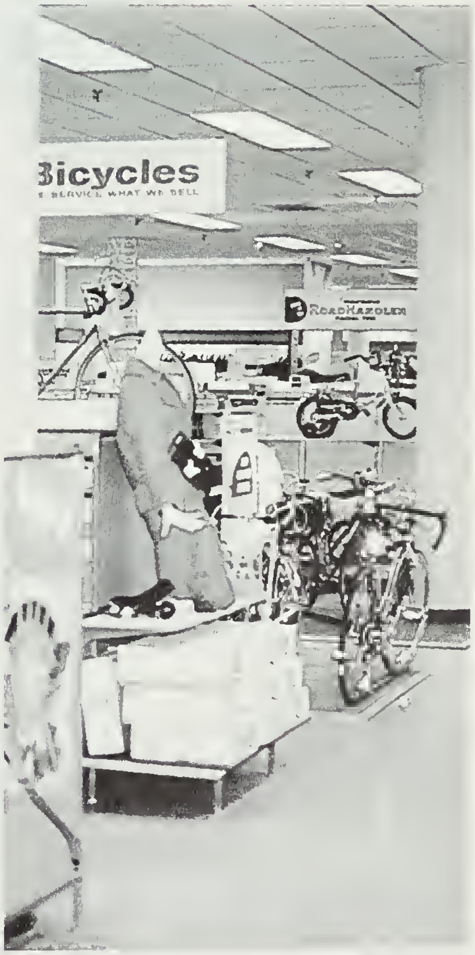
	b Amount borrowed	n Number of payments	a Monthly payment	$C = na - b$ Credit cost	$R = \frac{24C}{b(n + 1)}$ Rate
11.	\$300	15	\$22	\$30	15%
12.	\$700	14	\$55	\$70	16%
13.	\$600	12	\$53.25	\$39	12%
14.	\$400	12	\$35.50	\$26	12%
15.	\$720	15	\$54.40	\$96	20%
16.	\$360	12	\$33.25	\$39	20%

17–22. First find the amount borrowed. Then find the dollar cost of credit. Finally find the annual percentage rate.

	p Cash price	d Down payment	$b = p - d$ Amount borrowed	n Number of payments	a Monthly payment	$C = na - b$ Credit cost	$R = \frac{24C}{b(n+1)}$ Rate
17.	\$345	\$45	\$300	12	\$28.25	\$39	24%
18.	\$459	\$59	\$400	16	\$29.25	\$68	24%
19.	\$244	\$94	\$150	15	\$11.20	\$18	18%
20.	\$355	\$75	\$280	14	\$22.00	\$28	16%
21.	\$528	\$48	\$480	14	\$37.95	\$51.30	17.1%
22.	\$479	\$79	\$400	15	\$29.60	\$44	16.5%

23–26. Solve in the same way as Ex. **17–22.**

23. A 10-speed bicycle is on sale for \$99. If it is bought on the installment plan, there is a down payment of \$14 and 15 monthly payments of \$6.46. Find both the dollar cost of credit and the annual percentage rate. \$11.90, 21%

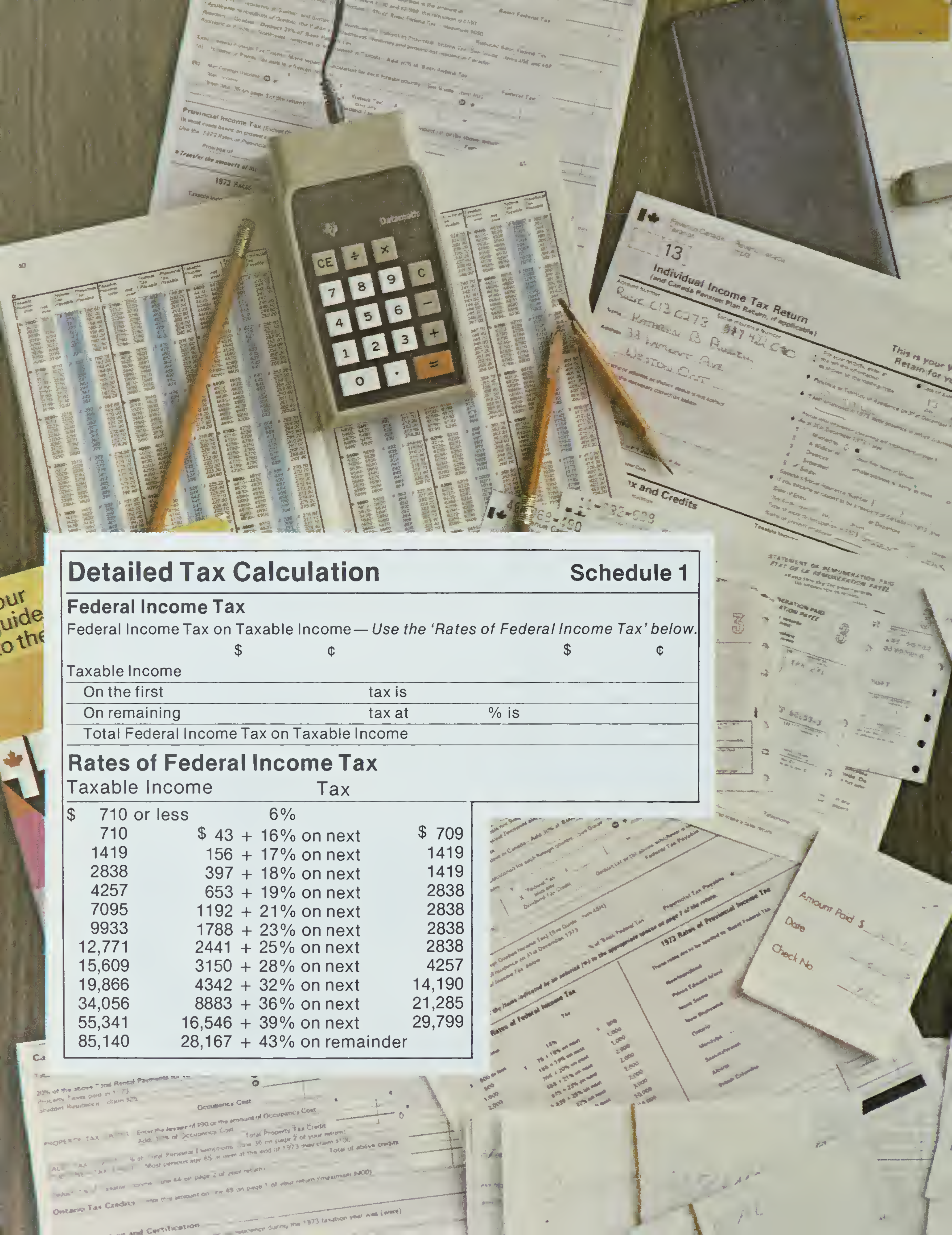


William Parker

24. Nora bought a pair of skis on credit for \$119. She made a down payment of \$11 and then paid \$6.95 a month for 18 months. What was the credit cost? What was the true annual percentage rate? \$17.10, 20%

25. Mike bought a \$388 amplifier for his guitar on the easy-payment plan. The down payment was \$52 and the 12 monthly payments were each \$30.73. What was the credit cost? What was the annual percentage rate? \$32.76, 18%

26. A \$339 microwave oven is bought on credit with a \$51 down payment. There are 12 monthly payments of \$26.47. Find both the credit cost and the annual percentage rate. \$29.64, 19%


Individual Income Tax Return
(and Canada Pension Plan Return, if applicable)
This is your
Return for
Tax and Credits
Amount Paid $
Date
Check No

Detailed Tax Calculation — Schedule 1

Federal Income Tax

Federal Income Tax on Taxable Income — *Use the 'Rates of Federal Income Tax' below.*

	$	¢			$	¢
Taxable Income						
On the first			tax is			
On remaining			tax at	% is		
Total Federal Income Tax on Taxable Income						

Rates of Federal Income Tax

Taxable Income	Tax		
$ 710 or less		6%	
710	$ 43	+ 16% on next	$ 709
1419	156	+ 17% on next	1419
2838	397	+ 18% on next	1419
4257	653	+ 19% on next	2838
7095	1192	+ 21% on next	2838
9933	1788	+ 23% on next	2838
12,771	2441	+ 25% on next	2838
15,609	3150	+ 28% on next	4257
19,866	4342	+ 32% on next	14,190
34,056	8883	+ 36% on next	21,285
55,341	16,546	+ 39% on next	29,799
85,140	28,167	+ 43% on remainder	

Taxes 8.9

Performance Objective: The student can use a Tax Rate Schedule to determine the income tax on a given taxable income.

Figuring income tax does not have to be a difficult job. If you keep track of your information slips (T-4, etc.) and your eligible deductions, the income tax return with its clearly itemized lines should not present a problem.

Everyone in Canada uses the same federal income tax form. However, within this federal tax form, there is a section for the calculation of the provincial tax that is due the provinces. This will be different for each province. The federal tax payable is calculated from your taxable income. The provincial tax payable is a percentage of the federal tax amount.

A Tax Rate Schedule is printed on the opposite page. It gives the tax rates on taxable income.

NOTE: This rate schedule is one used by persons who do a Detailed Tax Calculation, Schedule 1.

Example: Find the tax for a taxable income of $11,350.

The first step is to find the right line on the Tax Rate Schedule on the opposite page.
$11,350 is more than $9933 but less than $12 771.
The line is marked in red.

According to the schedule

Tax = $1788 + 23% of the amount over $9933

To find the amount over $9933, subtract.

11,350 — 9933 = 1417

Tax = $1788 + 23% of $1417

To find the percent, multiply.

0.23 × 1417 = 325.91

Tax = $1788 + $325.91

Finally, do the addition.

Tax = $2113.91

Exercises

1—10. Use the Tax Rate Schedule on page 370. Give the numbers between which each amount below should be placed.

Example: $25,436
$19,866 and $34,056

1. $4600 $4257 and $7095
2. $5800 $4257 and $7095
3. $13,310 $12,771 and $15,609
4. $9240 $7095 and $9933
5. $8416 $7095 and $9933
6. $28,673 $19,866 and $34,056
7. $21,214.70 $19,866 and $34,056
8. $36,561.80 $34,056 and $55,341
9. $42,875.22 $34,056 and $55,341
10. $58,928.74 $55,341 and $85,140

11—30. Use the Tax Rate Schedule on page 370. Find the tax for each taxable income.

11. $2300 $305.77
12. $2700 $373.77
13. $4800 $756.17
14. $3600 $534.16
15. $7800 $1340.05
16. $9200 $1634.05
17. $21,400 $4832.88
18. $15,300 $3073.25
19. $36,200 $9654.84
20. $37,800 $10,230.84
21. $16,470 $3391.08
22. $15,280 $3068.25
23. $27,680 $6842.48
24. $21,410 $4836.08
25. $15,374 $3091.75
26. $18,962 $4088.84
27. $46,195 $13,253.04
28. $33,774 $8792.56
29. $29,524 $7432.56
30. $37,146 $9995.40

31—36. In computing taxes, you are allowed to round all figures to the nearest whole dollar. Find the tax to the nearest $1.00 for each taxable income.

31. $5342.50 $859
32. $7319.40 $1239
33. $17,876.17 $3785
34. $28,291.82 $7038
35. $21.475.87 $4857
36. $16,249.08 $3329

CLASS PROJECTS

In the last section, you used one of the Tax Rate Schedules to find the tax on a given taxable income. But how do you find the taxable income? What happens after you find the tax? What happens if you do not use any of the Tax Rate Schedules?

These and other questions should concern you since you will probably pay income tax someday. You can get the latest information from your District Taxation Office.

DISTRICT TAXATION OFFICES

NEWFOUNDLAND
Sir Humphrey Gilbert Bldg.
165 Duckworth St.
St. John's, Nfld.
A1C 5X6

ONTARIO
Century Blvd
360 Lisgar St.,
K1A 0L9

PRINCE EDWARD ISLAND
Dominion Bldg.
97 Queen St.
Charlottetown, P.E.I.
C1A 7N1

MANITOBA
Revenue Bldg.
391 York Ave.
Winnipeg, Man.
R3C 0P5

NOVA SCOTIA
Ralston Blvd.
1557 Hollis St.
Halifax, N.S.
B3J 2T5

SASKATCHEWAN
Income Tax Bldg.
1955 Smith St.
Regina, Sask.
S4P 2N9

NEW BRUNSWICK
65 Canterbury St.
Saint John, N.B.
E2L 4H9

ALBERTA
Federal Public Bldg
9820 — 107th St.
Edmonton, Alta.
T5K 1E8

QUEBEC
National Revenue Bldg.
305 Dorchester Blvd. W.
Montreal, Que.
H2Z 1A6

BRITISH COLUMBIA
1166 West Pender St.
Vancouver, B.C.
V6E 3H8

CONSUMER SKILLS KIT

Taxation: Your district Taxation Office

Insurance: Education Division
The Canada Life Insurance Association
44 King Street West
Toronto 1, Ontario
M5H 1E2

Banking: Your Money Matters
Teachers' Kit and Consumer Kit
Community Relations Officers
Consumer Education Program
Royal Bank Building
Box 6001
Montreal, Quebec
H3C 3A9

A Student Guide to Banking
The Canadian Imperial Bank of Commerce
Ask at your local branch.

Financing: Money Management Institute
Household Finance Corporations of Canada
85 Bloor Street East
Toronto, Ontario
M4W 1B4

Miscellaneous: Learning From Newspapers Dr. Hugh Partlon
Canadian Daily Newspaper Publishing Association
250 Bloor Street East
Toronto, Ontario
M4W 1E7

Forms in Your Life Wyle Elrick
Globe/Modern Curriculum Press
200 Steelcase Road East
Markham, Ontario
L3R 1G2

8.10 Time

Performance Objective: The student can find sums or differences of times.

We think of time in two different ways.

A. Time as the hour of the day (What time is it?)

"The time is now 32 minutes after 7 a.m."

B. Time as a duration (How long was it?)

"I worked 7 hours and 32 minutes today."

In both cases, we can write 7:32. Whether 7:32 means a *time of day* or a *length of time* can be told from the situation.

There are three ways in which you might work with time.

You might have two lengths of time.

Example 1: The two movies at a double feature have lengths of 1 h 46 min and 2 h 28 min. How long is the whole show?

The two periods of time should be added.

$$\begin{array}{r} 1:46 \\ +\ 2:28 \\ \hline 3:74 \end{array}$$

Add the minutes; add the hours.

Add 1 h = Subtract 60 min

When there are more than 60 min, "carry" an hour.

$$4:14$$

Change 60 minutes to 1 hour.

The whole show lasts 4 h 14 min.

Example 2: The class decided the graduation exercises, which lasted 4 h 12 min, were too long. So they decided to cut 1 h 25 min. How long will the shortened exercises be?

The length of time cut should be subtracted.

$$\begin{array}{r} 4:12 \\ -1:25 \\ \hline \end{array}$$

But you cannot subtract 25 from 12. You must "borrow" an hour from the 4.

$$\begin{array}{rl} \overset{3}{\cancel{4}}:\overset{72}{\cancel{12}} & \text{Change 1 hour to 60 minutes.} \\ -1:25 & \\ \hline 2:47 & \text{Subtract.} \end{array}$$

The shortened exercises will be 2 h 47 min.

You might have a time of day and a length of time.

Example 3: A plane starts a flight lasting 2 h 57 min at 6:50 a.m. Find its arrival time.

Think of a.m. as *after midnight.*

Think of p.m. as *after noon.*

Add the period of time to the starting time.

$$\begin{array}{rl} 6:50\ \text{a.m.} & \\ +\,2:57\phantom{\ \text{a.m.}} & \\ \hline 8:107\ \text{a.m.} & \text{Add.} \\ & \\ 9:47\ \text{a.m.} & \text{Change 60 minutes to 1 hour.} \end{array}$$

The plane will arrive at 9:47 a.m. (assuming it lands in the same time zone).

Notice that the work in this example is just like that in Example **1**, even though the ideas are different.

You might have two times of day.

Example 4: Your car was in a parking lot from 10:58 a.m. to 2:36 p.m. How long was that?

Subtract the starting time from the ending time.

$$\begin{array}{rl} \overset{\overset{13}{\cancel{1}}}{\cancel{2}}:\overset{\overset{96}{96}}{\cancel{36}}\ \overset{\overset{\text{a.m.}}{\cancel{\text{p.m.}}}}{\text{p.m.}} & \begin{array}{l}\text{2. Change } \frac{1}{2} \text{ day to 12 hours.} \\ \text{1. Change 1 hour to 60 minutes.}\end{array} \\ -\,10:58\ \text{a.m.} & \\ \hline 3:38\phantom{\ \text{a.m.}} & \text{Subtract.} \end{array}$$

Your car was parked 3 h 38 min.

Exercises

1–6. Tell whether each statement contains a *time of day* or a *length of time.*

1. It is 2:30 in the afternoon. Time of day
2. Jan spent 4 h and 56 min in school today. Length of time
3. Pat left at 3:07 p.m. Time of day
4. Pat arrived at 5:12 p.m. Time of day
5. Pat's trip took 2 h 5 min. Length of time
6. Jeff woke up 7 min before 8 o'clock. Time of day

7–12. Add the times. "Carry" where necessary.

7. $$\begin{array}{r} 6:02 \\ +3:18 \\ \hline 9:20 \end{array}$$

8. $$\begin{array}{r} 7:12 \\ +2:24 \\ \hline 9:36 \end{array}$$

9. $$\begin{array}{r} 4:38 \\ +2:32 \\ \hline 7:10 \end{array}$$

10. $$\begin{array}{r} 4:16 \\ +3:57 \\ \hline 8:13 \end{array}$$

11. $$\begin{array}{r} 10:54 \\ +\ \ 1:20 \\ \hline 12:14 \end{array}$$

12. $$\begin{array}{r} 3:42 \\ +3:36 \\ \hline 7:18 \end{array}$$

13–18. Subtract the times. "Borrow" where necessary.

13. $$\begin{array}{r} 7:38 \\ -4:20 \\ \hline 3:18 \end{array}$$

14. $$\begin{array}{r} 10:46 \\ -\ \ 2:12 \\ \hline 8:34 \end{array}$$

15. $$\begin{array}{r} 8:12 \\ -4:58 \\ \hline 3:14 \end{array}$$

16. $$\begin{array}{r} 9:05 \\ -5:23 \\ \hline 3:42 \end{array}$$

17. $$\begin{array}{r} 11:42 \\ -\ \ 2:56 \\ \hline 8:46 \end{array}$$

18. $$\begin{array}{r} 6:34 \\ -4:48 \\ \hline 1:46 \end{array}$$

19–24. Add the second length of time to the first.

	First length	Second length	
19.	1 h 18 min	2 h 12 min	3:30
20.	3 h 22 min	4 h 48 min	8:10
21.	2 h 40 min	6 h 28 min	9:08
22.	3 h 10 min	4 h 34 min	7:44
23.	2 h 56 min	3 h 18 min	6:14
24.	7 h 36 min	2 h 28 min	10:04

25–30. Subtract the second length of time from the first.

	First length	Second length	
25.	9 h 43 min	5 h 16 min	4:27
26.	6 h 12 min	2 h 45 min	3:27
27.	12 h 10 min	7 h 38 min	4:32
28.	6 h 32 min	4 h 18 min	2:14
29.	8 h 11 min	3 h 37 min	4:34
30.	8 h 25 min	3 h 56 min	4:29

31—36. Find each ending time after the given length of time.

	Starting time	Length of time	
31.	2:16 p.m..	3 h 50 min	p.m. 6:06
32.	5:40 p.m.	2 h 34 min	8:14
33.	10:24 p.m.	4 h 18 min	2:42
34.	4:50 a.m.	2 h 26 min	7:16
35.	6:24 a.m.	1 h 55 min	8:19
36.	9:08 p.m.	5 h 30 min	2:38 a.m.

37—42. Find each length of time in hours and minutes.

	Starting time	Ending time	
37.	4:56 p.m.	8:12 p.m.	3:16
38.	8:20 a.m.	11:45 a.m.	3:25
39.	7:46 a.m.	11:20 a.m.	3:34
40.	15:19	20:42	5:23
41.	10:18 a.m.	5:38 p.m.	7:20
42.	7:50 a.m.	4:22 p.m.	8:32

43—48. Solve each problem by adding or subtracting times.

43. A bus schedule shows a departure time of 14 47 and an arrival time of 16 18. How long does the trip take? 1 h 31 min

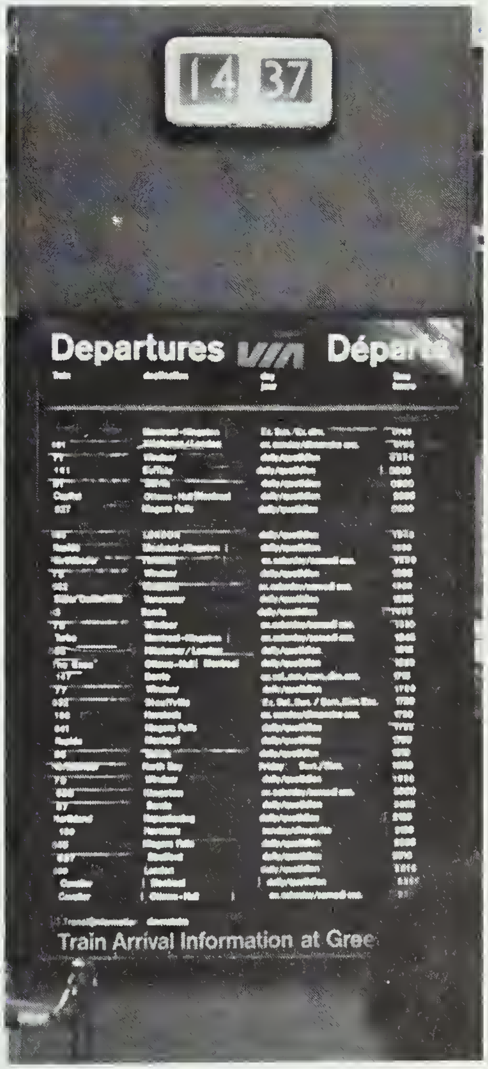
44. The notice on a theater box office gives a starting time of 5:45 p.m. If the movie runs 1 h 42 min, at what time does it end? 7:27 p.m.

45. Some companies have a flexible-hour plan in which you must work a certain number of hours a week but can vary your starting and quitting times. If you get to work at 9:42 a.m. and you must work $7\frac{1}{2}$ h, at what time should you leave? 5:12 p.m.

46. Your ticket from a parking lot shows you drove in at 8:24 a.m. If you leave at 5:10 p.m., how long was your car parked? 8 h 46 min

47. The newspaper ad for a movie theater says the first feature starts at 6:45 and the second feature starts at 8:05. How long is the first feature? 1 h 20 min

48. A train ride of 2 h 47 min is followed by a bus ride of 1 h 16 min. How long is the whole trip? 4 h 3 min



Robert Brown

Test for Chapter 8

Find each cash-on-hand amount. (8.1)

Income	Expense	Cash on hand
		72.40
	32.85	**1.** 39.55
	4.12	**2.** 35.43
25.00		**3.** 60.43
	19.67	**4.** 40.76

Find each percent. (8.2)

Find each amount. (8.3)

Amount	Percent	Budget category	Percent	Amount
\$21	**5.** 28%	◀ food ▶	24%	**9.** \$108
\$12	**6.** 16%	◀ clothing ▶	18%	**10.** \$81
\$9	**7.** 12%	◀ savings ▶	10%	**11.** \$45
\$33	**8.** 44%	◀ other ▶	48%	**12.** \$216
\$75	100%	◀ totals ▶	100%	\$450

Using unit costs, find the better buys. (8.4)

13. 75 g for 39¢ 0.52 ←
150 g for 70¢ 0.47

14. 275 g for 89¢ 0.32
400 g for \$1.19 0.3 ←

Cheque No	Description	Amount	Deposit	Bal
	To For			Bal 135.15
	To For			Bal 85.15
	To For			Bal 147.65
	To For			Bal 119.49

15. Make a cheque record (see form at left) and fill it in. Starting balance: \$157.62
Cheque No. 426: Sim's Gift Shop, \$22.47, *gift*
427: Local Bank, \$50.00, *savings*
Deposit: \$62.50
428: A&BMart, \$28.16, *food* (8.5)

	Bank balance		Cheque record balance
−	Outstanding cheques	−	Service charges
74.55		94.55	Corrected balance
+	Outstanding deposits		
94.55	Corrected balance		

16. Make a form (see left) to reconcile the account. Bank balance: \$172.81; cheque record balance: \$96.05; outstanding cheques: \$98.26; outstanding deposit: \$20.00; service charge: \$1.50 (8.6)

$$C = na - b$$

17. Find C (credit cost) if n (number of payments) is 15, a (amount of each payment) is \$24.60, and b (amount borrowed) is \$340. \$29 (8.7)

$$R = \frac{24C}{b(n+1)}$$

18. Find R (annual percentage rate) if n (number of payments) is 15, b (amount borrowed) is \$200, and C (credit cost) is \$24. 18% (8.8)

RATE YOURSELF

Number Correct	
21–22	Superior
19–20	Excellent
16–18	Good
12–15	Fair
0–11	Poor

19. Find the tax on \$16,240 if the Tax Rate Schedule reads \$3830 + 34% of the amount over \$16,000. \$3911.60 (8.9)

Add or subtract the times. (8.10)

20. 6:38 + 2:45 = 9:23

21. 9:18 − 4:55 = 4:23

22. 3:15 p.m. − 8:52 a.m. = 6:23 or 6 h 23 min

Chapter 8 Review

1. Find each cash-on-hand amount. (8.1)

Day	Item	Income	Expense	Cash on hand
	Balance			29.85
Fri.	Paycheque	256.48		**a.** 286.33
	Groceries		20.75	**b.** 265.58
Sat.	Rent		145.00	**c.** 120.58
	Gasoline		8.50	**d.** 112.08
	Entertainment		12.60	**e.** 99.48
Mon.	Lunch		2.80	**f.** 96.68
	Savings		40.00	**g.** 56.68

2. Find each percent. (8.2) 3. Find each amount. (8.3)

Amount	Percent	Budget category	Percent	Amount
$40	25%	◀ food ▶	22%	$121
$32	20%	◀ housing ▶	24%	$132
$16	10%	◀ transportation ▶	8%	$44
$8	5%	◀ savings ▶	10%	$55
$16	10%	◀ recreation ▶	12%	$66
$48	30%	◀ other ▶	24%	$132
$160	100%	◀ totals ▶	100%	$550

Using unit costs, find the better buys. (8.4)

4. 100 mL for 52¢ 0.52
175 mL for 75¢ 0.43 ←

5. 150 mL for 41¢ 0.27 ←
225 mL for 68¢ 0.3

6. 200 g for 93¢ 0.47
275 g for $1.25 0.45 ←

7. 225 g for $1.16 0.52
350 g for $1.69 0.48 ←

Make a cheque record like the one on the opposite page and fill it in. (8.5)

8. Starting balance: $286.25
Cheque No. 221: Reed Management Co., $175.00, *rent* 111.25
222: Bell Telephone, $10.28, *telephone bill* 100.97
Deposit: $78.00 178.97
223: A & B Garage, $42.63, *car repair* 136.34
224: B. Dobson & Co., $39.18, *clothing* 97.16

Make a form like the one at the left for each exercise. Reconcile each account. (8.6)

	Bank balance
−	Outstanding cheques
65.51	
+	Outstanding deposits
115.51	Corrected balance

	Cheque record balance
−	Service charges
115.51	Corrected balance

9. Bank balance: $128.91; cheque record balance: $118.01; outstanding cheques: $63.40; outstanding deposits: $50.00; service charge: $2.50

10. Bank balance: $245.68; cheque record balance: $199.28; outstanding cheques: $120.45; outstanding deposits: $72.25; service charge: $1.80

245.68	
− 120.45	199.28
125.23	− 1.80
+ 72.25	197.48
197.48	

Find each credit cost. (8.7)

$$C = na - b$$

	b Amount borrowed	n Number of payments	a Monthly payments	C Credit cost
11.	$180	12	$16	$12
12.	$270	14	$21	$24
13.	$335	15	$24.60	$34
14.	$206	12	$18.75	$19

Find each annual percentage rate. (8.8)

$$R = \frac{24C}{b(n+1)}$$

	b Amount borrowed	n Number of payments	C Credit cost	R Rate
15.	$300	12	$26	16%
16.	$400	12	$39	18%
17.	$480	24	$85	17%
18.	$320	14	$40	20%

$ 710 or less	6%	
710	$ 43 + 16% on next	$ 709
1419	156 + 17% on next	1419
2838	397 + 18% on next	1419
4257	653 + 19% on next	2838
7095	1192 + 21% on next	2838
9933	1788 + 23% on next	2838
12,771	2441 + 25% on next	2838
15,609	3150 + 28% on next	4257
19,866	4342 + 32% on next	14,190
34,056	8883 + 36% on next	21,285
55,341	16,546 + 39% on next	29,799
85,140	28,167 + 43% on remainder	

Use the Tax Rate Schedule at the left to find the tax on each taxable income. (8.9)

19. $5200 $832.17 **20.** $9500 $1697.05

21. $11,480 $2143.81 **22.** $7240 $1222.45

Add or subtract the times. (8.10)

23. 2:47 + 3:54 = 6:41

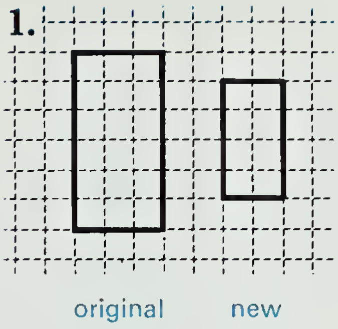
24. 8:20 − 4:55 = 3:25

25. 4:50 p.m. − 9:27 a.m. = 7:23 or 7 h 23 min

26. 11:20 a.m. + 3:48 = 3:08 p.m.

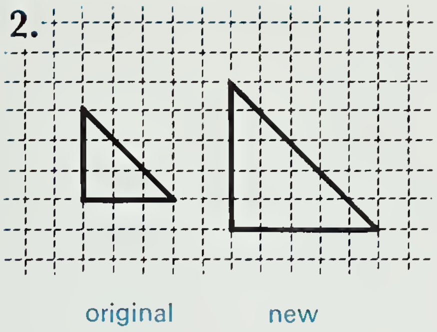
Give each ratio, new figure: original figure. (7.1)

1.


original
new

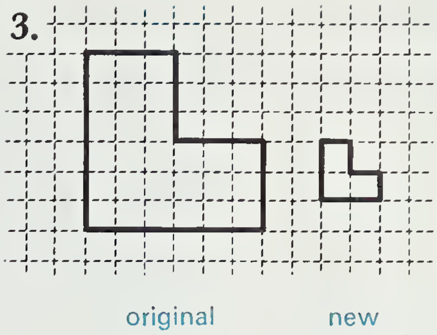
2:3

2.


original
new

5:3

3.

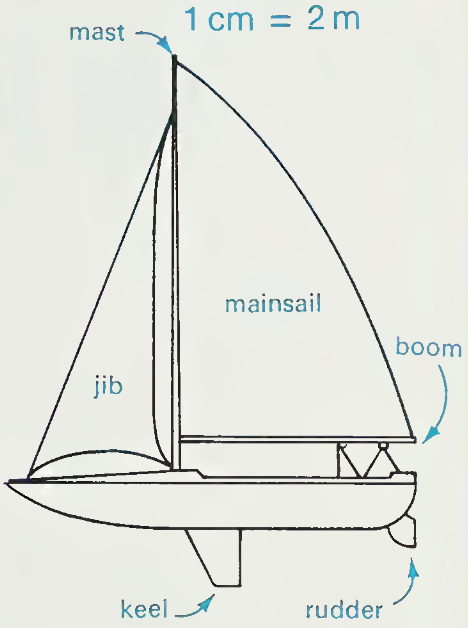

original
new

1:3

Copy and complete the chart. Use the scale drawing of a sailboat at the right. (7.2)

	Measurement	Drawing (cm)	Real (metres)
4.	total length of boat	4.5	9
5.	height of mast	4.4	8.8
6.	length of boom	2.5	5
7.	depth of keel below bottom of boat	0.6	1.2
8.	height on mast at which jib is attached	3.8	7.6

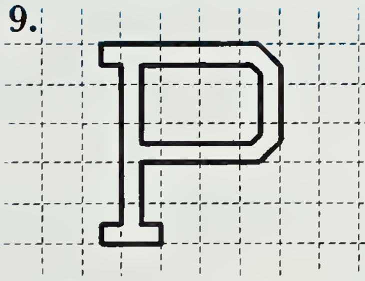
Scale:
1 cm = 2 m


mast
mainsail
boom
jib
keel
rudder

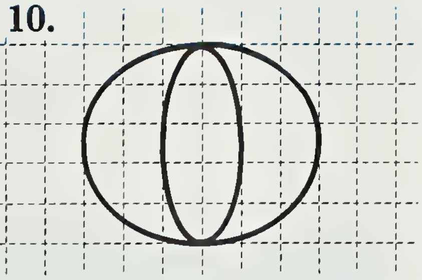
9–12. *Drawings*

Use the grid method to make a 3:2 enlargement. Use a grid with 2 squares to the centimetre. (7.3)

9.



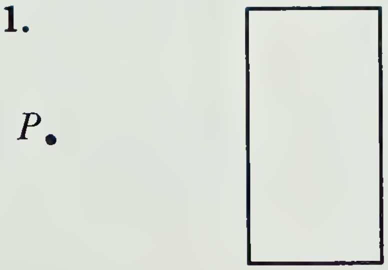
10.



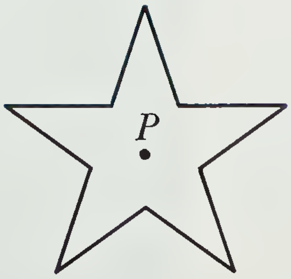
This grid has 3 squares per centimetre

Use the point method to make a 2:1 enlargement. (7.4)

11.



12.



Find the real distance for each map distance. (7.5)

Scale: 1 cm = 64 km

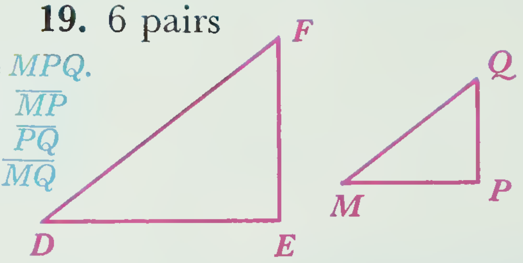
13. 2 cm 128 km 14. 1.75 cm 112 km 15. 2.375 cm 152 k

Scale: 1 cm = 28 km

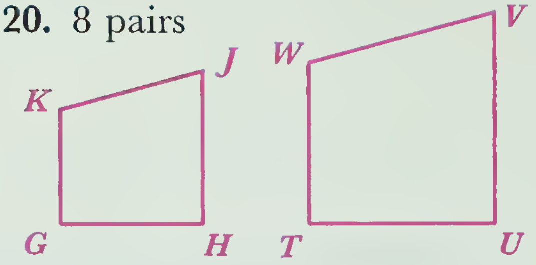
16. 4 cm 112 km 17. 2.5 cm 70 km 18. 6.3 cm 176.4 k

For each pair of similar figures, write a similarity statement and give all pairs of corresponding parts. (7.6)

19. 6 pairs



20. 8 pairs

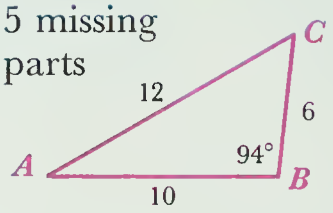


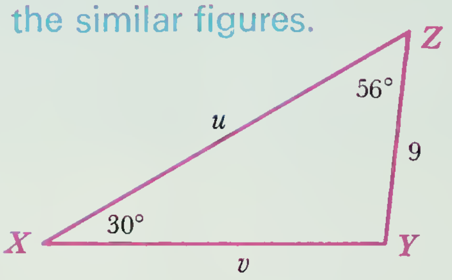
19. Triangle *DEF* is similar to triangle *MPQ*.
$\angle D \leftrightarrow \angle M$ $\overline{DE} \leftrightarrow \overline{MP}$
$\angle E \leftrightarrow \angle P$ $\overline{EF} \leftrightarrow \overline{PQ}$
$\angle F \leftrightarrow \angle Q$ $\overline{DF} \leftrightarrow \overline{MQ}$

20. Figure *GHJK* is similar to figure *TUVW*.

Find all the missing parts of the similar figures. (7.7)

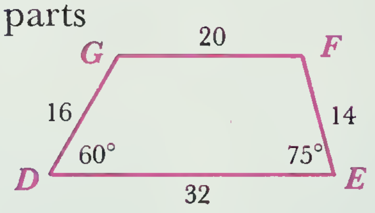
21. 5 missing parts

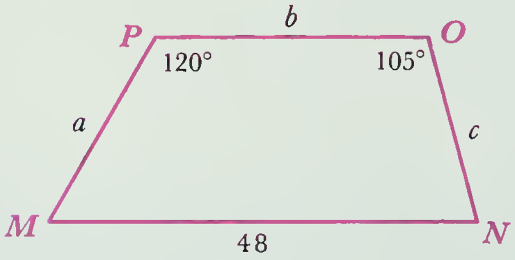




21.
$\angle A = 30°$ $u = 18$
$\angle C = 56°$ $v = 15$
$\angle Y = 94°$

22. 7 missing parts





22.
$\angle F = 105°$ $a = 24$
$\angle G = 120°$ $b = 30$
$\angle M = 60°$ $c = 21$
$\angle N = 75°$

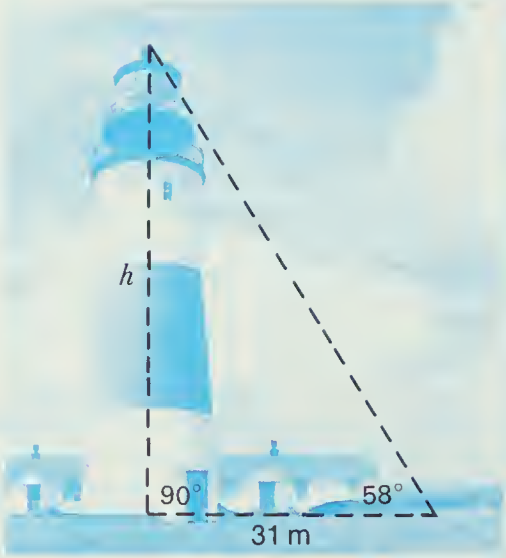
Construct each triangle from measurements. (7.8)

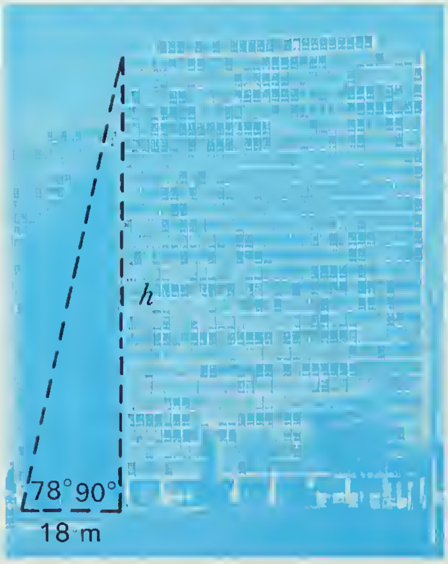
23–24. *Drawings*

23. Triangle *ABC*: *AB* is 4 cm, $\angle A$ is 60°, and $\angle B$ is 40°.

24. Triangle *XYZ*: *XY* is 5.5 cm, $\angle X$ is 80°, and $\angle Y$ is 30°.

Make a scale drawing, measure, and use a proportion. (7.9)





25. How tall is the lighthouse? About 50 m

26. How tall is the building? About 85 m

27. Find each cash-on-hand amount. (8.1)

Day	Item	Income	Expense	Cash on hand
	Balance			$116.30
Wed.	Lunch		2.56	**a.** 113.74
	Clothing		49.19	**b.** 64.55
Thurs.	Gasoline		8.50	**c.** 56.05
	Paycheque	132.68		**d.** 188.73
Fri.	Groceries		21.56	**e.** 167.17

28. Find each percent. (8.2) **29.** Find each amount. (8.3)

Amount	Percent	Budget category	Percent	Amount
$33	22%	◀ food ▶	21%	$84
$36	24%	◀ housing ▶	23%	$92
$12	8%	◀ clothing ▶	10%	$40
$18	12%	◀ recreation ▶	11%	$44
$9	6%	◀ savings ▶	8%	$32
$42	28%	◀ other ▶	27%	$108
$150	100%	◀ totals ▶	100%	$400

Using unit costs, find the better buy. (8.4)

30. 225 g for 52¢ 0.231
275 g for 63¢ 0.229 ←

31. 75 g for 42¢ 0.56
125 g for 66¢ 0.53 ←

32. 250 mL for $1.71 0.68 ←
375 mL for $2.73 0.73

33. 300 mL for $1.62 0.54
400 mL for $1.98 0.5 ←

Make a cheque record like the one at the right and fill it in. (8.5)

34. Starting balance: $132.58
Cheque No. 305: Midtown Bank, $50.00, *savings*
306: AA Mart, $22.89, *food*
307: Cash, $30.00
Deposit: $46.28
308: Northern Power Co., $34.22, *electric bill*

Cheque No.	Description	Amount	Deposit	Bal
	To For			Bal 82.58
	To For			Bal 59.69
	To For			Bal 29.69
	To For			Bal 75.97

41.75

Make a form like the one at the left for each exercise. Reconcile each account. (8.6)

	Bank balance
–	Outstanding cheques
54.13	
+	Outstanding deposits
129.13	Corrected balance

	Cheque record balance
–	Service charges
129.13	Corrected balance

156.14
− 38.70
117.44
+ 47.92
165.36

167.16
− 1.80
165.36

35. Bank balance: \$106.40; cheque record balance: \$131.63; outstanding cheques: \$52.27; outstanding deposits: \$75.00; service charge: \$2.50

36. Bank balance: \$156.14; cheque record balance: \$167.16; outstanding cheques: \$38.70; outstanding deposits: \$47.92; service charge: \$1.80

$C = na - b$

Find each credit cost. (8.7)

	b Amount borrowed	n Number of payments	a Monthly payment	C Credit cost
37.	\$90	8	\$12	\$6
38.	\$165	12	\$15	\$15
39.	\$275	14	\$22	\$33
40.	\$470	18	\$28	\$34

$$R = \frac{24C}{b(n + 1)}$$

Find each annual percentage rate. (8.8)

	b Amount borrowed	n Number of payments	C Credit cost	R Rate
41.	\$500	15	\$60	18%
42.	\$200	14	\$20	16%
43.	\$320	14	\$34	17%
44.	\$600	18	\$95	20%

\$ 710 or less	6%	
710	\$ 43 + 16% on next	\$ 709
1419	156 + 17% on next	1419
2838	397 + 18% on next	1419
4257	653 + 19% on next	2838
7095	1192 + 21% on next	2838
9933	1788 + 23% on next	2838
12,771	2441 + 25% on next	2838
15,609	3150 + 28% on next	4257
19,866	4342 + 32% on next	14,190
34,056	8883 + 36% on next	21,285
55,341	16,546 + 39% on next	29,799
85,140	28,167 + 43% on remainder	

Use the Tax Rate Schedule at the left to find the tax on each taxable income. (8.9)

45. \$8600 \$1508.05 **46.** \$12,500 \$2378.41

47. \$11,260 \$2093.21 **48.** \$13,740 \$2683.25

Add or subtract the times. (8.10)

49. 3:16 + 4:55 = 8:11

50. 8:42 − 4:56 = 3:46

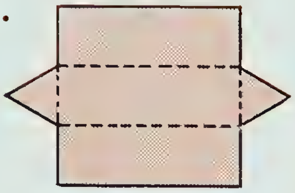
51. 3:15 p.m. − 8:55 a.m. = 6 h 20 min

52. 10:25 a.m. + 3:48 = 2:13 p.m.

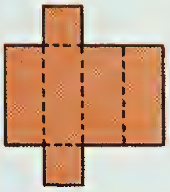
Preview Test for Chapter 9

Refer to the patterns below to answer questions 1 and 2.

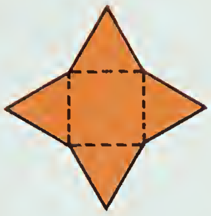
a.



b.



c.

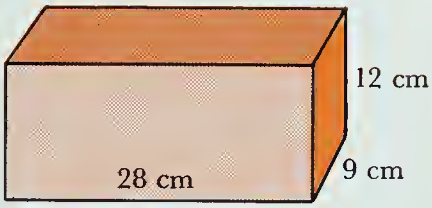


1. Which pattern could be folded to form a rectangular solid? b (9.1)

2. Which patterns could be folded to form right prisms? a, b (9.2)

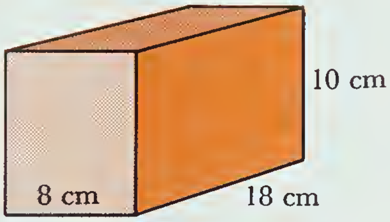
How many 1-centimetre cubes would each box hold? (9.3)

3.



3024

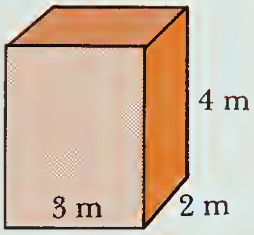
4.



1440

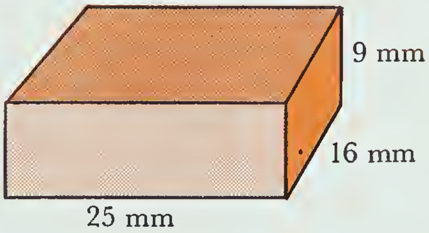
What is the volume of each rectangular solid? (9.4)

5.



$24\ m^3$

6.



$3600\ mm^3$

Change each volume. (9.5)

7. $3\ m^3$ to cm^3 $3\ 000\ 000\ cm^3$ 8. $3000\ cm^3$ to litres 3 L

9. Find the surface area of the solid in Exercise 5. $52 m^2$ (9.6)

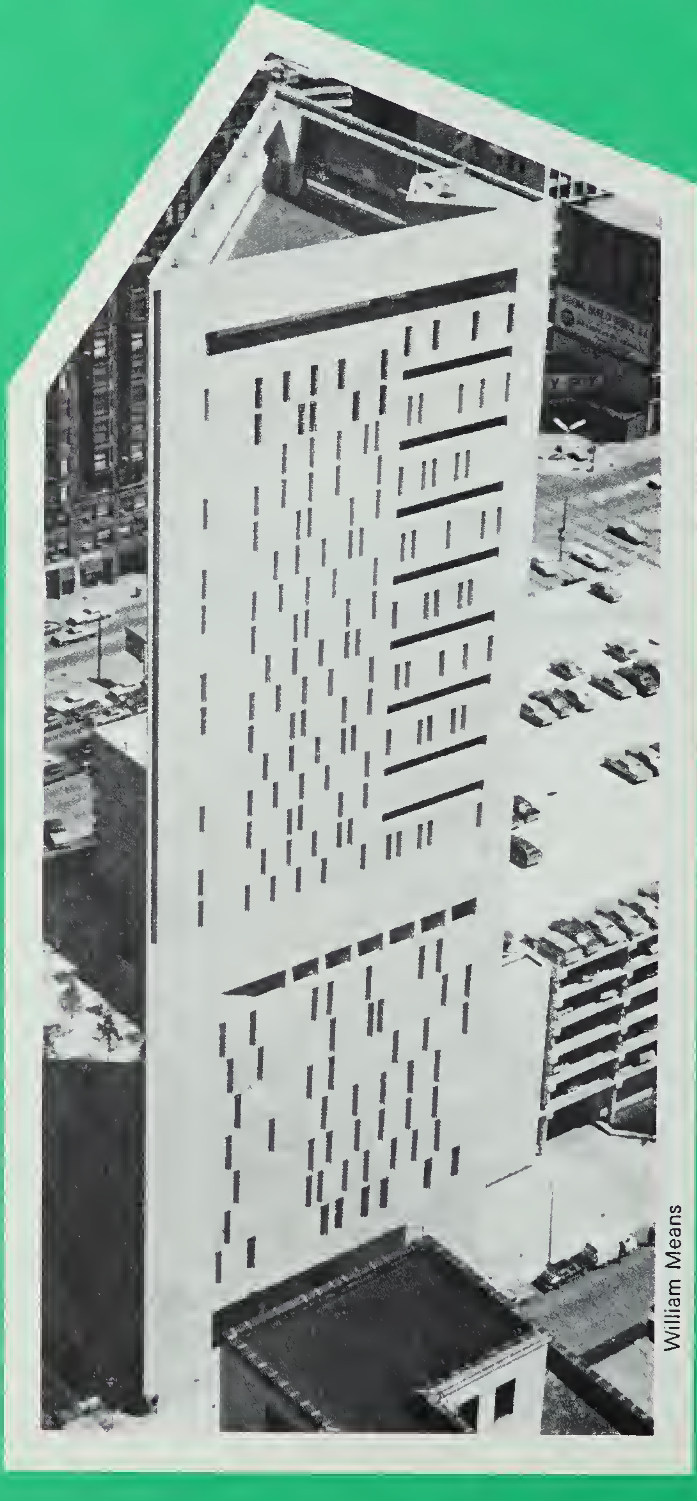
10. Find the volume of a cylinder whose base has a radius of 5 cm. The height of the cylinder is 10 cm. Use 3.14 for π . $785\ cm^3$ (9.7)

11. Find the volume of a sphere whose radius is $\frac{1}{2}$ m. Use $\frac{22}{7}$ for π . $\frac{11}{21} m^3$ (9.8)

9 SOLIDS

PICTURE PREVIEW: Solid figures are all around us. For example, the boxes and the tin cans our food comes in are models of the two most common solids—rectangular solids and cylinders. Other solids, like cones, pyramids, and spheres, are not so common. But they occur often in designs, furniture, and buildings. How many can you recognize on these pages?

Camerique



William Means

Prince/De Wys, Inc.

Johnson/De Wys, Inc.

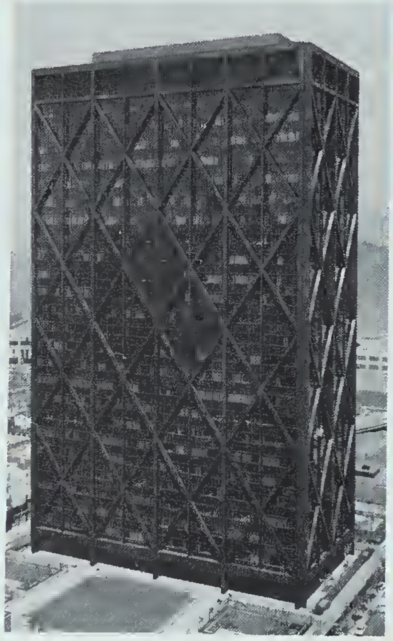
Courtesy Virginia Burdick Associates

9.1 Models of Rectangular Solids

Plane figures such as squares, triangles, and circles are flat. They have only two dimensions. However, **solid figures**, or **solids**, have three dimensions.

A common kind of solid is the **rectangular solid**. It has six **faces**, each a rectangle. The sides of the rectangles are called **edges**. Most cardboard boxes are rectangular solids. Some other examples are shown below.

H. Armstrong Roberts

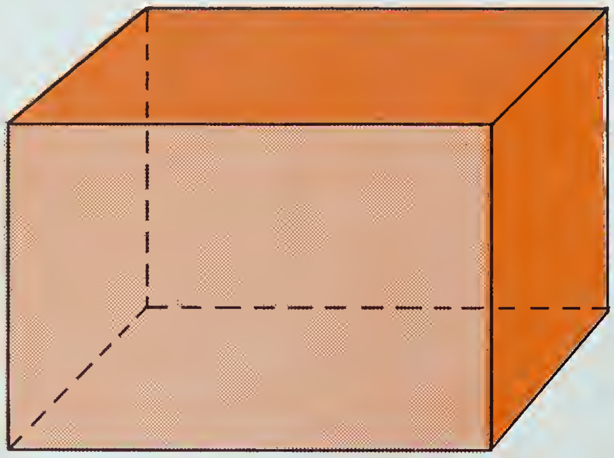


A. Devaney, Inc.

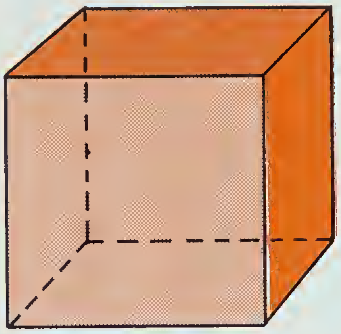
Courtesy Virginia Burdick As

When you look at a rectangular solid, you cannot see all the faces and edges at once. Therefore, dashed lines are used below to show the hidden edges. Notice that a rectangular solid has 12 edges.

Performance Objective:
The student can
- identify properties of rectangular solids.
- build models of rectangular solids by using patterns.



A special kind of rectangular solid is the **cube**. Each face of a cube is a square.



United Press International

Exercises

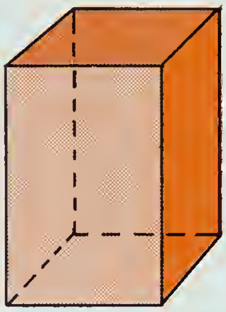
1. How many faces does a rectangular solid have? 6

2. How many edges does a rectangular solid have? 12

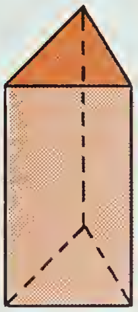
3. A rectangular solid in which each face is a square is called a Cube.

4. Which solid below is not a rectangular solid? b

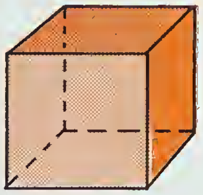
a.



b.

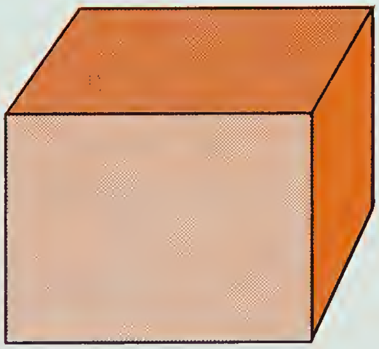


c.

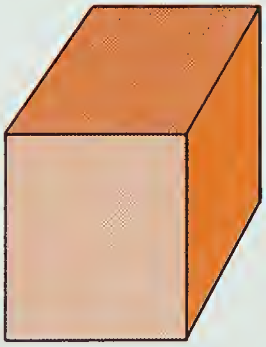


5–6. Draw solids like the following. Include dashed lines to show the hidden edges. 5–6. *Drawings*

5.

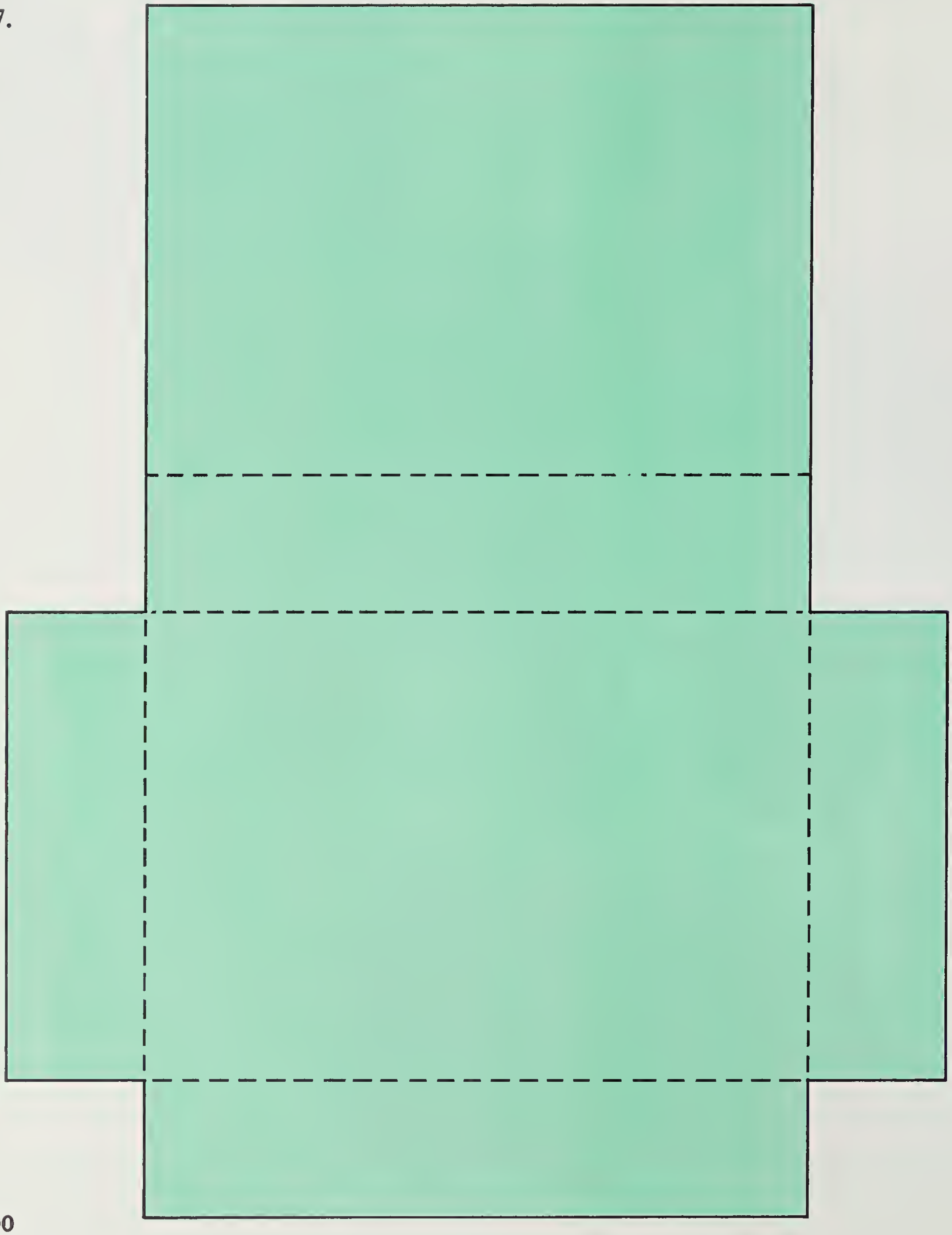


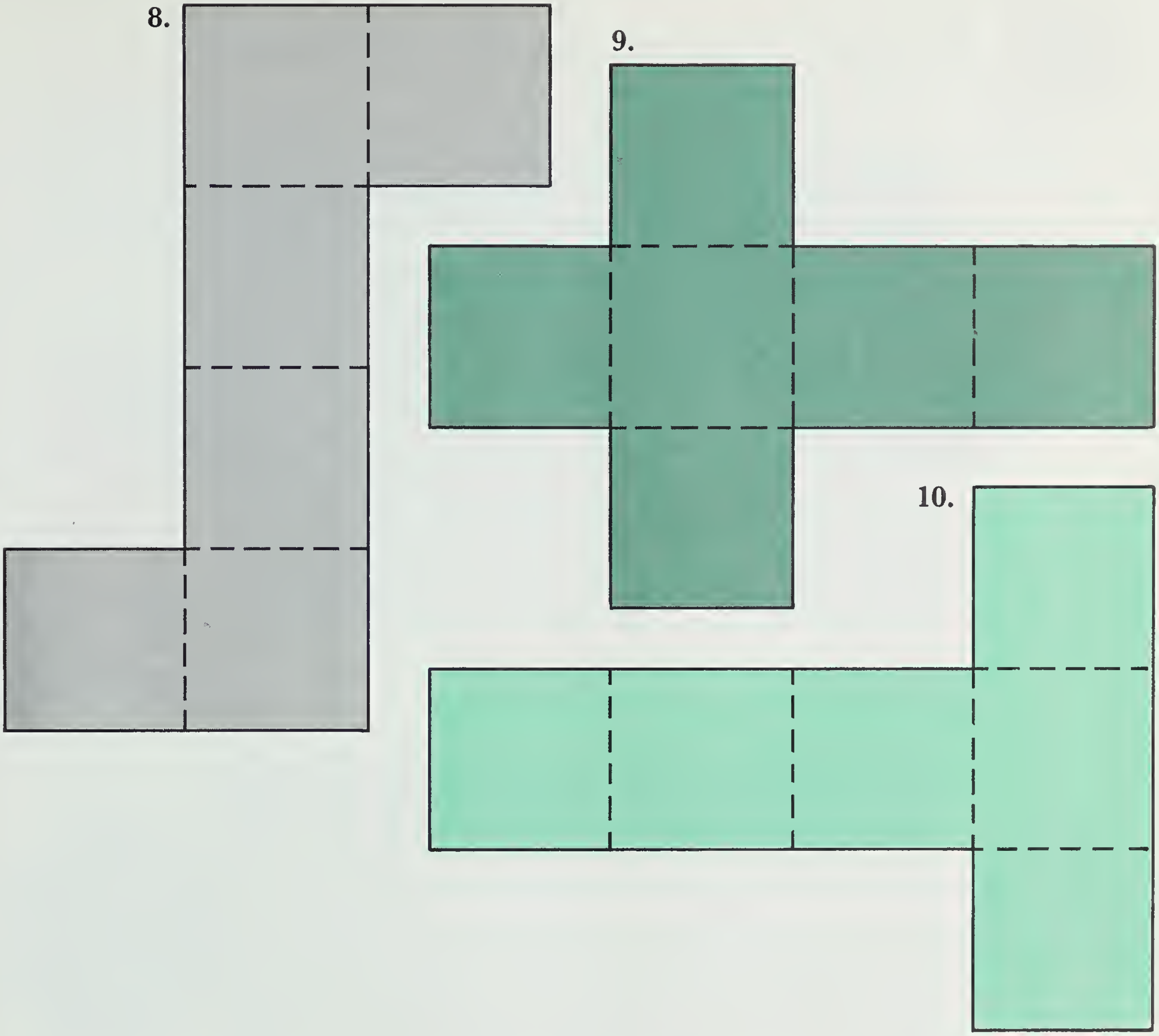
6.



7–10. Trace each pattern on a sheet of paper. Cut out each pattern along the solid lines and fold along the dashed lines. Use tape to hold the solids together.

7.





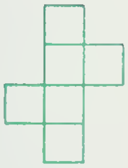
8.

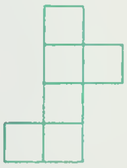
9.

10.

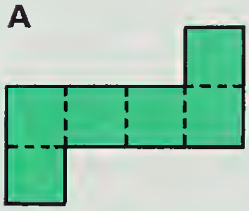
Patterns with Cubes

Notice that all three patterns in Exercises **8**, **9**, and **10** can be folded to form a cube. There are eight other patterns that can be used to form a cube. Draw as many as you can. Use 2 cm squares. (NOTE: **A** and **B** below show the *same* pattern in two different positions, not two different patterns. Also, patterns like **C** are not permitted.)

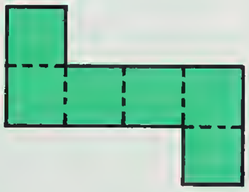




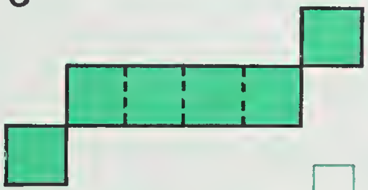
A

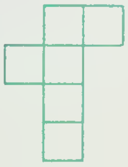


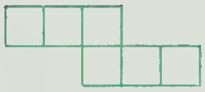
B

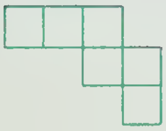


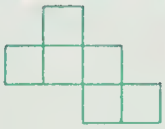
C

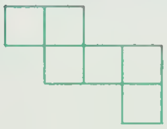


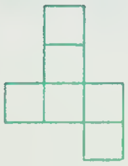










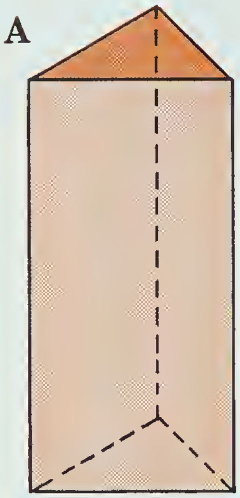


9.2 Models of Right Prisms

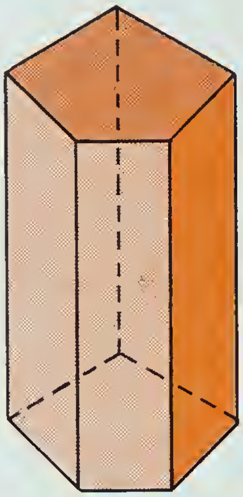
Performance Objective: The student can
- identify properties of right prisms.
- build models of right prisms by using patterns.

The **right prism** is a special type of solid. Each prism has two **bases** that are polygons. These bases are the same shape and size and would never meet, no matter how far extended. The other faces of a right prism are rectangles. Here are three examples:

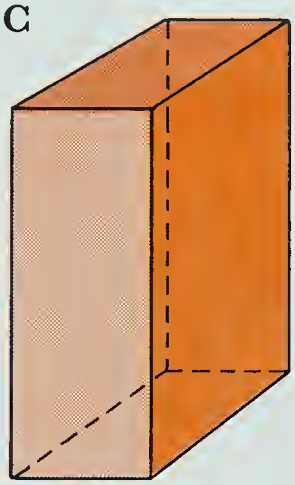
A



B



C

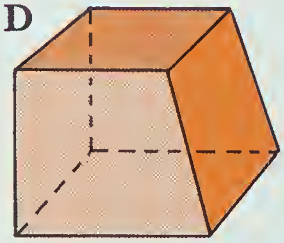


Notice from Example **C** that a rectangular solid is a special kind of right prism in which the two bases are rectangles.

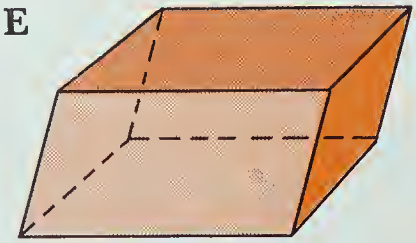
Exercises

1. How many bases does a right prism have? 2
2. How many faces (including the bases) does each right prism above have? A, 5; B, 7; C, 6
3. What kind of polygons are the bases of each right prism shown above? A, triangle; B, pentagon; C, rectangle
4. Which solid figures below are not right prisms? D, E

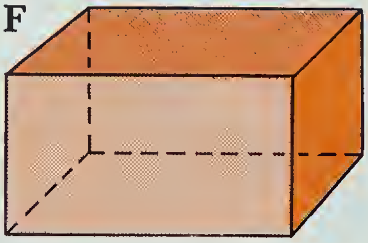
D



E

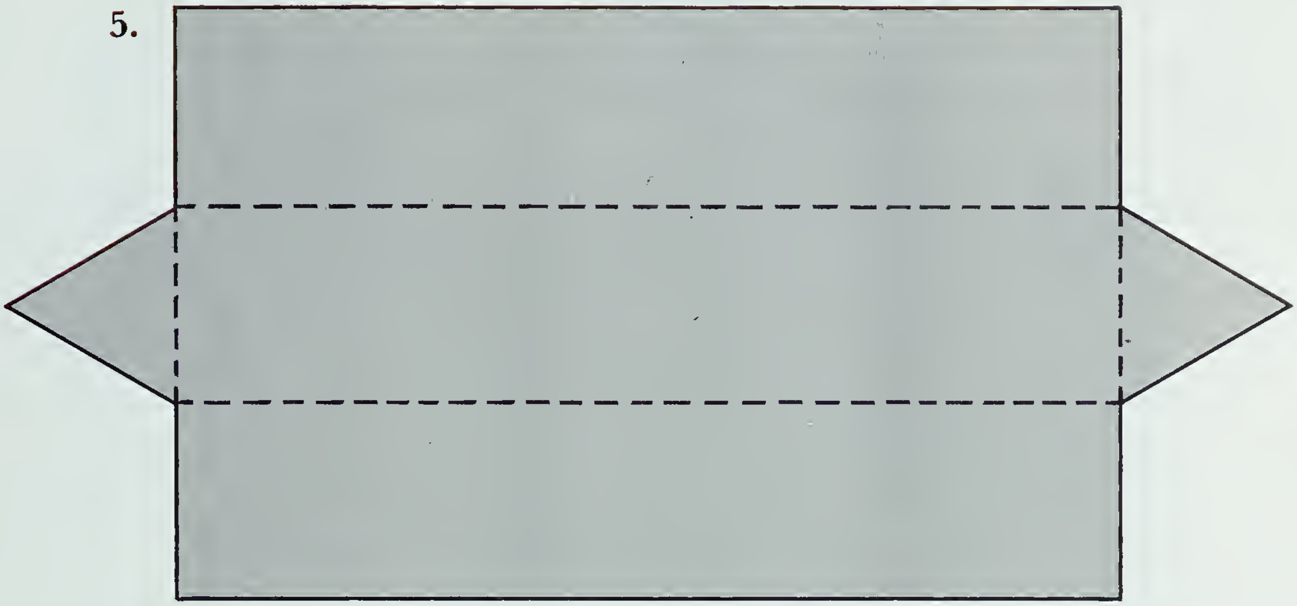


F

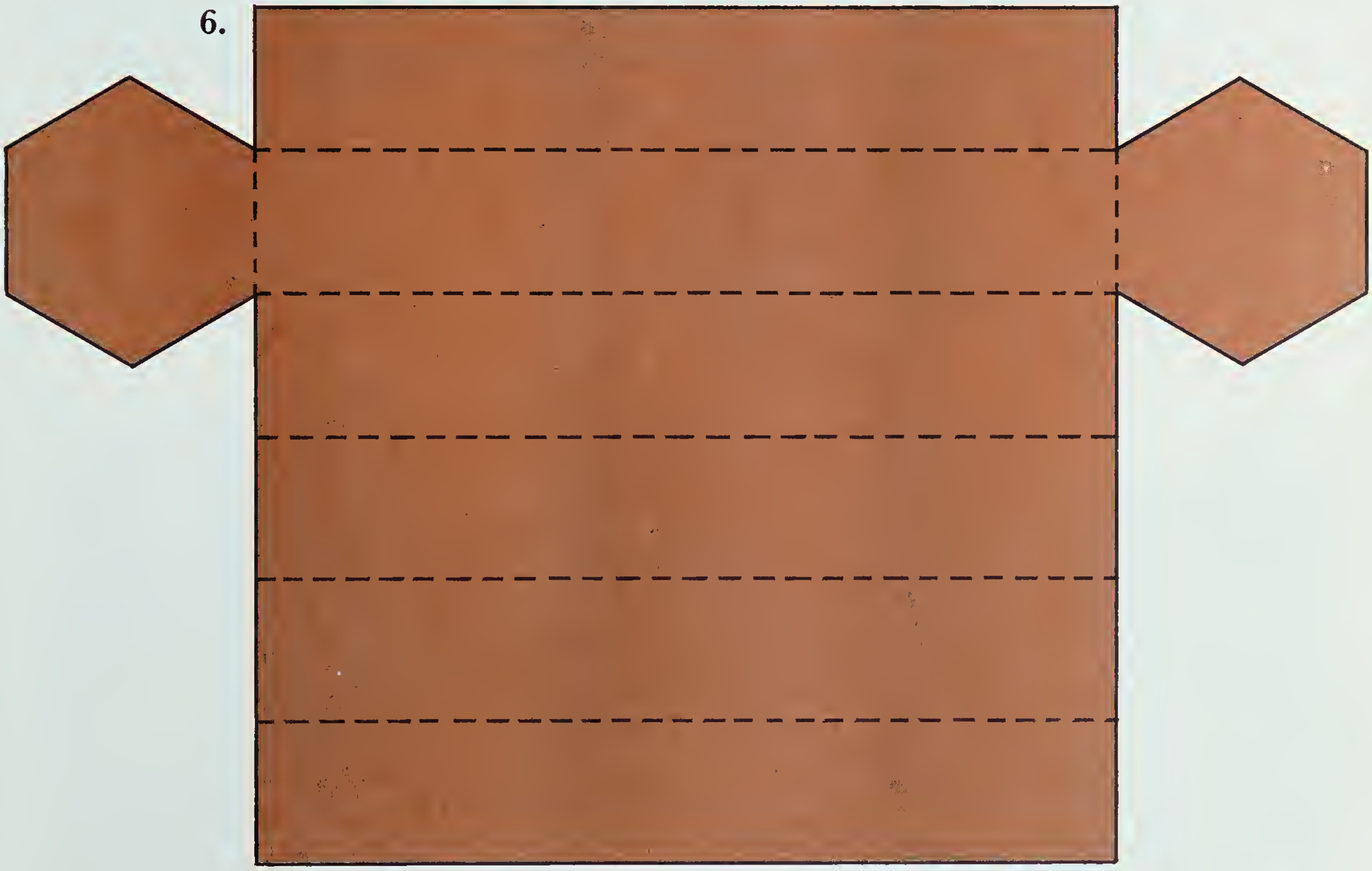


5–7. Copy each pattern on a sheet of paper. Cut the patterns along the solid lines and fold along the dashed lines. Use tape to hold the right prisms together.

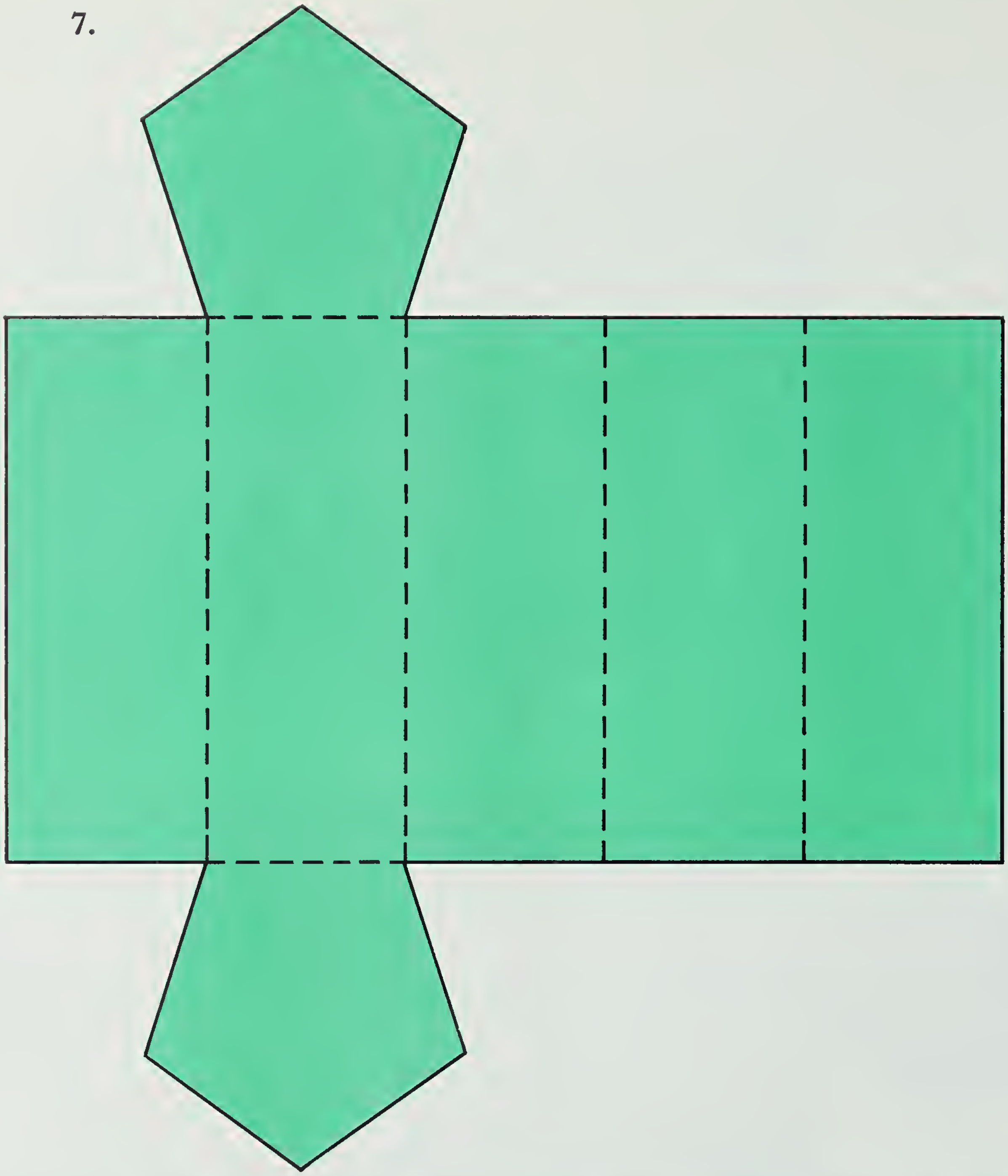
5.



6.



7.



Quick Quiz

Check this skill:

- *multiplying 3 factors*

1–12. Multiply.

1. $18 \times 25 \times 15$ 6750

2. $72 \times 43 \times 19$ 58 824

3. $8.4 \times 12 \times 26$ 2620.8

4. $41.6 \times 24 \times 34$ 33 945.6

5. $4.8 \times 3.5 \times 15$ 252

6. $8.7 \times 1.9 \times 20$ 330.6

7. $\frac{1}{2} \times 10 \times 16$ 80

8. $\frac{1}{4} \times 12 \times 20$ 60

9. $\frac{1}{6} \times 2\frac{1}{2} \times 6$ $2\frac{1}{2}$

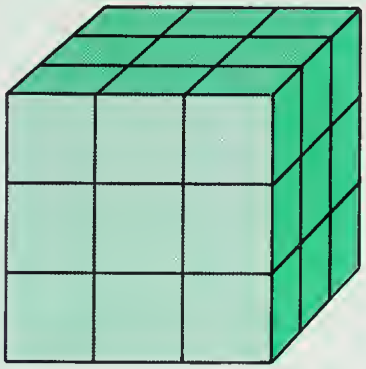
10. $\frac{1}{3} \times 3\frac{1}{3} \times 9$ 10

11. $2\frac{1}{4} \times 2\frac{1}{2} \times 8$ 45

12. $3\frac{1}{5} \times \frac{1}{5} \times 10$ $6\frac{2}{5}$

SLICING CUBES

1. A cube is painted green and cut into smaller cubes as shown. Complete the chart.

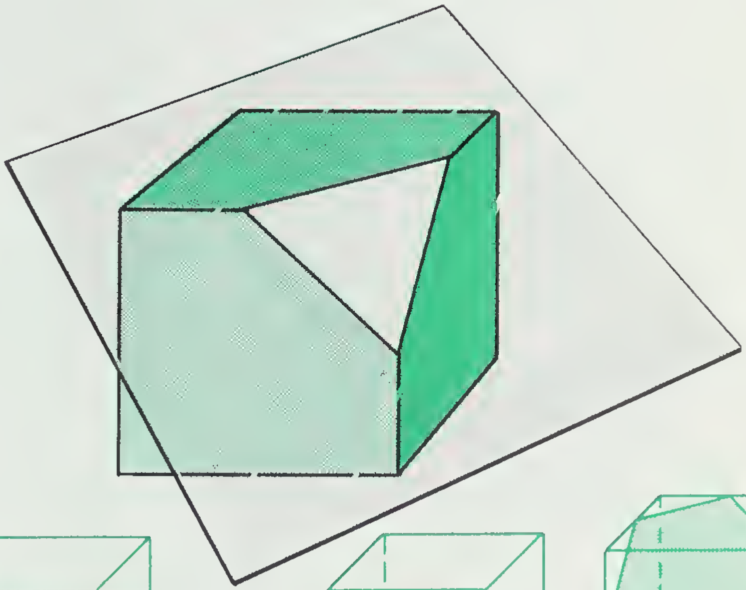


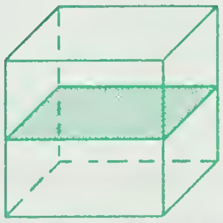
number of smaller cubes having	
4 green faces	0
3 green faces	8
2 green faces	12
1 green face	6
0 green faces	1

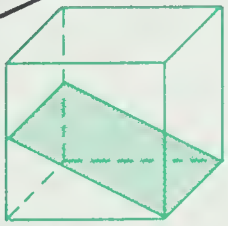
2. A corner of the cube shown at the right was sliced off by a plane. The cut left a triangle. A cube can be sliced in many ways.

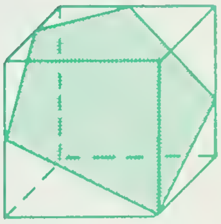
Can each shape be sliced from a cube? Complete the chart.

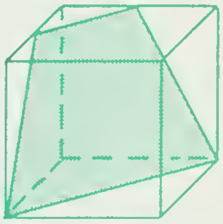
	Yes	No
square	Yes	
rectangle	Yes	
pentagon	Yes	
trapezoid	Yes	
hexagon	Yes	
circle		No

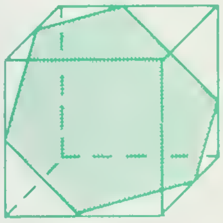












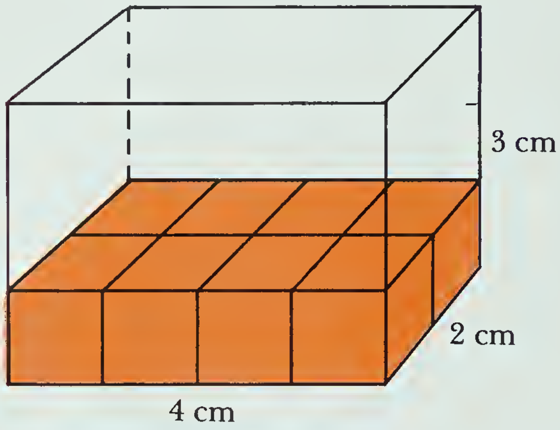
Courtesy University of Iowa

9.3 Measuring with Cubes

Performance Objective: The student can use unit cubes to find the volumes of rectangular solids.

Plane figures can be measured with squares. In a similar way, solid figures can be measured with cubes.

Suppose you want to see how many 1 cm cubes the following rectangular solid would hold. The solid is 4 cm long, 2 cm wide, and 3 cm high.



bottom layer: 2 rows of 4 cubes each

number of layers: 3

Think of filling the solid with layers of cubes. Each layer contains 8 cubes, and the solid would hold 3 layers. Therefore, it would take 24 cubes to fill the solid.

Exercises

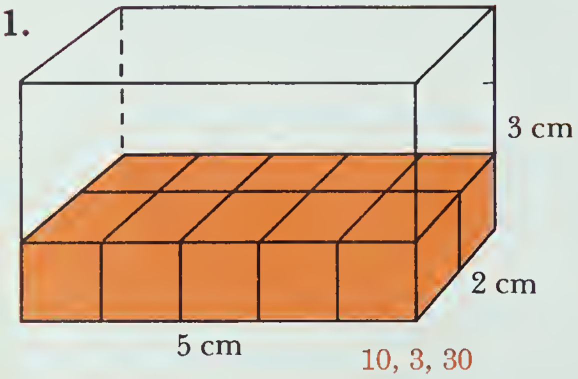
1–8. For each rectangular solid, find

a. the number of cubes in the bottom layer.

b. the number of layers the solid would hold.

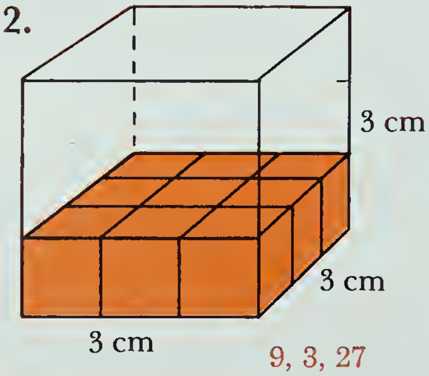
c. the number of cubes it would take to fill the solid.

1.



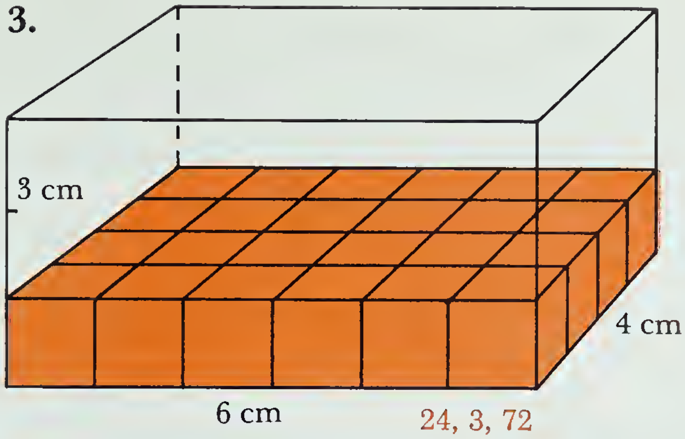
10, 3, 30

2.



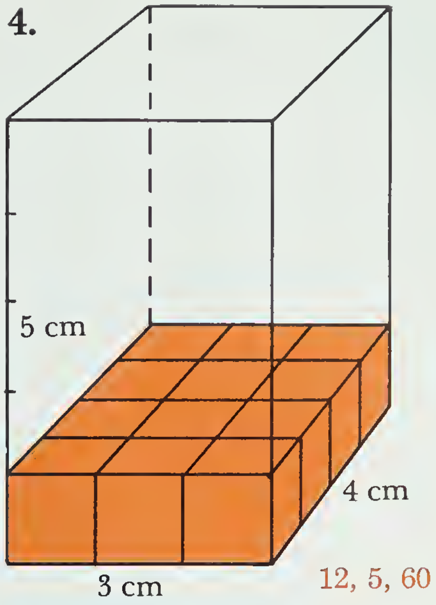
9, 3, 27

3.



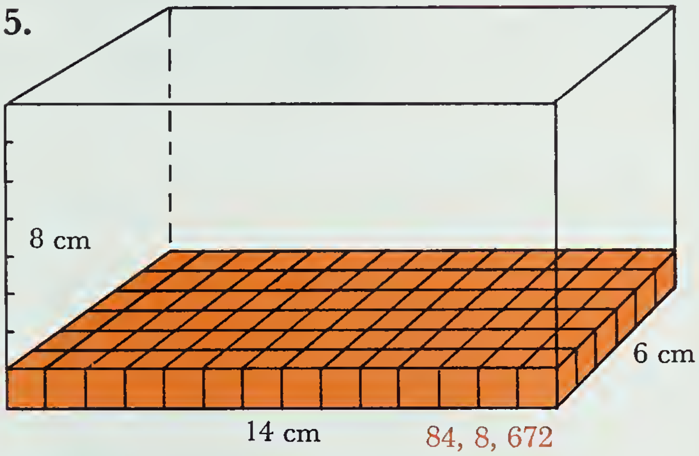
24, 3, 72

4.



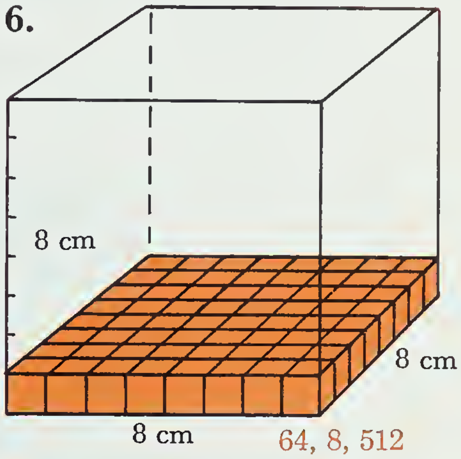
12, 5, 60

5.


8 cm
6 cm
14 cm

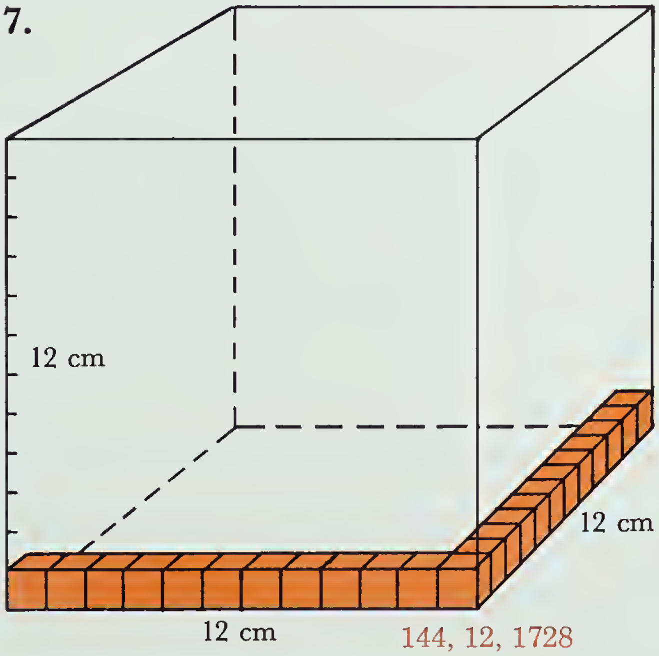
84, 8, 672

6.



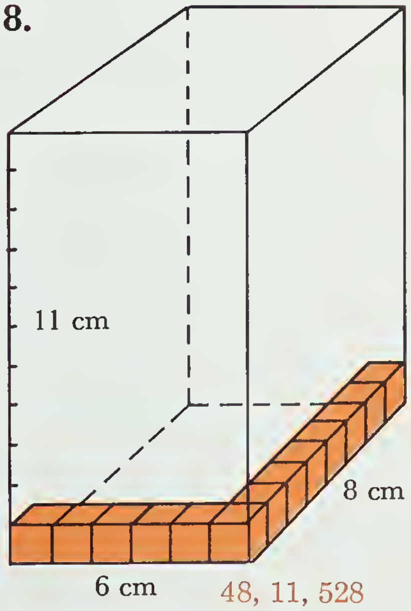
64, 8, 512

7.


12 cm
12 cm
12 cm

144, 12, 1728

8.


11 cm
8 cm
6 cm

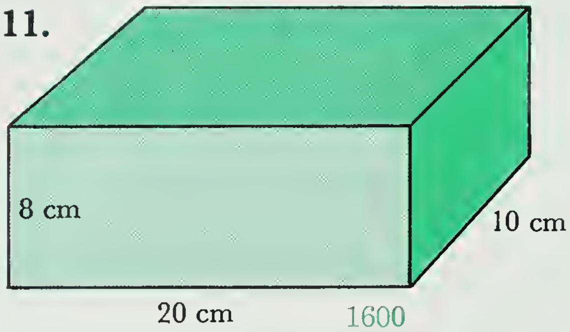
48, 11, 528

9. Without actually counting the cubes, how can you find the number of cubes in the bottom layer of a rectangular solid?
Multiply length times width.

10. Without actually counting the cubes, how can you find the total number of cubes in a rectangular solid?
Multiply the number of cubes in the bottom layer by the height.

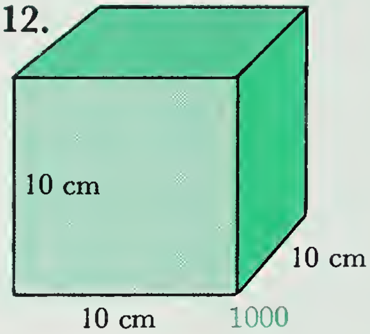
11—12. How many 1 cm cubes would it take to fill each solid?

11.



1600

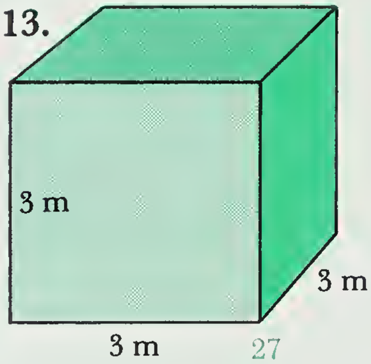
12.



1000

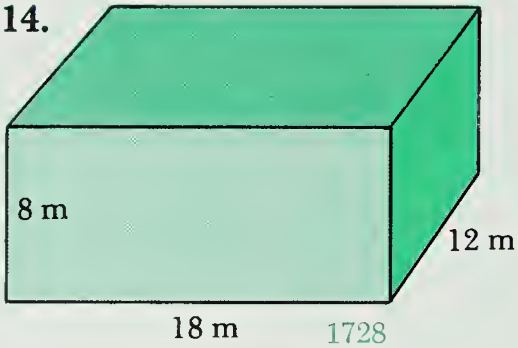
13—14. How many 1 m cubes would it take to fill each solid?

13.



27

14.



1728

15. A cardboard box is 10 cm long, 8 cm wide, and 5 cm high. It is to be filled with wooden blocks that are 1 cm on each side. How many blocks will it take to fill the box? 400

16. A packing crate is 3 m long, 4 m wide, and 2 m high. It is packed with boxes that are 1 m cubes. How many boxes will the crate hold? 24

Class Activity

Illustrate Exercises **1** and **2** on page 396 by using the cubes that all the members of your class made in Section 9.1.

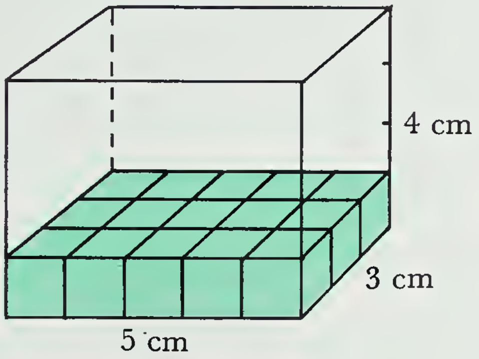
Volume 9.4

The amount of space that a solid contains is called its **volume.** Volume is usually measured in **cubic units.**

Performance Objective: The student can find the volumes of rectangular solids.

A cube that is 1 cm long, 1 cm wide and 1 cm high is called a **cubic centimetre** (cm^3). Another common unit of volume is the cubic metre (m^3). The cubic metre (m^3) is 1 m long, 1 m wide and 1 m high. Another unit of volume is the cubic millimetre (mm^3).

Example: 1 What is the volume of a rectangle solid that is 5 cm long, 3 cm wide, and 4 cm high?



number of cubes in bottom layer: 3 × 5 or 15

number of layers: 4

total number of cubes: 4 × 15

The volume of the box is 60 $cm.^3$

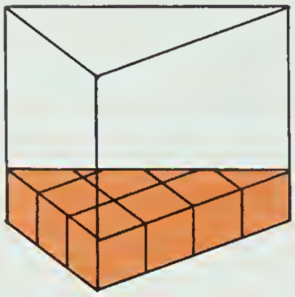
In Example 1, notice that the area of the base is 3 × 5 or 15 cm^2. Also, 15 is the number of cubes in the bottom layer. This suggests a way to find the volume of any rectangular solid. The same method applies to any right prism.

To find the volume of a rectangular solid or any right prism, multiply the area of the base times the height.

Volume = area of base × height

$$V = Bh$$

Example 2: Find the volume of the prism. The area of its base is 6 cm^2. Its height is 5 cm.



Since the area of the base is given, you know the number of cubes in the bottom layer.

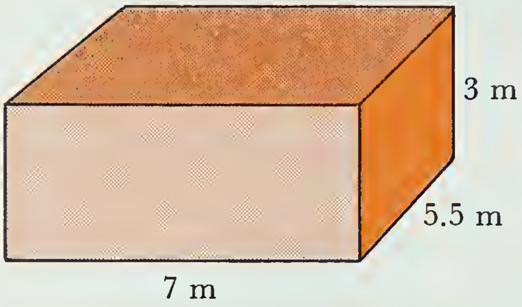
$V = Bh$

$V = 6 \times 5$

$V = 30$

The volume is 30 cm^3.

Example 3: A room is 7 m long, 5.5 m wide, and 3 m high. What is its volume?



Since the area of the base is not given, you must find it.

A room is a rectangular solid. So the area of its base is $B = lw$.

$V = Bh$ (or $V = lwh$)

$V = (7 \times 5.5) \times 3$

$V = 115.5$

The volume is 115.5 m^3.

From Example **3** notice the following:

> For a rectangular solid the volume formula $V = Bh$ is also stated as
>
> $$V = lwh.$$

Exercises

1. What is the formula for finding the volume of a rectangular solid or any right prism? $V = Bh$
2. What is another way to state the volume formula $V = Bh$ for a rectangular solid? $V = \ell wh$
3. If the length, width, and height of a rectangular solid are in metres, its volume will be in cubic metres

4. If the area of the base of a right prism is 10 cm^2 and the height is 5 cm, the volume is 50 ______ . cm^3

5–8. Find the volume of each right prism where B is the area of the base and h is the height.

5. $B = 9\text{ m}^2$, $h = 6\text{ m}$ 54 m^3

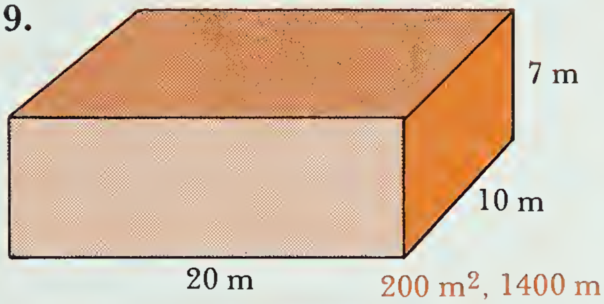
6. $B = 8\text{ cm}^2$, $h = 4\text{ cm}$ 32 cm^3

7. $B = 7\text{ m}^2$, $h = 9\text{ m}$ 63 m^3

8. $B = 6\text{ mm}^2$, $h = 2\text{ mm}$ 12 mm^3

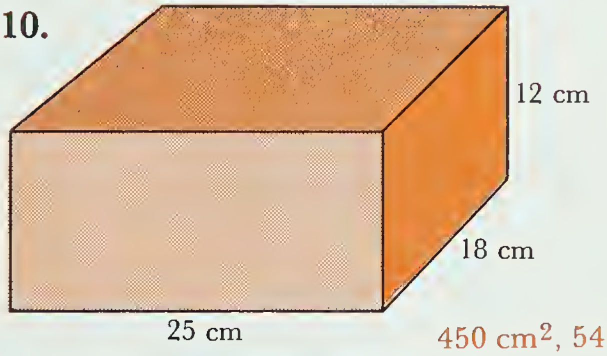
9–14. Find the area of the base of each rectangular solid. Then find the volume.

9.



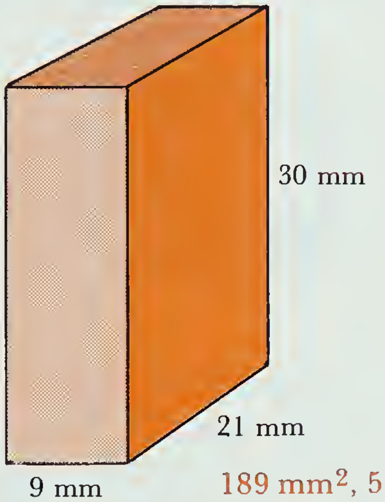
200 m^2, 1400 m^3

10.


12 cm
18 cm
25 cm

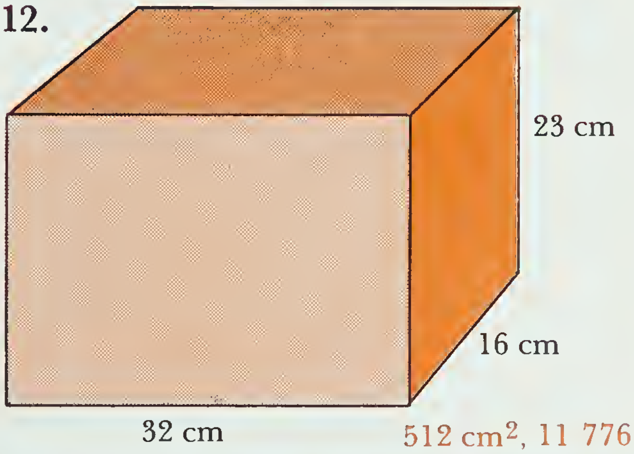
450 cm^2, 5400 cm^3

11.


30 mm
21 mm
9 mm

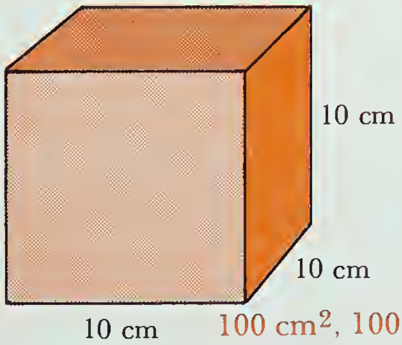
189 mm^2, 5670 mm^3

12.


23 cm
16 cm
32 cm

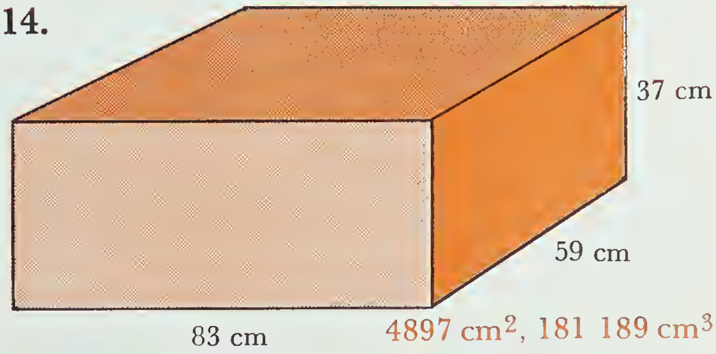
512 cm^2, 11 776 cm^3

13.

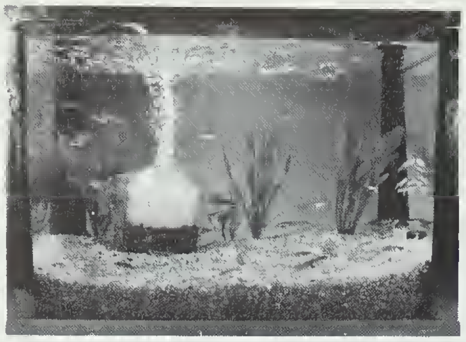

10 cm
10 cm
10 cm

100 cm^2, 1000 cm^3

14.


37 cm
59 cm
83 cm

4897 cm^2, 181 189 cm^3



Grant Heilman

15. An aquarium is 30 cm wide, 75 cm long and 34 cm high. What is its volume? 76 500 cm^3

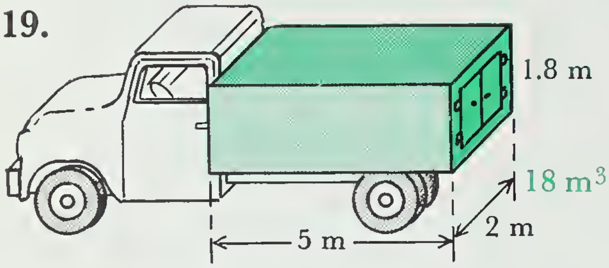
16. A contractor is digging a basement that is 6 m wide, 10 m long and 2.5 m deep. How many cubic metres of dirt must be removed? 150 m^3

17. A driveway is 3 m wide and 8 m long. It is covered with a concrete slab that is 15 cm (0.15 m) deep. How many cubic metres of concrete are needed for the driveway? 3.6 m^3

18. A classroom is 11 m long, 6 m wide and 3 m high. How many cubic metres of airspace are in the classroom? 198 m^3

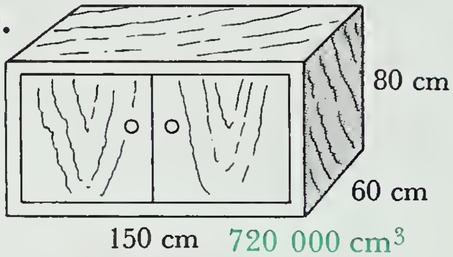
19–22. Find each volume.

19.



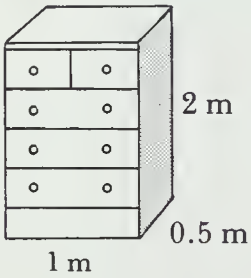
18 m^3

20.



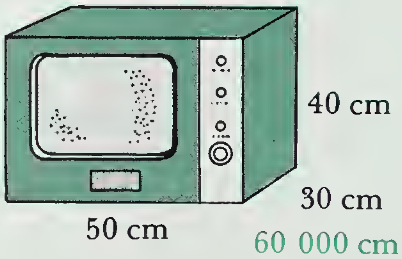
720 000 cm^3

21.



1 m^3

22.



60 000 cm^3

23–24. A factory makes auto parts that are packed in boxes 8 cm by 12 cm by 5 cm. How many boxes would fit into packing crates that have the following dimensions?

23. 90 cm by 50 cm by 56 cm 525

24. 30 cm by 36 cm by 24 cm 54

25. There are 1000 cm^3 of water in 1 L. How many litres of water would the aquarium in Exercise 15 hold? About 76.5 L

26. A litre of water has a mass of 1 kg. What is the mass of the water in the aquarium in Exercise 15? 76.5 kg

Performance Objective: The student can change a given volume to an equivalent volume.

Equivalent Volumes

9.5

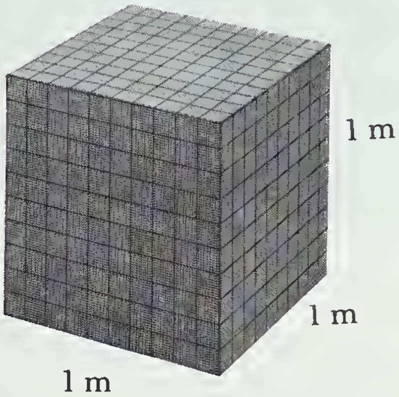
A cubic metre is 100 cm long, 100 cm wide and 100 cm high. So you can easily find how many cubic centimetres are in one cubic metre.

Volume = (length × width) × height

$V = (100 \times 100) \times 100$

$V = 10\ 000 \times 100$

$V = 1\ 000\ 000$



The volume is 1 000 000 cm^3.

1 000 000 cm^3 in 1 m^3

This fact is useful in finding **equivalent volumes.** Often you need to change a volume from cubic centimetres to cubic metres.

Example 1: Change 4 500 000 cm^3 to cubic metres.

There are 1 000 000 cm^3 in 1 m^3

4 500 000 in n m^3

$$\frac{1\ 000\ 000}{4\ 500\ 000} = \frac{1}{n}$$

$$n \times 1\ 000\ 000 = 4\ 500\ 000$$ Find the cross-products.

$$1\ 000\ 000\,n = 4\ 500\ 000$$

$$n = \frac{4\ 500\ 000}{1\ 000\ 000}$$ Divide by 1 000 000

$$n = 4.5$$

There are 4.5 m^3 in 4 500 000 cm^3.

Concrete is usually sold by the cubic metre.

Example 2: A large balcony is being built which is supported by concrete posts. The workers must pour concrete into forms which are rectangular. There are 12 posts and each is 20 cm by 20 cm by 120 cm. How many *cubic metres* of concrete are needed?

First find the volume in cubic centimetres of one post.

$$\text{Volume} = (\text{length} \times \text{width}) \times \text{height}$$

$$V = (20 \times 20) \times 240$$

$$V = 400 \times 240$$

$$V = 96\ 000 \text{ cm}^3$$

There are twelve posts

$$\text{Total volume} = \text{number of posts} \times \text{volume of one post}$$

$$V = 12 \times 96\ 000$$

$$V = 1\ 152\ 000 \text{ cm}^3$$

The volume is 1 152 000 cm^3. Now you must change 1 152 000 cm^3 to cubic metres.

1 000 000 cm^3 in 1 m^3
1 152 000 cm^3 in n m^3

$$\frac{1\ 000\ 000}{1\ 152\ 000} = \frac{1}{n}$$

$$1\ 000\ 000\, n = 1\ 152\ 000$$

$$n = \frac{1\ 152\ 000}{1\ 000\ 000}$$

$$n = 1.152$$

So 1.152 m^3 of concrete are needed. (In actual use, such a result would be rounded)

Exercises

1. A cube which is 1 cm long, 1 cm wide and 1 cm high is called a cubic centimetre.

2. A volume of one cubic metre is equivalent to 1 000 000 cubic centimetres.

3–10. Change each volume to cubic centimetres.

3. 16 m^3 16 000 000 cm^3
4. 7 m^3 7 000 000 cm^3
5. 6.2 m^3 6 200 000 cm^3
6. 8.1 m^3 8 100 000 cm^3
7. 0.3 m^3 300 000 cm^3
8. 0.7 m^3 700 000 cm^3
9. 0.012 m^3 12 000 cm^3
10. 0.002 m^3 2 000 cm^3

11–18. Change each volume to cubic metres.

11. 17 000 000 cm^3 17 m^3
12. 9 000 000 cm^3 9 m^3
13. 2 400 000 cm^3 2.4 m^3
14. 8 300 000 cm^3 8.3 m^3
15. 5 420 000 cm^3 5.42 m^3
16. 31 200 000 cm^3 31.2 m^3
17. 83 000 cm^3 0.083 m^3
18. 500 000 cm^3 0.5 m^3

19. A basement is dug that is 11 m long, 5.5 m wide and 2.5 m deep. How many *cubic metres* of dirt must be removed? 151.25 m^3

20. A concrete pad to support a heavy machine is poured into a form that is 8.5 m long, 5 m wide and 500 cm deep. How many *cubic metres* of concrete are needed? 21.25 m^3

21. A concrete patio is 5.5 m long, 3 m wide and 180 cm deep. How many *cubic metres* of concrete are needed for the patio? 2.97 m^3

22. A concrete supporting column has a 30 cm by 30 cm base and is 3 m high. What is its volume in cubic centimetres? If 12 of these columns are to be poured, how many *cubic metres* of concrete are needed? 270 000 cm, 3.24 m^3

CAREER CORNER

MATH IN PARAPROFESSIONS

You know that *paramedics* are specially trained helpers who work under a doctor's orders. There are other *paraprofessionals* too. These trained helpers work with some professional like an engineer, a lawyer, or a teacher. Here are just a few:

Air-conditioning and Heating Technician
Agricultural Technician
Automotive Technician
Community Aide
Dental Assistant

Ecology Technician
Forestry Technician
Safety Technician
Social Work Aide
Teacher Aide
Urban Planning Associate

Most paraprofessions are very new. Some are just getting started. Others are jobs of the future. Probably the best way to become a paraprofessional is to finish high school and then take a special two-year course in a community college.

Some paraprofessions need no more math than you will get in this course. Others need more math in high school or in a community college. Here are some volume problems from the paraprofessions:

1. *Agricultural Technician:* During a rainfall of 37.5 mm, what volume of water would fall on a field that measures 0.5 km by 0.8 km?

150 000 m^3

2. *Air-conditioning and Heating Technician:* Find the total volume of an apartment, given the dimensions below.

	length	width	height
living room	6 m	4 m	2.5 m
kitchen-dining room	4 m	4 m	2.5 m
bedroom	4 m	3 m	2.5 m
bathroom	2.5 m	2 m	2.5 m

142.5 m^3

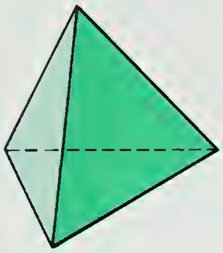
3. *Automotive Technician:* In a certain 4-cylinder engine, each cylinder has a bore (diameter) of 7.8 cm. The piston in each cylinder has a stroke (total travel distance) of 8.2 cm. Find the total engine displacement.

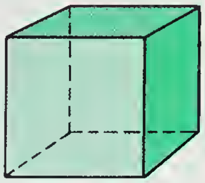
$$\text{Eng Disp} = \text{no. of cyl} \times \pi \times \left(\tfrac{1}{2}\,\text{bore}\right)^2 \times \text{stroke}$$

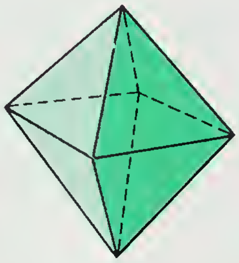
approx. 1566.5 cm^3

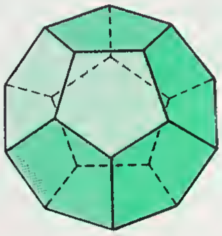
PLATONIC SOLIDS

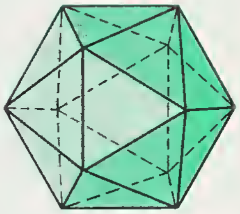
It has been proved that there are only five solids with these properties: (1) All edges are the same length, (2) all faces have the same size and shape, and (3) the same number of edges meet at each corner. Each solid is pictured below.


tetrahedron


cube


octahedron

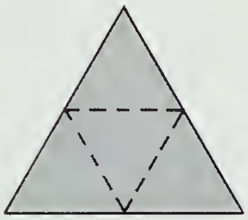

dodecahedron


icosahedron

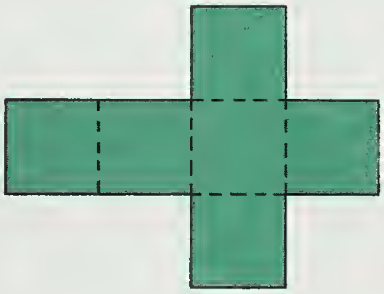
These solids are called **Platonic solids** (after Plato, a philosopher of ancient Greece). Plato felt that these solids stood for earth, fire, air, water, and the universe.

You can construct models of each Platonic solid by using the following patterns. However, each pattern should be enlarged. Cut on solid lines; fold on dashed lines.

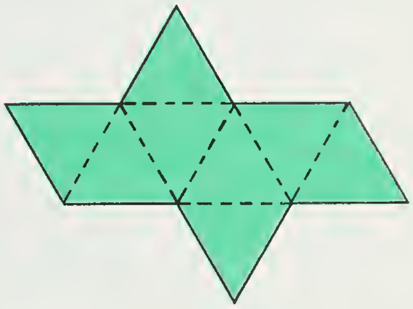
tetrahedron



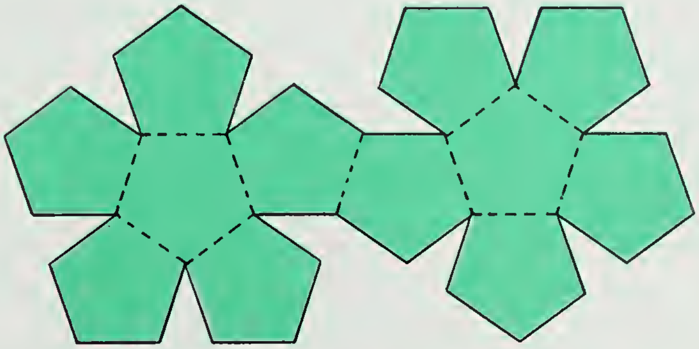
cube



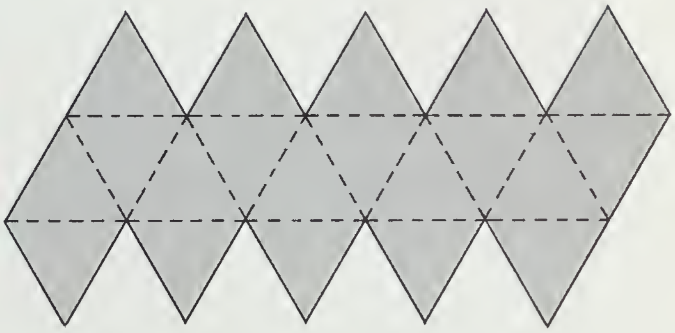
octahedron



dodecahedron



icosahedron

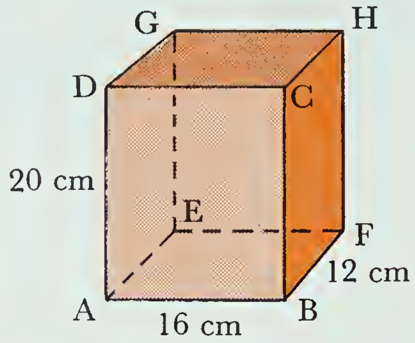


9.6

Surface Area

The total area of all the faces of a solid is called the **surface area**. To find the surface area of a rectangular solid, you must find the areas of six rectangles. For the solid below, the lengths of only three edges are labeled. From these, you know the lengths of all the other edges.

Performance Objective: The student can find the surface areas of rectangular solids.



AB, *EF*, *DC*, and *GH* are 16 cm.

BC, *FH*, *AD*, and *EG* are 20 cm.

BF, *AE*, *CH*, and *DG* are 12 cm.

area of $ABFE = 16 \times 12 = 192\text{ cm}^2$

area of $DCHG = 16 \times 12 = 192\text{ cm}^2$ — $192 \times 2 = 384$

area of $BFHC = 12 \times 20 = 240\text{ cm}^2$

area of $AEGD = 12 \times 20 = 240\text{ cm}^2$ — $240 \times 2 = 480$

area of $ABCD = 16 \times 20 = 320\text{ cm}^2$

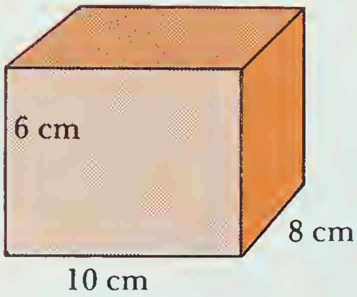
area of $EFHG = 16 \times 20 = 320\text{ cm}^2$ — $320 \times 2 = 640$

Add.

surface area $= 1504\text{ cm}^2$

For each rectangle in a rectangular solid, there is always another rectangle of equal area. For example, when you find the area of rectangle *ABFE*, you know the area of rectangle *DCHG*.

Example 1: A box is 10 cm long, 8 cm wide, and 6 cm high. How much paper is needed to wrap the box? (Assume none of the paper overlaps. In actual use you would allow a little extra for overlap.)

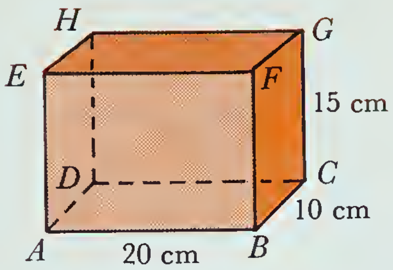


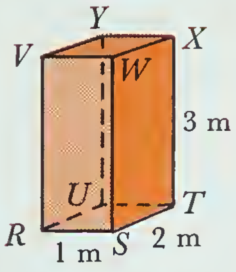
bottom $= 10 \times 8 = 80$	$80 \times 2 = 160$	
side $= 8 \times 6 = 48$	$48 \times 2 = 96$	Add.
front $= 10 \times 6 = 60$	$60 \times 2 = 120$	
	376	

376 cm^2 of wrapping paper are needed.

Exercises

1–16. Give the length of each edge.


H
G
E
F
15 cm
D
C
10 cm
A
20 cm
B


Y
V
X
W
3 m
U
T
R
1 m
S
2 m

1. *AD* 10 cm
2. *DC* 20 cm
3. *BF* 15 cm
4. *AE* 15 cm
5. *EF* 20 cm
6. *HG* 20 cm
7. *EH* 10 cm
8. *FG* 10 cm
9. *YU* 3 m
10. *VW* 1 m
11. *YX* 1 m
12. *YV* 2 m
13. *WS* 3 m
14. *RU* 2 m
15. *UT* 1 m
16. *VR* 3 m

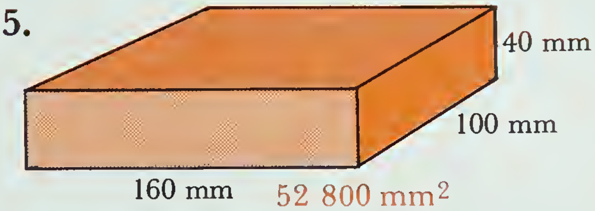
17–22. Use the solids above. For each rectangle, name another rectangle with equal area.

17. *ABCD* *EFGH*
18. *ADHE* *BCGF*
19. *ABFE* *DCGH*
20. *VWXY* *RSTU*
21. *VWSR* *YXTU*
22. *WSTX* *VRUY*

23–24. Find the surface area of each rectangular solid above.
23. 1300 cm^2 24. 22 m^2

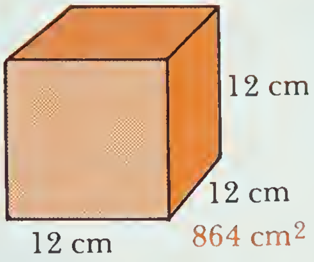
25–28. Find the surface area of each rectangular solid below.

25.


40 mm
100 mm
160 mm

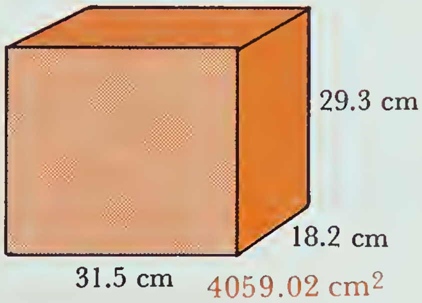
52 800 mm^2

26.


12 cm
12 cm
12 cm

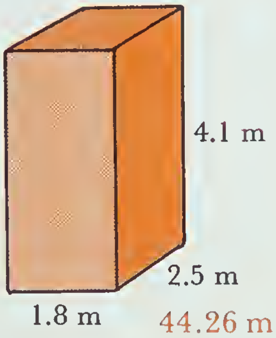
864 cm^2

27.


29.3 cm
18.2 cm
31.5 cm

4059.02 cm^2

28.

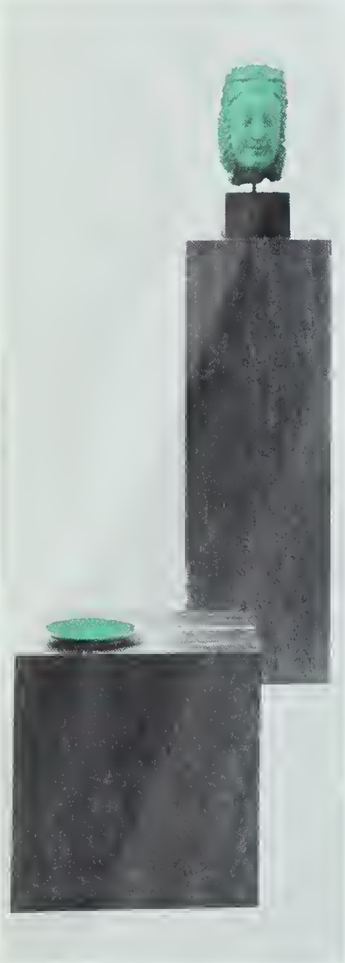


44.26 m^2

29—36. Solve the problems. Assume there is no waste or overlap.

29. How many square centimetres of cardboard are needed to make a box 12 cm long, 8 cm wide and 6 cm wide? 432 cm²

30. How many square centimetres of paper are needed to wrap a box that is 30 cm long, 24 cm wide and 15 cm deep? 3060 cm²



Courtesy The Pace Collection, Inc.
New, York, New York
Design by: Pace Design Staff

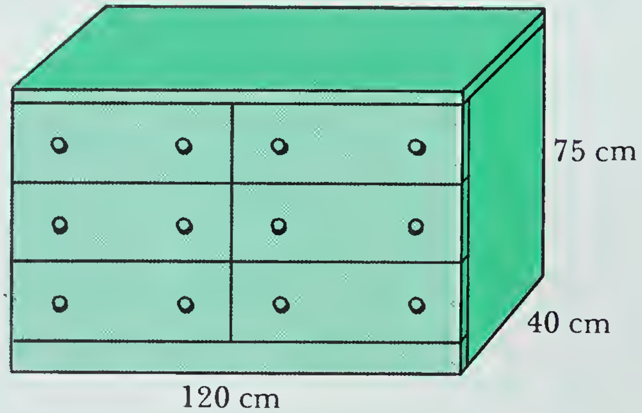
31. A coffee table in the form of a rectangular solid is 60 cm long, 45 cm wide and 45 cm high. The table (except for the base) is covered with a mirror surface. How many square centimetres of mirror surface are needed? 121 500 m²

32. A tool shed in the form of a rectangular solid is covered with sheet metal. It is 2 m long, 2 m wide and 2 m high. How many square metres of sheet metal are needed? (Only the sides and roof are covered with sheet metal.) 20 m²

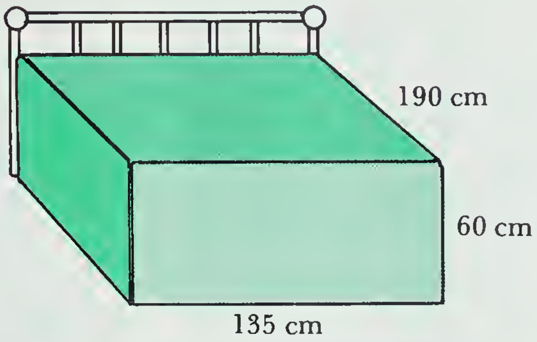
33. The top and sides of a rectangular solid are covered with wood veneer. The solid is 30 cm long, 30 cm wide and 75 cm high. How many square centimetres of wood veneer are needed? 9900 cm²

34. A jewelry box in the form of a rectangular solid is covered with gold leaf. How many square centimetres of gold leaf are needed if the jewelry box is 20 cm long, 10 cm wide and 7 cm deep? 820 cm²

35. An unpainted chest is to be painted on the top, the sides and the front. How many square centimetres of surface need to be painted? 28 080 cm²

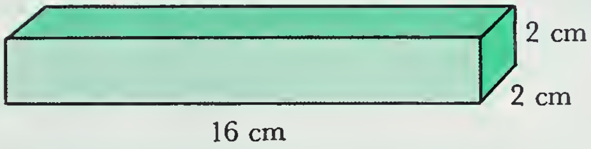


36. A bedspread is to be made for the bed shown below. The spread is to cover the top, the sides, and the front (but not the back) of the bed. How much cloth is needed to make the spread? Neglect the amount of cloth needed for the hem. 56 550 cm^2

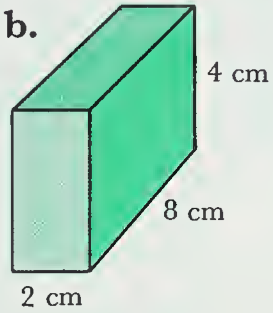


37. A cardboard-box manufacturer wants to make a box with a *volume* of 64 cm^3. Show that each box below has the correct volume.

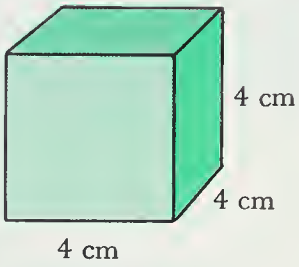
a.



b.



c.



Which box would require the least amount of cardboard? (Now you must find the surface area of each box.) c, 96 cm^2

38. Can two boxes have the same volume but different surface areas? (See Exercise **37**.) Find the dimensions of a fourth box that would have the same volume as those in Exercise **37** but a different surface area from all of them. Yes, 32 × 2 × 1

1–4. Find each product.

1. $3.14 \times 7 \times 7 \times 25$ 3846.5

2. $3.14 \times 12 \times 12 \times 20$ 9043.2

3. $\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6$

4. $\frac{4}{3} \times \frac{22}{7} \times 11 \times 11 \times 11$

Quick Quiz

Check this skill:

- *multiplying four or more factors*

3. $905\frac{1}{7}$

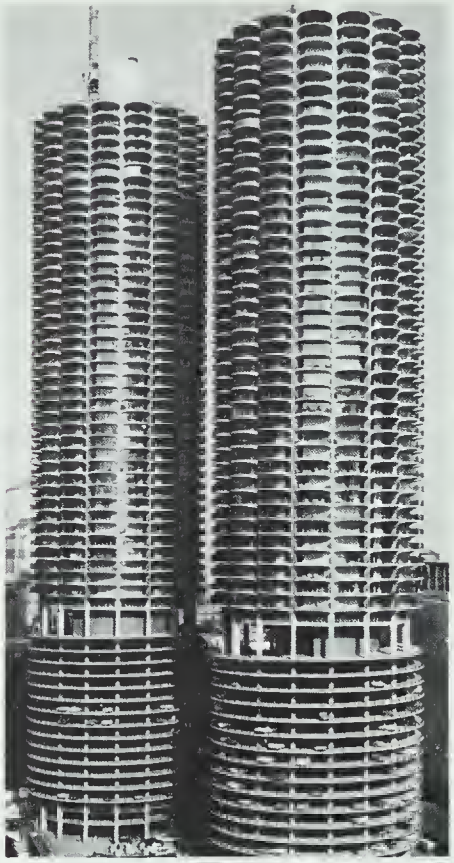
4. $5577\frac{11}{21}$

9.7 Cylinders

A common solid figure is the **cylinder**. A tin can is one example. Some others are pictured below.

Performance Objective:
The student can find the volumes of cylinders.

Photo Research International



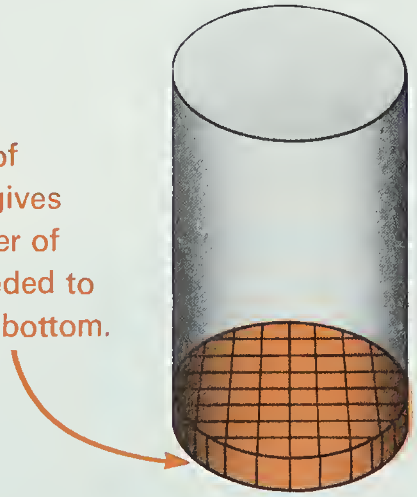
A. Devaney, Inc.

A. Devaney, Inc.

A cylinder has two circular bases. Each base is the same size. To find the volume of a cylinder, you multiply the area of the base times the height.

$$\text{Volume of cylinder} = \overbrace{\text{area of base}}^{\text{The number of cubes in bottom layer}} \times \underbrace{\text{height}}_{\text{The number of layers}}$$

Since the base is a circle, the area of the base is πr^2. (r is the radius of the base and $r^2 = r \times r$.)

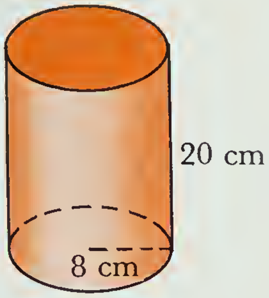


The area of the base gives the number of cubes needed to cover the bottom.

$$\text{Volume of cylinder} = \pi r^2 \times \text{height}$$

$$V = \pi r^2 h$$

Example 1: Find the volume of a cylinder with height 20 cm. The radius of the base is 8 cm. Use 3.14 for π.



$$V = \pi r^2 h$$
$$V = 3.14 \times 8 \times 8 \times 20$$
$$V = 4019.2$$

The volume is 4019.2 cm^3

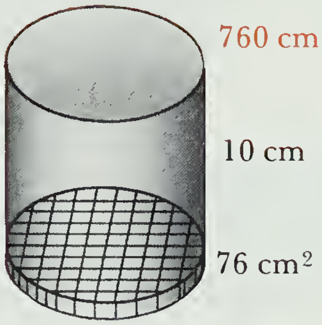
Exercises

1. What is the formula for finding the volume of a cylinder? $V = \pi r^2 h$

2. In the formula $V = \pi r^2 h$, the r^2 means $\underline{\quad r \quad}$ × $\underline{\quad r \quad}$.

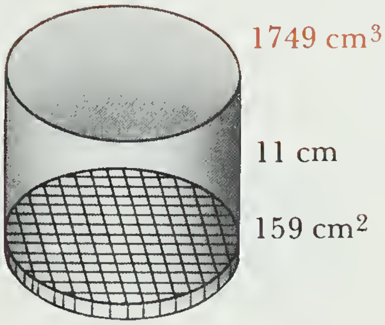
3–4. For each cylinder, the area of the base is given. Find the volume.

3.



$760\ \text{cm}^3$

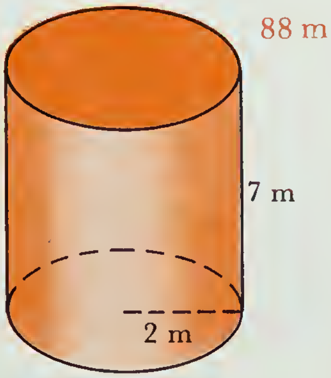
4.



$1749\ \text{cm}^3$

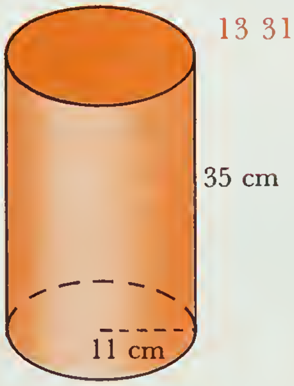
5–8. For each cylinder, the radius of the base and the height are given. Find the volume. Use $\frac{22}{7}$ for π.

5.



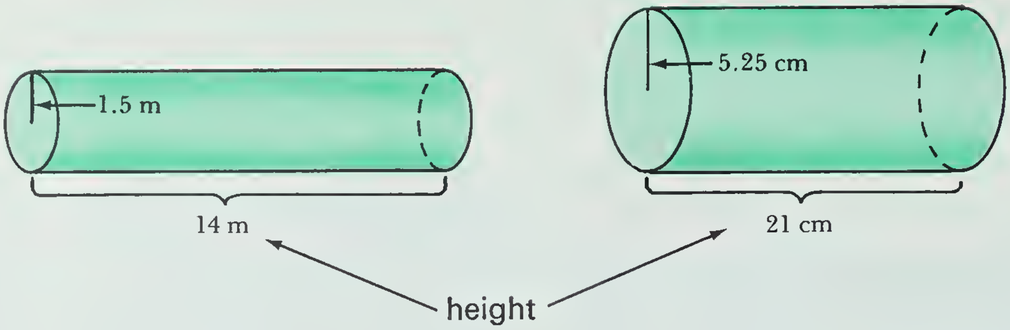
$88\ \text{m}^3$

6.



$13\,310\ \text{cm}^3$

7. 99 m^3 **8.** 1819.125 cm^3



9–14. Find the volume of each cylinder. Use 3.14 for π if necessary.

9. area of base = 18.3 m^2, height = 5.2 m 95.16 m^3

10. area of base = 47.2 mm^3, height = 31.5 mm 1486.8 mm^3

11. radius of base = 5.1 cm, height = 18 cm 1470.0852 cm^3

12. radius of base = 1.5 m, height = 1 m 7.065 m^3

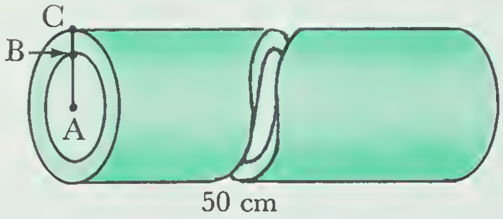
13. diameter of base = 10 cm, height = 9 cm 706.5 cm^3

14. diameter of base = 280 mm, height = 60 mm 3 692 640 mm^3

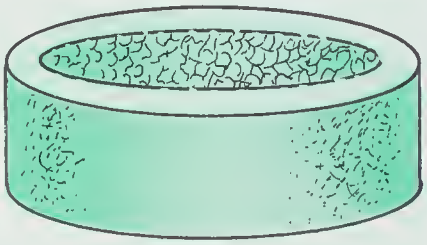
15–18. Use 3.14 for π .

15. A tin can has a height of 15 cm. The radius of the base is 5 cm. What is its volume? 1181.25 cm^3

16. An oil drum is 90 cm high. The radius of its base is 30 cm. What is its volume? 254 340 cm^3



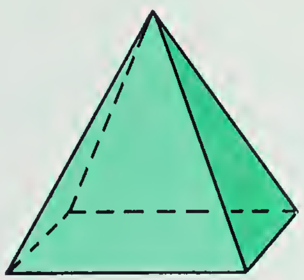
17. The inner radius (AB) of a steel pipe is 2 cm. The outer radius (AC) is 2.5 cm. If the length of the pipe is 50 cm, what is the volume of steel needed to make the pipe? 353.25 cm^3



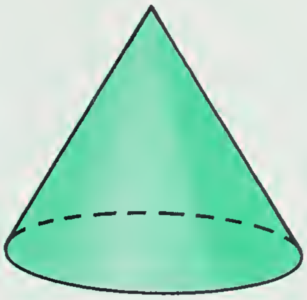
18. A concrete tree container is shaped like a cylinder. The inner radius is 50 cm and the outer radius is 55 cm. What is the volume of concrete needed to make the container if the height is 30 cm? 49 455 cm^3

VOLUME PROJECTS

Two solid figures that you may have heard of are the **pyramid** and the **cone.**

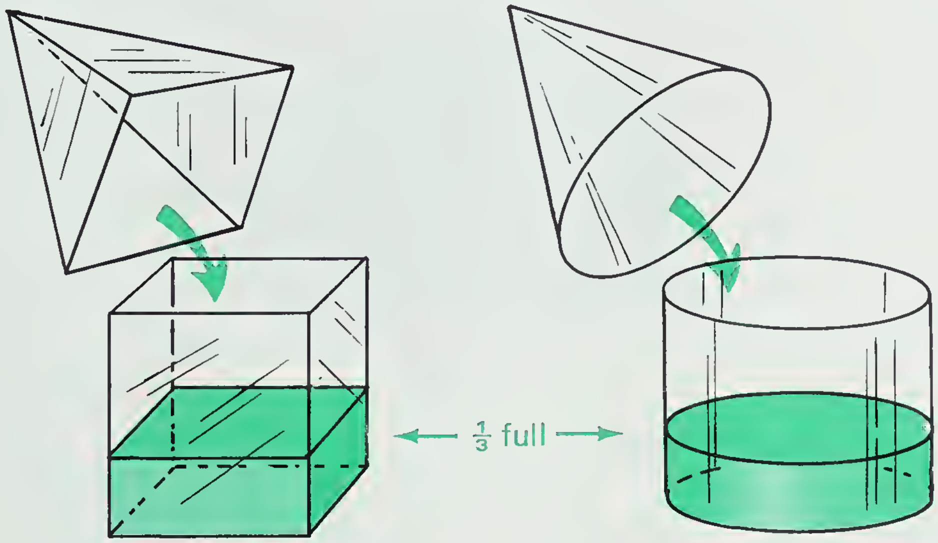


pyramid



cone

Think of filling each solid with liquid. The pyramid will be emptied into a *prism* of *equal* base and *equal* height. The cone will be emptied into a *cylinder* of *equal* base and *equal* height.



For the right prism,

$V = Bh.$

For the cylinder,

$V = \pi r^2 h.$

For the pyramid,

$V = \frac{1}{3}Bh.$

For the cone,

$V = \frac{1}{3}\pi r^2 h.$

As a project, use plastic models if they are available to show that these formulas are true. If models are not available, you can make models from cardboard and use sand to fill the models.

9.8 Spheres

Performance Objective: The student can find the volumes of spheres.

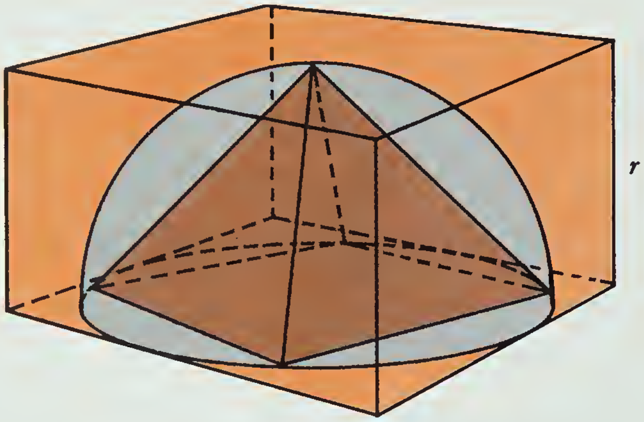
Another common solid is the sphere. Basketballs and golf balls are just two examples.

The *radius* of a sphere is the distance from the center to any point on the outside. The *diameter* of a sphere is twice as long as the radius. A **hemisphere** is half a sphere.

It can be shown that the volume of a sphere is equal to $\frac{4}{3}$ times π times the radius cubed. (radius cubed $= r^3 = r \times r \times r$)

The pyramid inside the hemisphere below has height r.

The rectangular solid around the hemisphere also has height r.



If the hemisphere were a pyramid, its volume would be

$$V = \tfrac{1}{3}Bh$$
$$V = \tfrac{1}{3} \times \pi r^2 \times r$$
$$V = \tfrac{1}{3}\pi r^3$$

If the hemisphere were a rectangular solid, its volume would be

$$V = Bh$$
$$V = \pi r^2 \times r$$
$$V = \pi r^3$$

Take the average.

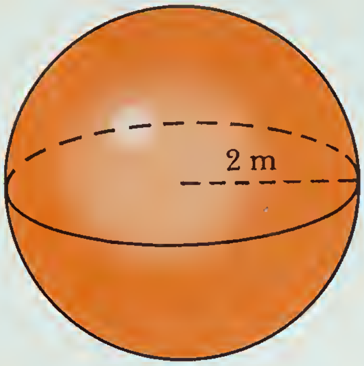
$$(\tfrac{1}{3}\pi r^3 + \pi r^3) \div 2 = (\tfrac{4}{3}\pi r^3) \div 2$$
$$= \tfrac{2}{3}\pi r^3$$

This average should be close to the volume of the hemisphere. (In this case it happens to turn out to be exact.)

Double the last result to get the volume of the whole sphere.

$$V = \tfrac{4}{3}\pi r^3$$

Example 1: A sphere has a radius of 2 m. What is its volume? Use $\frac{22}{7}$ for π .



$$V = \tfrac{4}{3}\pi r^3$$

$$V = \tfrac{4}{3} \times \tfrac{22}{7} \times 2 \times 2 \times 2$$

$$V = \tfrac{88}{21} \times 8$$

$$V = \tfrac{704}{21}$$

$$V = 33\tfrac{11}{21}$$

The volume is $33\frac{11}{21}$ m^3.

Exercises

1. What is the formula for the volume of a sphere? $V = \frac{4}{3}\pi r^3$

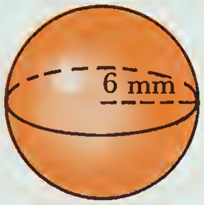
2. In the formula $V = \frac{4}{3}\pi r^3$, the r^3 means $\underline{\ r\ } \times \underline{\ r\ } \times \underline{\ r\ }$.

3. If the diameter of a sphere is 20 cm, the radius is $\underline{\quad 10 \quad}$ centimetres.

4. A fraction that is used for π is $\underline{\ \frac{22}{7}\ }$.

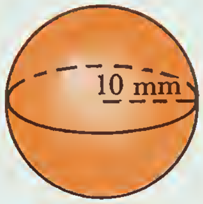
5–8. What number should replace each blank? Use 3.14 for π.

5.



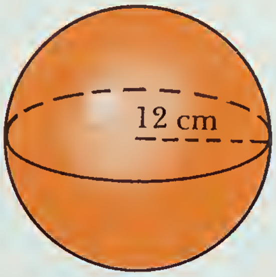
$V = \frac{4}{3} \times \pi \times \underline{\ 6\ } \times \underline{\ 6\ } \times \underline{\ 6\ }$

6.



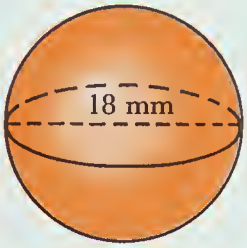
$V = \underline{\ \frac{4}{3}\ } \times \pi \times 10 \times 10 \times 10$

7.



$V = \frac{4}{3} \times \underline{\ 3.14\ } \times 12^3$

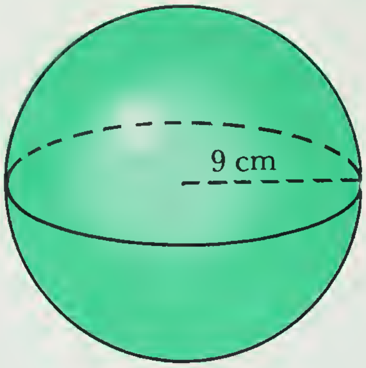
8.



$V = \frac{4}{3} \times \pi \times \underline{\ 9\ } \times \underline{\ 9\ } \times \underline{\ 9\ }$

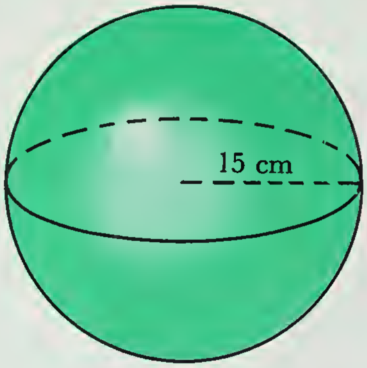
9–10. Find the volume of each sphere. Use 3.14 for π.

9.



3052.08 cm^3

10.



14 130 cm^3

11–16. Find the volume of each sphere. Use $\frac{22}{7}$ for π

11. radius = 0.5 cm 0.52 cm^3

12. radius = 0.25 m 0.065 m^3

13. radius = 1.75 m 22.46 m^3

14. radius = 5.25 mm 606.37 mm^3

15. diameter = 21 cm
4851 cm^3

16. diameter = 84 mm
310 464 mm^3

HINT: $d = 2r$

17–20. Use 3.14 for π .

17. A clear paperweight is shaped like a sphere and is filled with liquid. How much liquid is needed to fill the paperweight? The inside radius is 3 cm. 113.04 cm^3

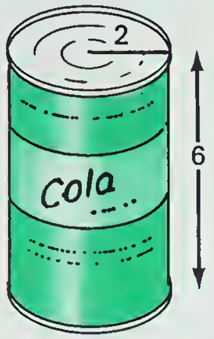
18. A goldfish bowl is shaped like a sphere and has an inside radius of 15 cm. How much water is needed to fill the bowl? 14 130 cm^3

19. A steel ball bearing has a radius of 1 cm. How much steel is needed to make the ball bearing? approx. 4.187 cm^3

20. The earth has a radius of 6400 km. What is the volume of the earth? 1 098 508 096 000 km^3

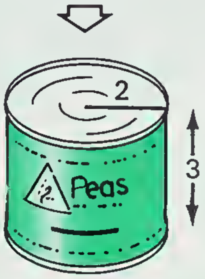
PACKAGING CAN FOOL YOU

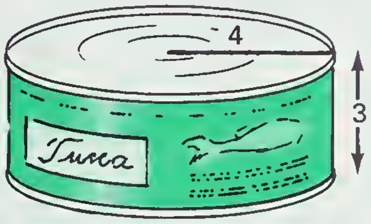
Being familiar with solids and knowing how to find volumes can be helpful when shopping for packaged materials.



Double the height.

Compare the other cans to this one.





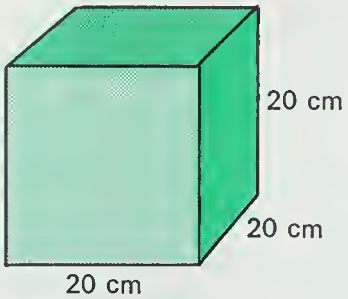
Double the radius.

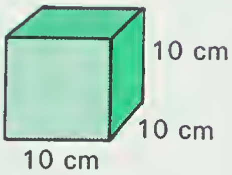
The can on the left gives twice as much volume as the one in the middle. But the can on the right gives four times as much. Why?

If you find the volume of each cylinder by using the formula $V = \pi r^2 h$, you have

Can 1:	Can 2:	Can 3:
$V = \pi \times 2^2 \times 6$	$V = \pi \times 2^2 \times 3$	$V = \pi \times 4^2 \times 3$
$V = 3.14 \times 4 \times 6$	$V = 3.14 \times 4 \times 3$	$V = 3.14 \times 16 \times 3$
$V = 75.36$	$V = 37.68$	$V = 150.72$

Look at another example.





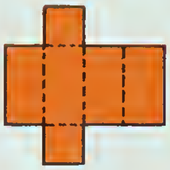
The package on the right does NOT give $\frac{1}{2}$ as much volume as the one on the left. It gives only $\frac{1}{8}$ as much. Why?

For the 20 cm cube:	For the 10 cm cube:
$V = (20 \times 20) \times 20$	$V = (10 \times 10) \times 10$
$V = 8000$	$V = 1000$

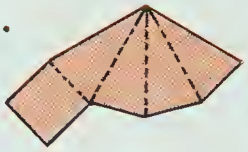
Test for Chapter 9

Refer to the patterns below to answer questions **1** and **2**.

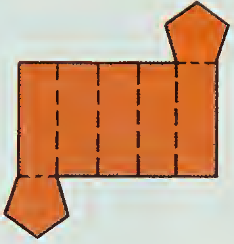
a.



b.



c.

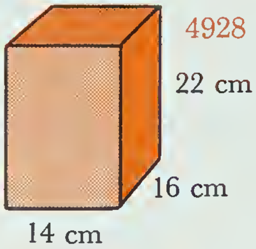


1. Which pattern could be folded to form a rectangular solid? a (9.1)

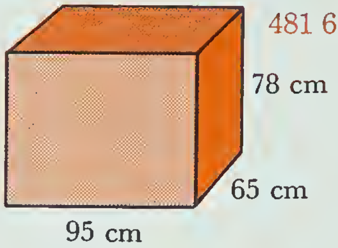
2. Which patterns could be folded to form a right prism? a, c (9.2)

How many 1 cm cubes would each box hold? (9.3)

3. 4928

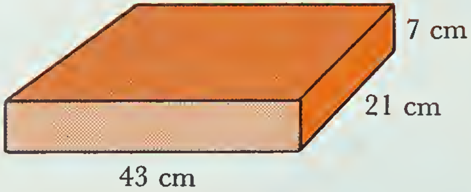


4. 481 650

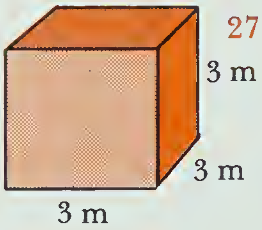


What is the volume of each rectangular solid? (9.4)

5. 6321 cm^3



6. 27 m^3



RATE YOURSELF

Number Correct	
11	Superior
10	Excellent
8-9	Good
6-7	Fair
0-5	Poor

Change each volume.

7. 0.025 m^3 to cm^3 — 25 000 cm^3

8. 6 m^3 to kL — 6 kL (9.5)

9. Find the surface area of the solid in Exercise 5. 2702 cm^2 (9.6)

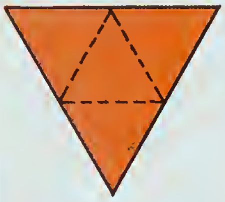
10. Find the volume of a cylinder whose base has a radius of 9 cm. The height of the cylinder is 15 cm. Use 3.14 for π. 3815.1 cm^3 (9.7)

11. Find the volume of a sphere whose radius is 3 cm. Use $\frac{22}{7}$ for π. $113\frac{1}{7}$ cm^3 (9.8)

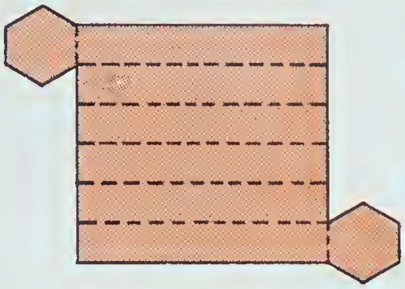
Chapter 9 Review

1. Which pattern below could be folded to form a rectangular solid? c (9.1)

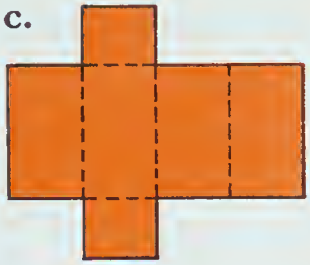
a.



b.

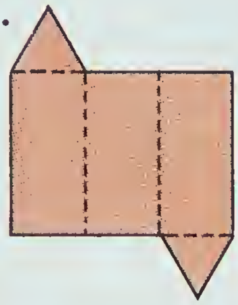


c.

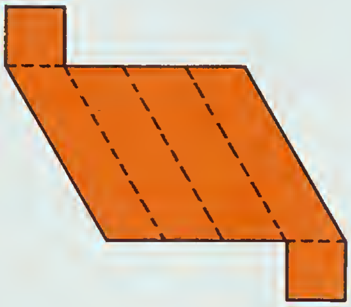


2. Which pattern below could be folded to form a right prism? a (9.2)

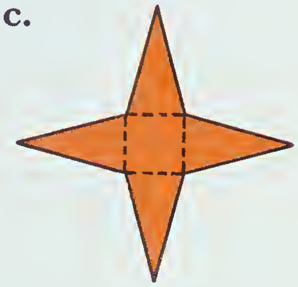
a.



b.

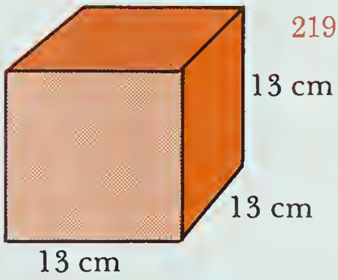


c.

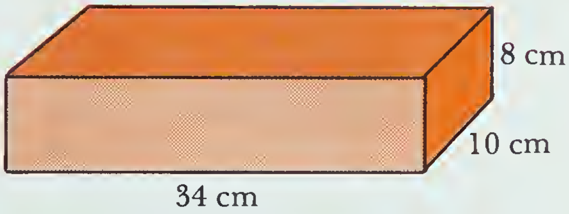


How many 1 cm cubes would each box hold? (9.3)

3. 2197

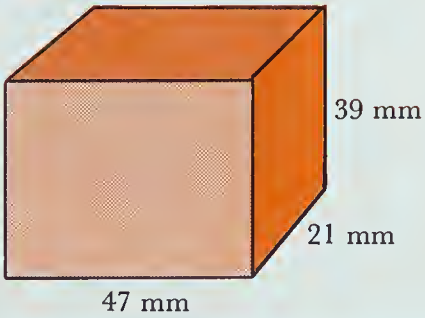

13 cm
13 cm
13 cm

4. 2720

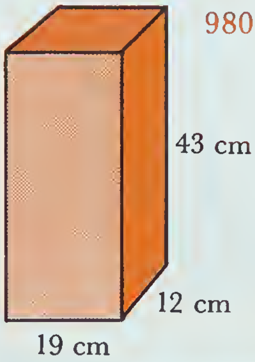

8 cm
10 cm
34 cm

What is the volume of each rectangular solid? (9.4)

5. 38 493 mm^3


39 mm
21 mm
47 mm

6. 9804 cm^3


43 cm
12 cm
19 cm

Change each volume. (9.5)

7. $0.043\ m^3$ to cm^3 $43\ 000\ cm^3$

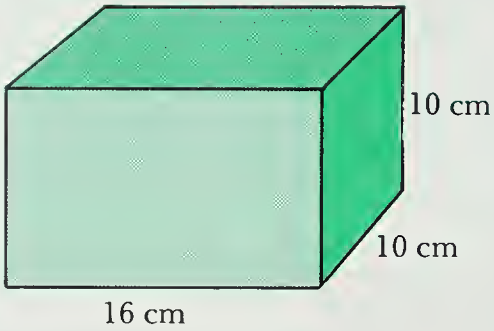
8. $3000\ cm^3$ to L 3 L

9. $25\ 000\ 000\ cm^3$ to m^3 $25\ m^3$

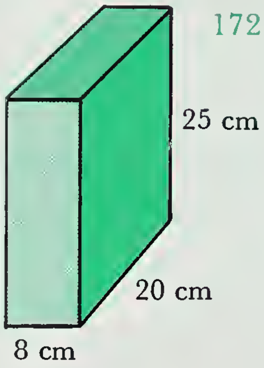
10. 9 kL to m^3 $9\ m^3$

Find the surface area of each rectangular solid. (9.6)

11. $840\ cm^2$

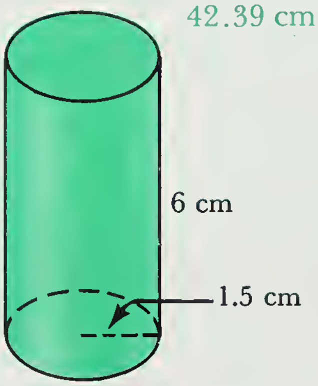

10 cm
10 cm
16 cm

12. $1720\ cm^2$

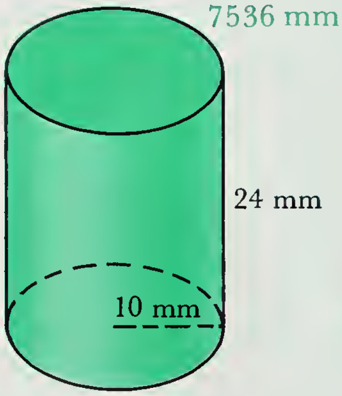

25 cm
20 cm
8 cm

Find the volume of each cylinder. Use 3.14 for π. (9.7)

13. $42.39\ cm^3$

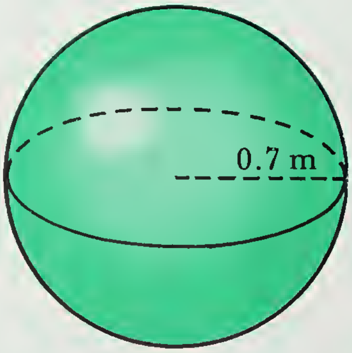


14. $7536\ mm^3$

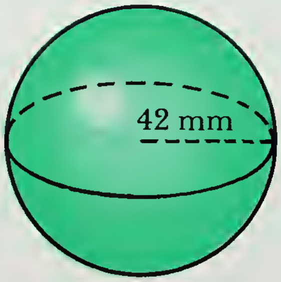


Find the volume of each sphere. Use $\frac{22}{7}$ for π. (9.8)

15. $1.44\ m^3$



16. $310\ 464\ mm^3$



Preview Test for Chapter 10

1. In an experiment, a single marble was drawn from a bag containing 10 marbles—4 red and 6 white. After each draw, the marble was replaced. What is the experimental probability of drawing a red marble if red occurred 21 times out of 50? Express it as a fraction, a decimal, and a percent. $\frac{21}{50}$, 0.42, 42% (10.1)

2. In Exercise **1**, what is the mathematical probability of drawing a red marble? Express it as a fraction, a decimal, and a percent. $\frac{2}{5}$, 0.4, 40% (10.1)

3. In an experiment, a single coin is to be tossed 50 times. In how many tosses would you expect heads to occur? 25 (10.2)

4. A bag contains 20 marbles—10 blue, 6 yellow, and 4 white. If a single marble is to be drawn, what is the mathematical probability of drawing a blue or a white marble? Express it as a fraction. $\frac{7}{10}$ (10.3)

5. A single die is to be tossed and a coin flipped at the same time. What is the mathematical probability of getting 1 on the die and tails on the coin? Express it as a fraction. $\frac{1}{12}$ (10.4)

6–9. Estimate each answer by rounding each number to the nearest ten. Then find the exact answer.

6. $57 + 61$ 120; 118 **7.** $91 + 18 + 62$ 170; 171 (10.5)

8. $79 - 28$ 50; 51 **9.** $159 - 47$ 110; 112 (10.6)

10–13. Estimate each answer by rounding each number to the nearest hundred. Then find the exact answer.

10. 131×287 30 000; 37 597 **11.** 615×407 240 000; 250 305 (10.7)

12. $812 \div 203$ 4; 4 **13.** $1764 \div 294$ 6; 6 (10.8)

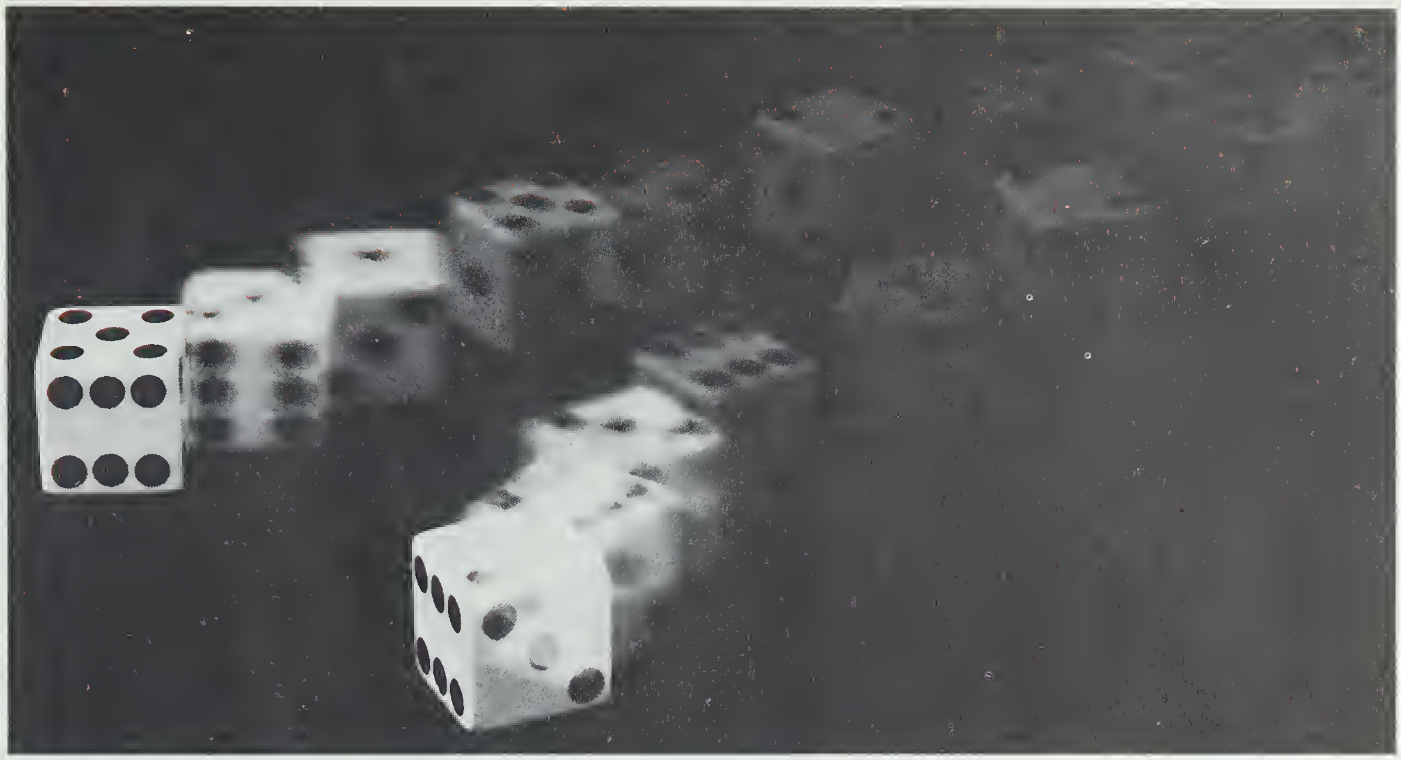
14. A worker is paid a weekly salary of $185. Estimate how much the worker makes in a year (52 weeks in a year). Round $185 to the nearest hundred and 52 to the nearest ten. $10,000 (10.9)

10

Camerique

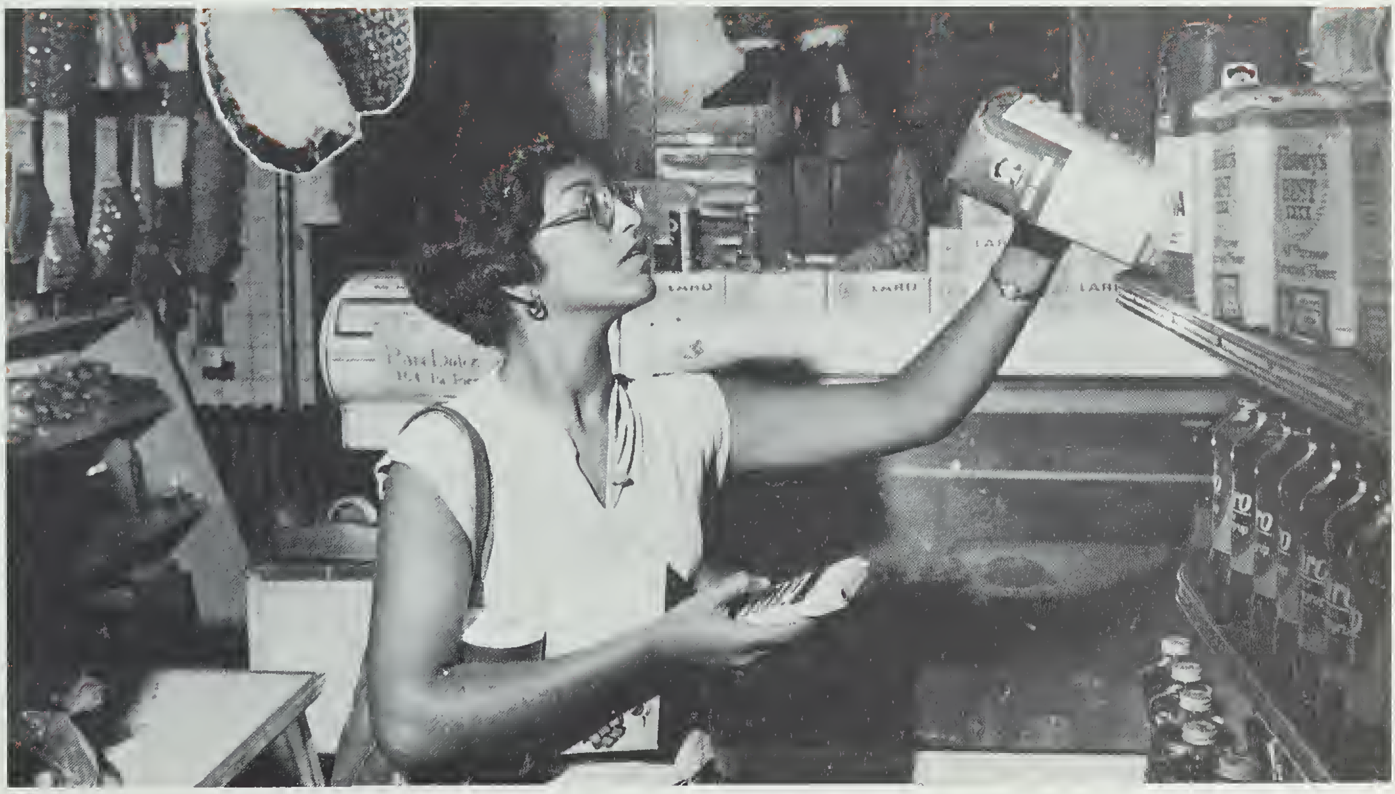
Brent Jones

Chance and Estimates



Camerique

PICTURE PREVIEW: This chapter begins by looking at the mathematics of chance, answering questions like What are the chances of getting "heads" when tossing a fair coin? Of rolling a 7 on a pair of honest dice? Of drawing a queen from a deck of ordinary cards? The chapter ends by considering how estimated answers can be useful in many everyday activities, including using a calculator.

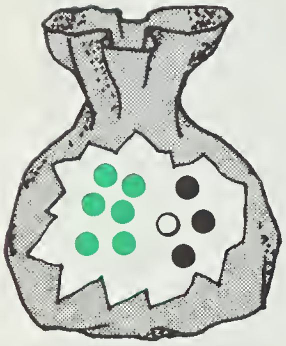


Brent Jones

10.1 Outcomes and Probability

Suppose 10 marbles of the same size—6 green, 3 black, and 1 white—are placed in a bag and thoroughly mixed. If you reach into the bag without looking and pick one marble, there are three possible outcomes:

picking a green marble
picking a black marble
picking a white marble



What are the chances of picking a green marble? You could perform an experiment to answer this question. You would pick one marble, record its color, replace the marble in the bag, and thoroughly mix them up, repeating the process 80 times. The results might be listed in a table as follows.

Performance Objective: The student can find experimental and mathematical probabilities.

Notice that the experiment is done in four sets of 20 trials each to make keeping track easier.

Trials	Number of green marbles drawn	Number of black marbles drawn	Number of white marbles drawn
First 20:	13	5	2
Second 20:	11	8	1
Third 20:	9	8	3
Fourth 20:	11	6	3
Total 80:	**44**	**27**	**9**

The idea of **probability** is useful in describing the outcomes of this experiment. A green marble was drawn 44 times out of 80. We can say the *experimental probability of drawing a green marble is* $\frac{44}{80}$. A short way to write this is

$$\text{ex pr (green)} = \frac{44}{80} = 0.55.$$

A probability can also be given as a percent. For example, in the experiment above, a green marble was drawn 55% of the time. So, ex pr (green) = 55%.

An experimental probability can be found only after an experiment has been performed. For the experiment above, we also have

$$\text{ex pr (black)} = \frac{27}{80} = 0.34 \text{ approx.}$$

$$\text{ex pr (white)} = \frac{9}{80} = 0.11 \text{ approx.}$$

Suppose an experiment *consisted* of n trials and a particular outcome *happened* s times by chance. Then the **experimental probability** of that outcome is $\frac{s}{n}$.

Sometimes we want to find the probability of an outcome *before doing an experiment* or *without doing one at all.* Such a probability is called a **mathematical probability.** We will let pr stand for mathematical probability.

Suppose a trial in an experiment *can happen* in n equally likely ways. If s of these ways *would result* in a particular outcome, then the **mathematical probability** of that outcome is $\frac{s}{n}$.

Example: For the marble experiment on page 426, find these *mathematical* probabilities.

pr (green) pr (black) pr (white)

A trial consists of picking one marble. There are 10 marbles. So the number n of ways a trial can happen is 10. (We assume that the method of drawing would make all 10 equally likely.) There are 6 green marbles. So the number of ways a trial could result in the outcome "picking a green marble" is 6.

Therefore, pr (green) $= \frac{6}{10} = 0.6$

Similarly, pr (black) $= \frac{3}{10} = 0.3$

pr (white) $= \frac{1}{10} = 0.1$

Notice that these mathematical probabilities are close to the experimental probabilities on page 426. It has been proved that experimental probabilities get very close to the mathematical probabilities if the number of trials is large enough. So experimental probabilities are sometimes used as estimates of mathematical probabilities.

For both experimental and mathematical probabilities:

1. Probabilities are never less than 0 nor greater than 1.
2. An impossible outcome has probability 0.
3. An outcome that is sure to happen has probability 1.

Exercises

1. A probability that results from an actual experiment is called an ______ probability. Experimental

2. A probability that is found without doing an experiment is called a ______ probability. Mathematical

3. An experiment consisted of 25 trials, and an outcome called "heads" happened 12 times by chance. Then ex pr (heads) is $\frac{12}{25}$.

4. Suppose a trial in an experiment can happen in 6 equally likely ways. If exactly 1 of these ways would result in an outcome called "4 dots," then pr (4 dots) = $\frac{1}{6}$.

5–8. Refer to the correct table below to find each *experimental* probability in three ways: as a fraction, a decimal, and a percent.

MARBLE EXPERIMENT

Trials	Number of blue	Number of red
1st 10:	6	4
2nd 10:	8	2
3rd 10:	5	5
Total 30:	**19**	**11**

COIN-TOSSING EXPERIMENT

Trials	Number of heads	Number of tails
1st 20:	13	7
2nd 20:	11	9
3rd 20:	8	12
Total 60:	**32**	**28**

5. ex pr (blue) $\frac{19}{30}$, 0.63, 63%

6. ex pr (red) $\frac{11}{30}$, 0.37, 37%

7. ex pr (tails) $\frac{7}{15}$, 0.47, 47%

8. ex pr (heads) $\frac{8}{15}$, 0.53, 53%

-8. Decimals and percents are approximate.

9–10. The experiment for Exercises **5** and **6** was done with 5 marbles—3 blue and 2 red. Use this information to find each *mathematical* probability as a fraction, a decimal, and a percent.

9. pr (blue) $\frac{3}{5}$, 0.6, 60%

10. pr (red) $\frac{2}{5}$, 0.4, 40%

11–12. In tossing a coin, two equally likely outcomes—heads and tails—are possible. Find each *mathematical* probability as a fraction, a decimal, and a percent.

11. pr (tails) $\frac{1}{2}$, 0.5, 50%

12. pr (heads) $\frac{1}{2}$, 0.5, 50%

13. If the experiment for Exercises **5–6** were repeated, would you expect exactly the same probabilities to result? No

14. A "fixed" coin has two heads (no tails). What is pr (heads)? What is pr (tails)? 1; 0

15–20. The table below gives the results of this experiment. Each of the ten letters in the word STATISTICS was printed on a 3-by-5 note card. The cards, placed facedown, were mixed and drawn one at a time and replaced as in the marble experiment on page 426. Find each *experimental* probability as a percent.

Number of times each letter drawn

Trials	S	T	A	I	C
1st 20:	8	4	3	3	2
2nd 20:	4	9	0	5	2
3rd 20:	7	8	1	1	3
4th 20:	5	7	3	4	1
5th 20:	5	5	4	5	1
Total 100:	**29**	**33**	**11**	**18**	**9**

15. ex pr (S) 29%

16. ex pr (T) 33%

17. ex pr (A) 11%

18. ex pr (I) 18%

19. ex pr (C) 9%

20. ex pr (Z) 0%

21–26. In the experiment for Exercises **15–20,** find each *mathematical* probability as a percent.

21. pr (S) 30%

22. pr (T) 30%

23. pr (A) 10%

24. pr (I) 20%

25. pr (Z) 0%

26. pr (C) 10%

27–34. A regular deck of 52 playing cards is shown below. Notice that there are 13 cards in each of four suits—hearts, diamonds, clubs, and spades. For the experiment of drawing one card from a full, thoroughly shuffled deck, find each *mathematical* probability as a fraction.

27. pr (the 2 of diamonds) $\frac{1}{52}$

28. pr (the 10 of clubs) $\frac{1}{52}$

29. pr (a card in spades) $\frac{1}{4}$

30. pr (a card in hearts) $\frac{1}{4}$

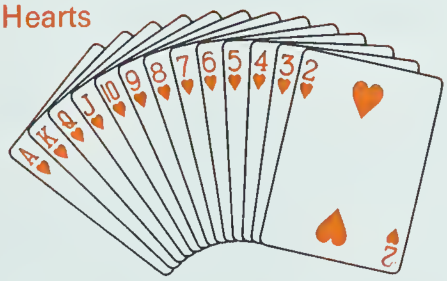
31. pr (a 2) $\frac{1}{13}$

32. pr (a 10) $\frac{1}{13}$

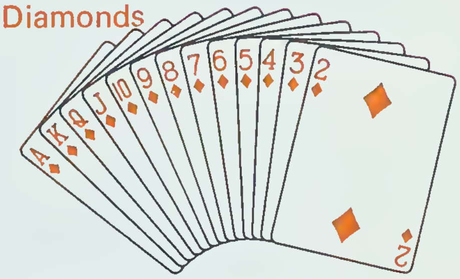
33. pr (a black 2) $\frac{1}{26}$

34. pr (a black 10) $\frac{1}{26}$

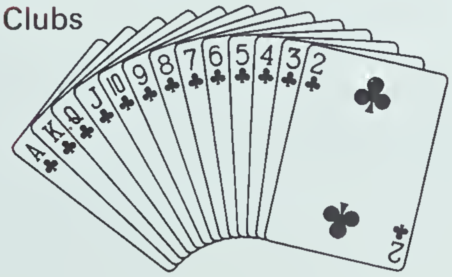
Hearts



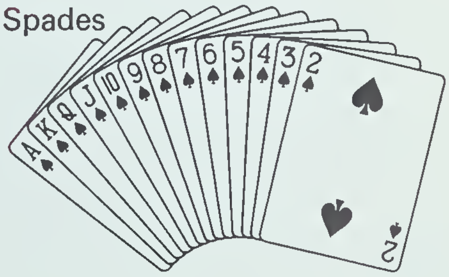
Diamonds



Clubs



Spades



10.2 Experiments with Chance

Performance Objective: The student can carry out probability experiments to find experimental probabilities.

In Section 10.1, you saw that experiments can be used to find probabilities. In this set of exercises, you will perform some of these experiments.

Exercises

EXPERIMENT 1 When a coin is tossed, we expect one of two equally likely outcomes, heads or tails. If the coin is tossed for many trials, it seems that heads would occur about half the time and tails about half the time. To test this, toss a coin for 50 trials and record the results in a table like the following:

Trials	Number of heads	Number of tails
1st 10:		
2nd 10:		
3rd 10:		
4th 10:		
5th 10:		
Total 50:		

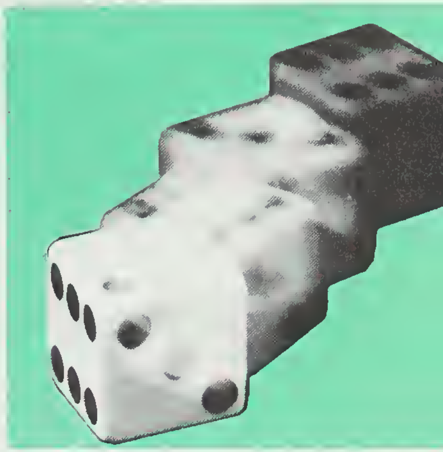
1–6. Using your completed table for Experiment 1, find each experimental probability as a percent.

1–12. *Answers will vary.*

1. ex pr (heads) for the 1st 10 trials
2. ex pr (tails) for the 2nd 10 trials
3. ex pr (heads) for the total 50 trials
4. ex pr (tails) for the total 50 trials
5. ex pr (heads) for 500 trials (NOTE: Combine your results with those of nine of your classmates.)
6. ex pr (tails) for 500 trials (See Note with Exercise **5.**)

EXPERIMENT 2 If a die (singular of *dice*) is rolled, we expect one of six equally likely outcomes: 1, 2, 3, 4, 5, or 6 dots. Roll a die 60 times and record your results in a table.

Trials	**Number of times you roll** **1 dot**	**2 dots**	**3 dots**	**4 dots**	**5 dots**	**6 dots**
1st 20:						
2nd 20:						
3rd 20:						
Total 60:						


Camerique

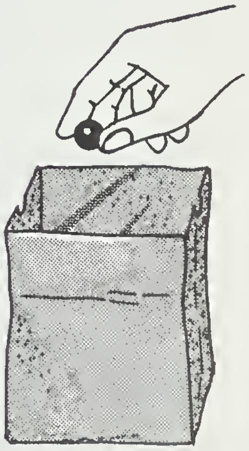
7–12. Using your completed table for Experiment 2, find each experimental probability (for all 60 trials) as a fraction.

7. ex pr (1 dot) **8.** ex pr (2 dots) **9.** ex pr (3 dots)

10. ex pr (4 dots) **11.** ex pr (5 dots) **12.** ex pr (6 dots)

EXPERIMENT 3 Put 7 marbles of one color (say red) and 3 marbles of another (say black) in a sack. Mix them up. Select one marble without looking. Record the color. Then replace the marble and mix again. Do this for 50 trials. All the marbles should be the same size. Why?

Trials	**Number of times you draw** **Red**	**Black**
1st 10:		
2nd 10:		
3rd 10:		
4th 10:		
5th 10:		
Total 50:		



13–16. Refer to Experiment 3 to answer these questions.

13. Which color is more likely to be selected in any one trial? Why? Red; there are more red marbles.

14. Are "selecting red" and "selecting black" equally likely outcomes? No

15–16. *Answers will vary.*

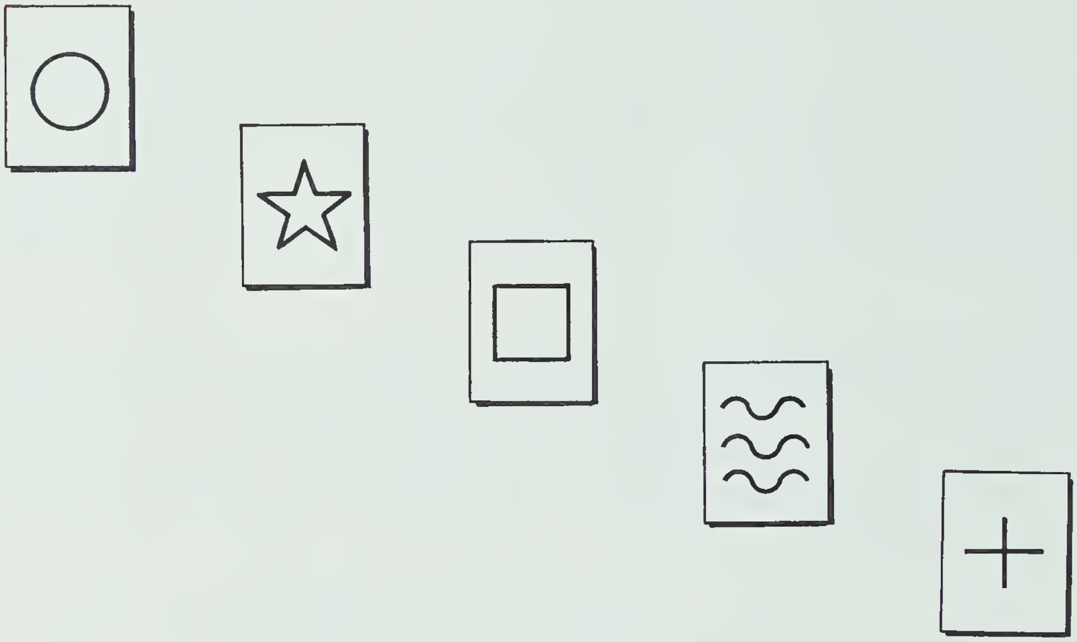
15. For all 50 trials, what is ex pr (red) as a fraction, a decimal, and a percent?

16. For all 50 trials, what is ex pr (black) as a fraction, a decimal, and a percent?

ESP AND PROBABILITY

You may have heard of extrasensory perception, often called ESP. People who claim to have ESP say that they can see things or gain information without using any known senses. Do you think you have ESP? Try the following experiment.

Take a 25-card Zeno deck of ESP symbol cards. There are five each of the following card types: circle, star, square, wavy lines, and plus sign. The backs of the cards are all the same. If you do not have a Zeno deck, make one with 3-by-5 note cards.



Out of 25 trials, how many times can you guess the top card? (Replace top card and shuffle after each guess.) Since there are five different symbols, the probability of guessing correctly is $\frac{1}{5}$. So you can expect to get 5 right by mere chance. If you get 10 right, would this mean you have ESP? No Compare your results with other people in the class. Who got the most right? Does this mean that person has ESP? If you repeated the experiment, would everyone still get the same results? No

Quick Quiz

Check this skill:

- *adding fractions*

1–6. Add.

1. $\frac{1}{5} + \frac{2}{5}$ $\frac{3}{5}$

2. $\frac{1}{4} + \frac{1}{4}$ $\frac{1}{2}$

3. $\frac{1}{6} + \frac{5}{6}$ 1

4. $\frac{1}{2} + \frac{1}{4}$ $\frac{3}{4}$

5. $\frac{1}{3} + \frac{1}{6}$ $\frac{1}{2}$

6. $\frac{1}{4} + \frac{1}{3}$ $\frac{7}{12}$

Adding Probabilities

10.3

Performance Objective: The student can find the probability that one or the other of two mutually exclusive events will happen.

Let's ask a different question about the experiment of drawing one marble from a bag of 10 marbles—6 green, 3 black, and 1 white.

> What is the probability of drawing **either** a green marble **or** a black marble?

The table below gives what these outcomes would have been for the 80 trials recorded on page 426.

Trials	Number of times green or black marble was drawn	Number of times white marble was drawn
First 20:	13 + 5 = 18	2
Second 20:	11 + 8 = 19	1
Third 20:	9 + 8 = 17	3
Fourth 20:	11 + 6 = 17	3
Total 80:	**44 + 27 = 71**	**9**

The *experimental* probabilities are therefore:

$$\text{ex pr (green } \textit{or} \text{ black)} = \frac{71}{80} \qquad \text{ex pr (white)} = \frac{9}{80}$$

In the marble experiment, the two outcomes "drawing a green marble" and "drawing a black marble" are **mutually exclusive events.** This means both outcomes could not happen at the same time (since only one marble is drawn at a time). We can reason as follows:

Number of ways of drawing a green marble	6
Number of ways of drawing a black marble	3
Number of ways of drawing *either* a green marble *or* a black marble	6 + 3 = 9

There are 10 marbles in all, and the number of ways of drawing a green marble or a black marble is 9. So the *mathematical* probability, pr (green *or* black), is $\frac{9}{10}$. For this experiment,

$$\text{pr (green } \textbf{or} \text{ black)} = \text{pr (green)} + \text{pr (black)}$$

$$\frac{9}{10} = \frac{6}{10} + \frac{3}{10}$$

If A and B are *mutually exclusive events*, then the *mathematical* probability of outcome "A **or** B" can be found by adding pr (A) and pr (B).

$$\text{pr}\ (A\ \textbf{or}\ B) = \text{pr}\ (A) + \text{pr}\ (B)$$

Exercises

1–4. Are outcomes A and B mutually exclusive events?

1. When tossing a single coin, A: throwing heads; B: throwing tails Yes

2. When drawing one card from an ordinary deck of 52 playing cards, A: getting a king; B: getting a spade No

3. A: winning a bet; B: losing a bet Yes

4. When throwing a single die, A: throwing an even number; B: throwing a 5 Yes

5–8. Outcomes A and B are mutually exclusive events. Find pr (A or B) as a fraction if

5. pr $(A) = \frac{1}{10}$, pr $(B) = \frac{7}{10}$ $\frac{4}{5}$

6. pr $(A) = \frac{1}{12}$, pr $(B) = \frac{5}{12}$ $\frac{1}{2}$

7. pr $(A) = \frac{1}{5}$, pr $(B) = \frac{2}{5}$ $\frac{3}{5}$

8. pr $(A) = \frac{1}{2}$, pr $(B) = \frac{1}{2}$ 1

9–14. The table at the left below gives the results of this experiment. Each of the letters A, B, and C was written on one of three identical tags. The tags were put into a hat, mixed well, drawn one at a time, and replaced after each draw. Find these *experimental* probabilities as fractions.

Trials	A	B	C
1st 20:	7	6	7
2nd 20:	5	7	8
Total 40:	**12**	**13**	**15**

9. ex pr (A) $\frac{3}{10}$

10. ex pr (B) $\frac{13}{40}$

11. ex pr (C) $\frac{3}{8}$

12. ex pr (A or B) $\frac{5}{8}$

13. ex pr (A or C) $\frac{27}{40}$

14. ex pr (B or C) $\frac{7}{10}$

15–20. For the tag experiment in Exercises **9–14,** find these *mathematical* probabilities as fractions.

15. pr (A) $\frac{1}{3}$ **16.** pr (B) $\frac{1}{3}$

17. pr (C) $\frac{1}{3}$ **18.** pr (A or B) $\frac{2}{3}$

19. pr (A or C) $\frac{2}{3}$ **20.** pr (B or C) $\frac{2}{3}$

21–26. The table below gives the results of an experiment. A bag contains 12 marbles—5 red, 4 white, and 3 blue. A marble was drawn and then replaced after each draw. Find these *experimental* probabilities as percents.

Trials	Red	White	Blue
1st 20:	8	6	6
2nd 20:	9	5	6
3rd 20:	7	6	7
4th 20:	8	5	7
5th 20:	9	4	7
Total 100:	**41**	**26**	**33**

21. ex pr (red) 41% **22.** ex pr (white) 26%

23. ex pr (blue) 33% **24.** ex pr (red or white) 67%

25. ex pr (red or blue) 74% **26.** ex pr (white or blue) 59%

27–32. For the experiment in Exercises **21–26,** find these *mathematical* probabilities as percents.

27. pr (red) $41\frac{2}{3}\%$ **28.** pr (white) $33\frac{1}{3}\%$

29. pr (blue) 25% **30.** pr (red or white) 75%

31. pr (red or blue) $66\frac{2}{3}\%$ **32.** pr (white or blue) $58\frac{1}{3}\%$

1–6. Multiply.

1. $\frac{1}{5} \times \frac{2}{5}$ $\frac{2}{25}$ **2.** $\frac{1}{4} \times \frac{1}{4}$ $\frac{1}{16}$ **3.** $\frac{1}{6} \times \frac{5}{6}$ $\frac{5}{36}$

4. $\frac{1}{2} \times \frac{1}{4}$ $\frac{1}{8}$ **5.** $\frac{1}{3} \times \frac{1}{6}$ $\frac{1}{18}$ **6.** $\frac{1}{4} \times \frac{1}{3}$ $\frac{1}{12}$

Quick Quiz

Check this skill:

- ***multiplying fractions***

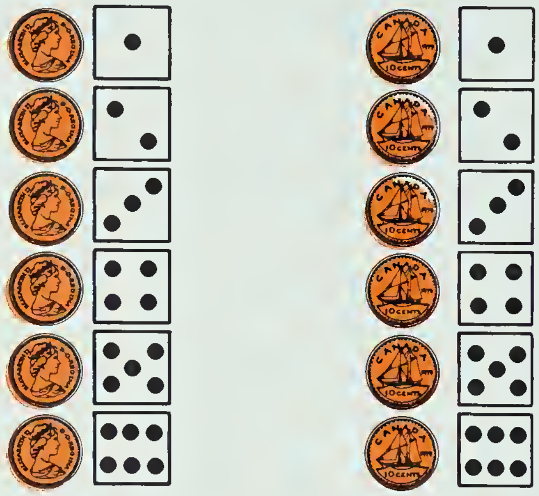
10.4 Multiplying Probabilities

Performance Objective: The student can find the probability that two independent events will both happen.

Suppose you are going to toss a coin, then roll a die.

What is the probability of getting **both** heads (on the coin) **and** 6 (on the die)?

You can reason as follows: There are two possible ways the coin could land. There are six possible ways the die could land. All possible matchings of the coin and die are shown below.



There are 12 possible outcomes. Only one outcome gives **both** heads **and** 6. So pr (heads *and* 6) = $\frac{1}{12}$.

You already know that pr (heads) = $\frac{1}{2}$ and pr (6) = $\frac{1}{6}$. Notice that

$$\frac{1}{12} = \frac{1}{2} \times \frac{1}{6}$$

$$\text{pr (heads } \textbf{and } 6) = \text{pr (heads)} \times \text{pr } (6)$$

The two outcomes "heads on the coin" and "6 on the die" have the property that neither outcome depends on the other. That is, either outcome can occur independently of the other.

Independent events A and B are two outcomes for which

$$\text{pr } (A \textbf{ and } B) = \text{pr } (A) \times \text{pr } (B).$$

Example: What is the probability of throwing heads on both coins when two coins are to be tossed?

The outcome on the first coin is independent of the outcome of the second coin. Therefore, you multiply the probabilities.

$$\begin{aligned} \text{pr (heads and heads)} &= \text{pr (heads)} \times \text{pr (heads)} \\ &= \tfrac{1}{2} \times \tfrac{1}{2} \\ &= \tfrac{1}{4} \end{aligned}$$

Exercises

1. Independent events A and B are two outcomes for which pr (A and B) equals pr (A) × pr (B).

2. When two outcomes A and B are independent events, you (add, multiply) the probabilities of A and B in order to find pr (A and B). Multiply

3–6. Are outcomes *A* and *B* independent events?

3. A: flipping tails on a coin; B: rolling a 5 on a die Yes

4. A: flipping heads on one toss of a coin; B: flipping tails on a second toss of the same coin Yes

5. A: rolling a 5 on one toss of a die; B: rolling a 4 on the second toss of the same die Yes

6. A: getting a 2 on one toss of a die; B: getting an even number on the same toss of a die No

7–14. Outcomes *A* and *B* are independent events. Find pr (*A* and *B*) as a fraction if

7. pr (A) $= \frac{1}{2}$, pr (B) $= \frac{1}{3}$ $\frac{1}{6}$ **8.** pr (A) $= \frac{1}{5}$, pr (B) $= \frac{1}{2}$ $\frac{1}{10}$

9. pr (A) $= \frac{1}{6}$, pr (B) $= \frac{1}{6}$ $\frac{1}{36}$ **10.** pr (A) $= \frac{1}{10}$, pr (B) $= \frac{2}{5}$ $\frac{1}{25}$

11. pr (A) $= \frac{1}{12}$, pr (B) $= \frac{4}{5}$ $\frac{1}{15}$ **12.** pr (A) $= \frac{2}{5}$, pr (B) $= \frac{1}{5}$ $\frac{2}{25}$

13. pr (A) $= \frac{1}{6}$, pr (B) $= \frac{3}{4}$ $\frac{1}{8}$ **14.** pr (A) $= \frac{5}{12}$, pr (B) $= \frac{2}{5}$ $\frac{1}{6}$

15–16. A coin is to be tossed. Then a die is to be rolled. Find these mathematical probabilities as fractions.

15. pr (heads and 5) $\frac{1}{12}$ **16.** pr (tails and 2) $\frac{1}{12}$

17–18. A die will be rolled twice in a row. Find these mathematical probabilities as fractions.

17. pr (6 and then 5) $\frac{1}{36}$ **18.** pr (1 and then 2) $\frac{1}{36}$

19–20. A coin is to be tossed twice in a row. Find these mathematical probabilities as decimals.

19. pr (heads and then tails) 0.25 **20.** pr (heads and then heads) 0.25

21. A basketball player fouled in the act of shooting is awarded two free throws. The player has made 33 of the last 50 free throws attempted. Using this information, find the probability (as a percent) that the player will make the first free throw. What is the probability (as a percent) that the player will make both free throws? (Assume they are independent events.) 66%, 43.56%

22. In Monopoly, two dice are rolled for each turn. The probability of rolling "doubles" is $\frac{1}{6}$. If doubles are rolled three times in a row, the player goes to jail. What is the probability (as a fraction) of rolling doubles three times in a row? (Assume each roll is independent of the next.) $\frac{1}{216}$

Quick Quiz

Check these skills:

- *rounding numbers*
- *computing with multiples of 10, 100, and 1000*

1–4. Round each number to the nearest ten.

1. 73 70 **2.** 28 30 **3.** 135 140 **4.** 162 160

5–8. Round each number to the nearest hundred.

5. 761 800 **6.** 107 100 **7.** 153 200 **8.** 641 600

9–12. Round each amount to the nearest dollar.

9. \$3.95 \$4.00 **10.** \$1.79 \$2.00 **11.** \$3.29 \$3.00 **12.** \$6.25 \$6.00

13–18. Find each answer.

13. 20×300 6000 **14.** $1600 \div 40$ 40 **15.** $1700 - 800$ 900

16. $50 + 70 + 90$ 210 **17.** 500×200 100 000 **18.** $21{,}000 \div 700$ 30

CONSUMER CORNER

WHAT ARE THE CHANCES?

Uses of probability occur in business, weather forecasting, games of chance, sports, and other activities.

1. *Quality control:* When a product is mass-produced by modern industry, it is often almost impossible to inspect every item. So most manufacturers use a sampling procedure. For example, a TV-tube factory might pick at random 100 tubes out of every 5000 and inspect them.

A certain amount of defective tubes in the sample might be acceptable. The manufacturer might figure it is not possible to improve production procedures, or it would be too costly to do so. The manufacturer would use probability ideas in deciding what number of defective items is acceptable and how big a sample to use.

For you as a consumer, this means you will sometimes buy a defective item. Suppose you buy a TV tube from a manufacturer whose 100-tube samples have, on the average, 3 defective tubes. What is the probability (percent) that the tube you buy will be defective? 3%

2. *Insurance:* In figuring life-insurance rates, companies use tables based on past records. One such table shows that for 14-year-olds, 1.4 deaths per 1000 people occur in a year. This same table shows that for 60-year-olds the annual death rate is 20 per 1000. Using this information, the probability of dying during the next year is $\frac{1.4}{1000}$ or 0.0014 for a 14-year-old. For a 60-year-old, the probability is $\frac{20}{1000}$ or 0.02. Who would pay a greater yearly premium in getting a policy? Why?

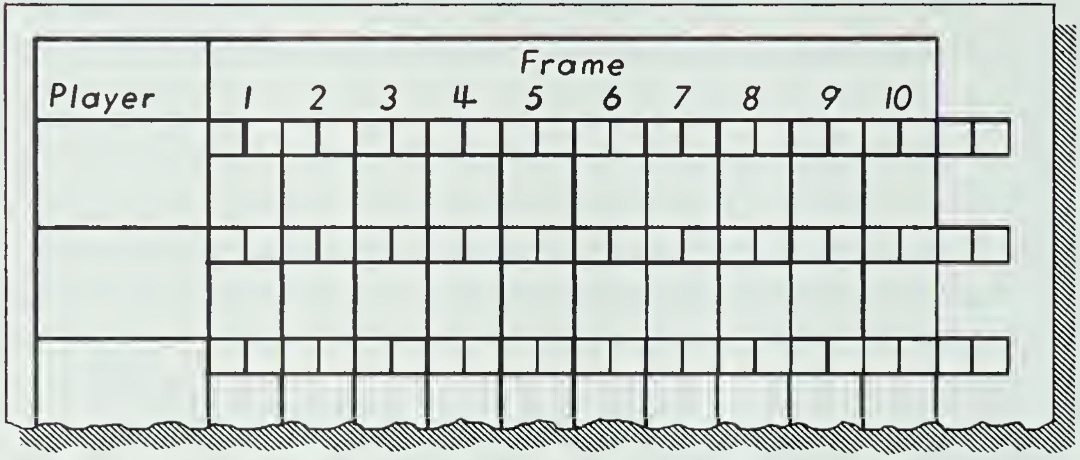
The 60-year-old; the probability of a 60-year-old dying within the year is much greater than that of a 14-year-old.

3. *Lotteries:* Many lotteries are conducted in order too raise money. In one, the winner receives one million dollars. If you buy one ticket, what is the probability of winning a million dollars if 2 000 000 tickets are sold? Do you think lottery tickets make a good investment? $\frac{1}{2\,000\,000}$, No.

MATH BOWL

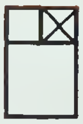
In actual bowling, a game consists of 10 frames. In each frame, the bowler tries to knock down all 10 pins with one or two rolls of the ball. Knocking *all* 10 pins down with one roll is a *strike*, with two rolls it's a *spare*.

Here's a math bowling game you can play with dice. The main rules are the same as for actual bowling. A score sheet should be made up as follows:

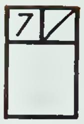


Roll the first "ball" in each frame with *two* dice. A sum of 10, 11, or 12 counts as a strike—the same as knocking all 10 pins down with one roll in actual bowling. A strike is scored by placing an × in the top right-hand corner for that frame.

A strike



A spare

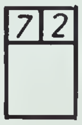


Brent Jones

If the sum on the first roll is less than 10, say 7, write that number in the top left-hand corner for that frame. Then roll a second ball with only *one* die. If the result of the second roll added to the first is 10 or more, place a ⁄ in the top right-hand corner for the frame. This is counted as a *spare*—the same as knocking all 10 pins down with two rolls in actual bowling.

For example, a 7 followed by a 4 is scored above.

If the total on both rolls is less than 10, record the number rolled on the second toss in the top right-hand corner. For example, a 7 followed by a 2 is scored below.



When a player rolls "snake eyes" (double-1) on the first toss, this is counted as a gutter ball. Mark it as 0 and then roll *both* dice again. (Remember that 10 or more on the second toss would be scored as a spare.)

Scoring is the same as in the actual game. A strike is scored as 10 plus the count on the next two balls. A spare is scored as 10 plus the count on the next ball. An open (neither a strike nor a spare) is scored as the total number of pins knocked down. A running total is kept by writing the score in each frame. A score sheet for an entire game is shown below.

Player	Frame									
	1	2	3	4	5	6	7	8	9	10
	☒	7 /	8 1	0 6	4 /	5 1	☒	8 /	5 2	☒ 6 2
Todd	20	38	47	53	68	74	94	109	116	134

MATH BOWL CHANCES

When two dice are to be tossed, there are 36 different ways the dice can land. Each face on one die can be paired with any of 6 faces on the other die. Use this fact in answering the following questions about the bowling game.

1. What is the probability of throwing a gutter ball? In other words, what is pr (sum of 2) when two dice are to be thrown? HINT: "Snake eyes" can be rolled in one way only: 1 and 1. $\frac{1}{36}$

2. What is pr (sum of 10)? HINT: A sum of 10 can be rolled in three ways: 4 and 6, 6 and 4, 5 and 5. What is pr (sum of 11)? What is pr (sum of 12)? What is the probability of rolling a strike in the bowling game? In other words, what is pr (sum of 10, 11, or 12)? $\frac{1}{12}, \frac{1}{18}, \frac{1}{36}, \frac{1}{6}$

3. One way to roll a spare is to roll a sum of 7 when the two dice are thrown and then roll 3 or greater when the single die is thrown. When two dice are to be thrown, what is pr (sum of 7)? When one die will be thrown, what is pr (3 or greater)? What is pr (sum of 7 with two dice ***and*** then 3 or greater when one die is tossed)? (Rolling two dice and then rolling one die are independent events, so you multiply probabilities.) $\frac{1}{6}, \frac{2}{3}, \frac{1}{9}$

10.5 Estimating Sums

Performance Objective: The student can estimate sums.

Many times you don't really need an exact answer. A number that's "in the ballpark" will do. Other times, you already have an answer. But it doesn't "look right." So you want to check it, at least roughly. For these purposes, estimates can be very handy.

Example 1: You are buying some groceries and you want the cost to be under $5.00. Estimate if this is so.

Milk, $1.65
Meat, $2.34

ESTIMATE:

Think

$1.65 about $1.70
$2.34 about $2.30

$$\begin{array}{r}\$1.00\\ \$3.00\\ \hline \$4.00\end{array}$$

So the cost is OK.

Photo Research International

Example 2: You add three numbers on a calculator, and you want to double-check by estimating.

$$\begin{array}{rl}\left.\begin{array}{r}573\\125\\314\end{array}\right\} & \leftarrow \text{the numbers you added}\\ \hline 3612 & \leftarrow \text{the calculator answer}\end{array}$$

ESTIMATE:

Think

$$\begin{array}{rcr}573 & \text{about} & 600\\ 125 & \text{about} & 100\\ 314 & \text{about} & 300\\ \hline & & 1000\end{array}$$

So the calculator answer should be close to 1000. The answer 3612 is wrong, and it is what you would get by accidentally recording 314 as 3014. You could add on the calculator again and find the correct sum (1012).

Exercises

1–12. Select the *best estimate* for each sum.

1. $.29 + $.29 **a.** $.40 **b.** $.50 [✓ **c.** $.60]

2. 127 + 486 [✓ **a.** 600] **b.** 700 **c.** 500

3. 7.8 + 3.3 **a.** 10 [✓ **b.** 11] **c.** 12

4. $1.39 + $2.19 **a.** $3.40 **b.** $3.00 [✓ **c.** $3.60]

5. 673 + 384 **a.** 900 **b.** 1000 [✓ **c.** 1100]

6. $1.29 + $3.19 [✓ **a.** $4.00] **b.** $5.00 **c.** $3.00

7. $.87 + $3.91 **a.** $4.00 [✓ **b.** $5.00] **c.** $6.00

8. $2.25 + $7.98 **a.** $9.00 [✓ **b.** $10.00] **c.** $11.00

9. $1.37 + $6.79 + $3.46 **a.** $10.00 [✓ **b.** $11.00] **c.** $12.00

10. $.28 + $.44 + $.59 **a.** $1.10 **b.** $1.20 [✓ **c.** $1.30]

11. 187 + 243 + 368 **a.** 700 [✓ **b.** 800] **c.** 900

12. 6.3 + 4.7 + 8.1 **a.** 18 [✓ **b.** 19] **c.** 20

13–18. Estimate each sum by rounding each addend to the nearest tenth of a dollar. Then find the exact sum.

Example: $\begin{array}{r} \$7.12 \\ +3.79 \\ \hline \end{array}$ Estimate: $10.90
Exact sum: $10.91

13. $\begin{array}{r} \$2.71 \\ +1.28 \\ \hline \end{array}$
$4.00; $3.99

14. $\begin{array}{r} \$3.43 \\ +4.58 \\ \hline \end{array}$
$8.00; $8.01

15. $\begin{array}{r} \$.39 \\ .19 \\ +.53 \\ \hline \end{array}$
$1.10; $1.11

16. $\begin{array}{r} \$1.28 \\ .43 \\ +\ \ .16 \\ \hline \end{array}$
$1.90; $1.87

17. $\begin{array}{r} \$.93 \\ .78 \\ .36 \\ +.23 \\ \hline \end{array}$
$2.30; $2.30

18. $\begin{array}{r} \$.37 \\ .86 \\ 1.29 \\ +2.17 \\ \hline \end{array}$
$4.80; $4.69

19–24. Estimate each sum by rounding each addend to the nearest hundred. Then find the exact sum.

19. $\begin{array}{r} 133 \\ +246 \\ \hline \end{array}$
300; 379

20. $\begin{array}{r} 281 \\ +176 \\ \hline \end{array}$
500; 457

21. $\begin{array}{r} 341 \\ +612 \\ \hline \end{array}$
900; 953

22.
$$\begin{array}{r} 466 \\ +199 \\ \hline \end{array}$$
700; 665

23.
$$\begin{array}{r} 387 \\ 247 \\ +112 \\ \hline \end{array}$$
700; 746

24.
$$\begin{array}{r} 257 \\ 398 \\ +156 \\ \hline \end{array}$$
900; 811

25–30. Estimate each sum by rounding each addend to the nearest whole number. Then find the exact sum.

25.
$$\begin{array}{r} 6.8 \\ +3.4 \\ \hline \end{array}$$
10; 10.2

26.
$$\begin{array}{r} 2.6 \\ +8.4 \\ \hline \end{array}$$
11; 11

27.
$$\begin{array}{r} 6.8 \\ 7.1 \\ +3.5 \\ \hline \end{array}$$
18; 17.4

28.
$$\begin{array}{r} 1.7 \\ 4.4 \\ +6.2 \\ \hline \end{array}$$
12; 12.3

29.
$$\begin{array}{r} 3.79 \\ 1.63 \\ +0.89 \\ \hline \end{array}$$
7; 6.31

30.
$$\begin{array}{r} 8.38 \\ 2.98 \\ +1.30 \\ \hline \end{array}$$
12; 12.66

31. At a restaurant, you pay the bill for yourself and two friends. The checks are for \$2.49, \$1.78, and \$3.10. Estimate the total bill by rounding each addend to the nearest dollar. \$7

32. On a calculator, you add the following numbers: 187, 621, 385, and 178. The sum you get is 1371. Check your answer by rounding each addend to the nearest hundred. 1400

33. On a trip, you record the number of litres of gasoline you bought: 56.4 L, 53.1 L, 49.8 L and 46.6 L. Estimate the total amount of gasoline you bought by rounding each addend to the nearest litre. 206 L

34. At a store, you purchase four items costing \$.39, \$.41, \$.28, and \$.69. Before you check out, estimate the total by rounding each addend to the nearest tenth of a dollar. \$1.80

35. On a trip, you record the number of kilometres you drove each day: 525 km, 603 km, 463 km and 599 km. Estimate the total kilometres you have driven by rounding each addend to the nearest hundred. 2200 km

36. A taxi driver records the tips she received in one morning: \$1.10, \$0.50, \$0.80, and \$0.25. Estimate her total tips by rounding each addend to the nearest dollar. \$3.00

Estimating Differences

10.6

The ideas you used when estimating sums can also be applied to estimating differences.

Performance Objective: The student can estimate differences.

Example 1: Your car's odometer shows 7298 when you leave on a trip. At the end of the trip it reads 7922. Estimate the distance you drove by rounding each number to the nearest hundred.

ESTIMATE:

Think

$$\begin{array}{rlr} 7922 & \text{about} & 7900 \\ \underline{-\,7298} & \text{about} & \underline{7300} \\ & & 600 \quad \text{difference} \end{array}$$

So the trip was about 600 km.

Example 2: At the grocery, the total bill is \$13.73. You give the cashier a \$20 bill and you receive \$5.27 in change. As an estimate you round \$13.73 to the nearest dollar before subtracting. Is your change correct?

ESTIMATE:

Think

$$\begin{array}{rlr} \$20.00 & & \$20 \\ \underline{-\,13.73} & \text{about} & \underline{14} \\ & & \$6 \quad \text{difference} \end{array}$$

Brent Jones

Your change is incorrect. You should receive \$6.27.

Exercises

1–12. Choose the *best estimate* for each difference.

1. 798 − 235	**a.** 400	**b.** 500	[✓ **c.**] 600
2. 19.8 − 3.5	**a.** 15	[✓ **b.**] 16	**c.** 17
3. $10.00 − $5.98	[✓ **a.**] $4.00	**b.** $5.00	**c.** $6.00
4. $5.00 − $.29	[✓ **a.**] $4.70	**b.** $4.80	**c.** $4.00
5. 8391 − 8123	**a.** 100	**b.** 200	[✓ **c.**] 300
6. $1.00 − $.41	**a.** $.40	**b.** $.50	[✓ **c.**] $.60
7. 934 − 178	**a.** 600	[✓ **b.**] 700	**c.** 800
8. $20.00 − $11.98	[✓ **a.**] $8.00	**b.** $9.00	**c.** $10.00
9. 28.8 − 7.7	**a.** 20	[✓ **b.**] 21	**c.** 22
10. 93 − 62	**a.** 20	[✓ **b.**] 30	**c.** 40
11. 818 − 515	**a.** 200	[✓ **b.**] 300	**c.** 400
12. 14815 − 14670	[✓ **a.**] 100	**b.** 200	**c.** 300

13–18. Estimate the difference by rounding each number to the nearest hundred. Then find the exact difference.

13. 478 − 123
400; 355

14. 913 − 261
600; 652

15. 827 − 415
400; 412

16. 1272 − 1121
200; 151

17. 6852 − 6231
700; 621

18. 11381 − 11128
300; 253

19–24. Estimate the difference by rounding each amount to the nearest dollar. Then find the exact difference.

19. $28.95 − 3.79
$25.00; $25.16

20. $1.84 − .79
$1.00; $1.05

21. $12.25 − 3.15
$9.00; $9.10

22. $44.31 − 13.87
$30.00; $30.44

23. $16.42 − 8.63
$7.00; $7.79

24. $27.68 − 14.28
$14.00; $13.4

25–30. Estimate the difference by rounding the amount being subtracted to the nearest tenth of a dollar. Then find the exact difference.

25. $\begin{array}{r} \$2.00 \\ -\ .49 \\ \hline \end{array}$

$1.50; $1.51

26. $\begin{array}{r} \$1.00 \\ -\ .63 \\ \hline \end{array}$

$.40; $.37

27. $\begin{array}{r} \$5.00 \\ -2.75 \\ \hline \end{array}$

$2.20; $2.25

28. $\begin{array}{r} \$10.00 \\ -\ 7.87 \\ \hline \end{array}$

$2.10; $2.13

29. $\begin{array}{r} \$3.00 \\ -\ .37 \\ \hline \end{array}$

$2.60; $2.63

30. $\begin{array}{r} \$25.00 \\ -11.77 \\ \hline \end{array}$

$13.20; $13.23

31. You buy $27.73 worth of groceries, but you have coupons that give a total discount of $1.75. Estimate how much the groceries will cost. Round each amount to the nearest dollar. $26.00

32. You do the following subtraction on a calculator: 783 − 377. The answer you get is 406. Check your answer by rounding each number to the nearest hundred. 400

33. At a restaurant your bill is $3.76. You give the cashier $10.00. Estimate how much change you should get. Round the amount being subtracted to the nearest dollar. $6.00

34. You buy a record player whose regular price is $94.99. But you get a 10% discount of $9.50. Estimate how much you pay. Round each amount to the nearest dollar. $85.00

"That's all we can buy; the batteries just went dead."

35. A salesperson takes in $897 in one week, but the expenses are $238. Estimate how much is actually earned for the week. Round each amount to the nearest hundred. $700

36. Your gross pay for a week is $237, but the payroll deductions are $78. Estimate how much you have left. Round each amount to the nearest $10. $160

10.7 Estimating Products

Sometimes it is helpful to estimate a product rather than to find the exact product.

Performance Objective: The student can estimate products.

Example 1: At a restaurant, you order three cheeseburgers that cost \$.79 apiece. Estimate the total cost.

ESTIMATE:

Think

\$.79 about

$$\begin{array}{r} \$.80 \\ \times \quad 3 \\ \hline \$2.40 \end{array}$$

So three cheeseburgers cost about \$2.40.

Example 2: The owner wants to fertilize a yard that is 63 m wide by 82 m long. Estimate the area by rounding each number to the nearest 10.

ESTIMATE:

Think

63 about 60
82 about 80

$$\begin{array}{r} 60 \\ \times 80 \\ \hline 4800 \end{array}$$

So the area is about 4800 m^2.

Example 3: By using a calculator, you get the following result: $287 \times 643 = 184541$. Estimate the product to check your answer.

ESTIMATE:

Think

287 about 300
643 about 600

$$\begin{array}{r} 300 \\ \times 600 \\ \hline 180000 \end{array}$$

The estimate makes your answer seem reasonable.

Exercises

1–12. Pick the *best estimate* for each product.

1. $.98 × 4 **a.** $3.60 [✓ **b.** $4.00] **c.** $5.00

2. 44 × 51 **a.** 200 [✓ **b.** 2000] **c.** 2500

3. 592 × 383 [✓ **a.** 240 000] **b.** 24 000 **c.** 2400

4. 12 × $1.14 **a.** $10.00 [✓ **b.** $12.00] **c.** $16.00

5. $6.95 × 4 **a.** $24.00 [✓ **b.** $28.00] **c.** $2.80

6. 88 × 68 **a.** 630 [✓ **b.** 6300] **c.** 63 000

7. $54.95 × 7 **a.** $300 [✓ **b.** $350] **c.** $3000

8. 759 × 195 **a.** 1600 **b.** 16 000 [✓ **c.** 160 000]

9. 97 × 97 **a.** 8100 [✓ **b.** 10 000] **c.** 1000

10. $1.98 × 12 **a.** $20.00 [✓ **b.** $24.00] **c.** $30.00

11. $27.98 × 24 **a.** $400 [✓ **b.** $600] **c.** $900

12. 186 × 466 **a.** 8000 **b.** 80 000 [✓ **c.** 100 000]

13–18. Estimate each product by rounding the factors to the nearest ten. Then find the exact product.

13. $\begin{array}{r} 98 \\ \times 47 \\ \hline \end{array}$

5000; 4606

14. $\begin{array}{r} 71 \\ \times 49 \\ \hline \end{array}$

3500; 3479

15. $\begin{array}{r} 56 \\ \times 84 \\ \hline \end{array}$

4800; 4704

16. $\begin{array}{r} 59 \\ \times 59 \\ \hline \end{array}$

3600; 3481

17. $\begin{array}{r} 12 \\ \times 37 \\ \hline \end{array}$

400; 444

18. $\begin{array}{r} 94 \\ \times 98 \\ \hline \end{array}$

9000; 9212

19–24. Estimate each product by rounding the factors to the nearest hundred. Then find the exact product.

19. $\begin{array}{r} 215 \\ \times 536 \\ \hline \end{array}$

100 000; 115 240

20. $\begin{array}{r} 627 \\ \times 181 \\ \hline \end{array}$

120 000; 113 487

21. $\begin{array}{r} 356 \\ \times 429 \\ \hline \end{array}$

160 000; 152 724

22. $\begin{array}{r} 723 \\ \times 610 \\ \hline \end{array}$

420 000; 441 030

23. $\begin{array}{r} 841 \\ \times 237 \\ \hline \end{array}$

160 000; 199 317

24. $\begin{array}{r} 920 \\ \times 375 \\ \hline \end{array}$

360 000; 345 000

25–30. Estimate each product by rounding the dollar-and-cents amount to the nearest dollar. Then find the exact product.

25. $\begin{array}{r} \$1.76 \\ \times \quad 4 \\ \hline \end{array}$ $8.00; $7.04

26. $\begin{array}{r} \$2.14 \\ \times \quad 5 \\ \hline \end{array}$ $10.00; $10.70

27. $\begin{array}{r} \$9.27 \\ \times \quad 8 \\ \hline \end{array}$ $72.00; $74.16

28. $\begin{array}{r} \$6.79 \\ \times \quad 3 \\ \hline \end{array}$ $21.00; $20.37

29. $\begin{array}{r} \$4.89 \\ \times \quad 7 \\ \hline \end{array}$ $35.00; $34.23

30. $\begin{array}{r} \$8.25 \\ \times \quad 6 \\ \hline \end{array}$ $48.00; $49.50

BEACH BOYS
JULY 16 & 17 8:00PM

Bill Smith Photography

William Means

31. At a rock concert, main floor tickets sell for $5.75. Estimate the cost of four tickets. Round the cost per ticket to the nearest dollar. $24.00

32. Ground beef sells for 5.18/kg. Estimate the cost of 3 kg of ground beef. (Round the cost per kilogram to the nearest tenth of a dollar). $15.60

33. A field is 57 m long by 92 m wide. Estimate the area of the field by rounding each number to the nearest ten metres. 5400 m^2

34. By using a calculator, you get the following result: 378 × 465 = 175 770. Check your result by estimating. Round each factor to the nearest hundred. 200 000

35. Carpeting sells for $14 a square metre and you need 28 m^2. Estimate the total cost by rounding each factor to the nearest ten. $300

36. A worker works 194 hours during a month and is paid $4.75 an hour. Estimate the total wages for a month. Round the hours to the nearest hundred and the wages to the nearest dollar. $1000

Estimating Quotients 10.8

In the sections before this, you estimated sums, differences, and products. In this section, you will estimate quotients.

Performance Objective: The student can estimate quotients.

Example 1: On a calculator, you get the following result: 8704 ÷ 32 = 272. Check your result by estimating. Round 8704 to the nearest thousand and round 32 to the nearest ten.

ESTIMATE:

Think

8704 about 9000
32 about 30

$$\begin{array}{r}300\\30\overline{)9000}\end{array} \quad \text{or} \quad \frac{900\cancel{0}}{3\cancel{0}} = 300$$

The estimate makes 272 seem reasonable.

Example 2: At a gas station, you have your car's tank filled with 80.8 L of gasoline. The total cost is \$20.15. Estimate the cost per litre.

ESTIMATE:

Think

\$20.15 about \$20.00
80.8 about 80

$$\begin{array}{r}\$0.25\\80\,\overline{)\,\$20.00}\end{array} \quad \text{or} \quad \frac{\overset{1}{\cancel{20}}}{\underset{4}{\cancel{80}}} = 0.25$$

The gasoline costs about \$0.25/L.

Exercises

1–12. Choose the *best estimate* for each quotient.

1. 6315 ÷ 71 [✓ **a.** 90] **b.** 900 **c.** 9000

2. 1423 ÷ 21 [✓ **a.** 70] **b.** 700 **c.** 7000

3. 163 ÷ 19 [✓ **a.** 8] **b.** 80 **c.** 90

4. 357 ÷ 41 [✓ **a.** 9] **b.** 80 **c.** 90

5. 199 ÷ 11.2 [✓ **a.** 20] **b.** 10 **c.** 100

6. 397 ÷ 18.6 [✓ **a.** 20] **b.** 200 **c.** 40

7. $7.95 ÷ 12 [✓ **a.** $.80] **b.** $.08 **c.** $1.00

8. $19.95 ÷ 5 **a.** $5 **b.** $6 [✓ **c.** $4]

9. $44.98 ÷ 9 **a.** $4 [✓ **b.** $5] **c.** $6

10. 3195 ÷ 319 [✓ **a.** 10] **b.** 100 **c.** 8

11. 8098 ÷ 905 **a.** 90 **b.** 100 [✓ **c.** 9]

12. 6291 ÷ 713 [✓ **a.** 9] **b.** 90 **c.** 100

13–24. Estimate each quotient by rounding each number to the nearest ten. Then find the quotient to the nearest tenth.

Example: $21\overline{)78}$ Estimate: 4
To nearest tenth: 3.6

13. $\overset{4;\ 4.3}{18\overline{)78}}$ **14.** $\overset{3;\ 3.2}{39\overline{)124}}$ **15.** $\overset{7;\ 6.7}{63\overline{)423}}$

16. $\overset{3;\ 3.0}{81\overline{)244}}$ **17.** $\overset{5;\ 5.2}{76\overline{)396}}$ **18.** $\overset{5;\ 4.9}{31\overline{)152}}$

19. $\overset{20;\ 15.5}{12.6\overline{)195}}$ **20.** $\overset{8;\ 8.6}{18.3\overline{)158}}$ **21.** $\overset{10;\ 11.3}{27.9\overline{)314}}$

22. $\overset{8;\ 8.2}{48.2\overline{)397}}$ **23.** $\overset{8;\ 7.1}{33.5\overline{)237}}$ **24.** $\overset{2;\ 2.1}{86.4\overline{)182}}$

25–36. Estimate each quotient by rounding each number to the nearest hundred. Then find the quotient to the nearest tenth.

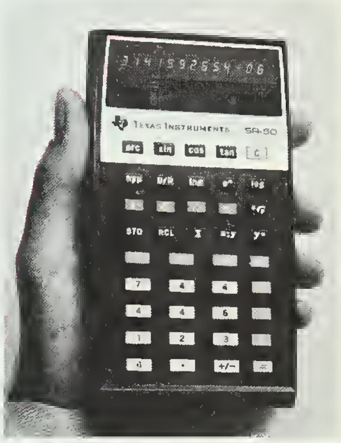
25. $\begin{array}{r} 4;\ 4.1 \\ 187\overline{)758} \end{array}$ **26.** $\begin{array}{r} 3;\ 3.0 \\ 314\overline{)927} \end{array}$ **27.** $\begin{array}{r} 4;\ 5.1 \\ 164\overline{)831} \end{array}$

28. $\begin{array}{r} 2;\ 1.8 \\ 216\overline{)398} \end{array}$ **29.** $\begin{array}{r} 8;\ 8.1 \\ 98\overline{)794} \end{array}$ **30.** $\begin{array}{r} 10;\ 9.6 \\ 104\overline{)998} \end{array}$

31. $\begin{array}{r} 5;\ 4.8 \\ 731\overline{)3542} \end{array}$ **32.** $\begin{array}{r} 5;\ 5.2 \\ 484\overline{)2521} \end{array}$ **33.** $\begin{array}{r} 6;\ 5.7 \\ 634\overline{)3620} \end{array}$

34. $\begin{array}{r} 2;\ 1.9 \\ 814\overline{)1560} \end{array}$ **35.** $\begin{array}{r} 6;\ 6.1 \\ 193\overline{)1185} \end{array}$ **36.** $\begin{array}{r} 4;\ 4.2 \\ 580\overline{)2430} \end{array}$

37. On a calculator, you get this result: 196.5 ÷ 39.3 = 5. Check your result by estimating. Round 196.5 to the nearest hundred and 39.3 to the nearest ten. 5



38. A chemical vat has a volume of 215 L. Estimate how many litres there are in 27 chemical vats. Round 27 to the nearest ten and round 215 to the nearest hundred. 6000 L

39. On a trip, you travel 337 km in 3 h 54 min. Estimate the average speed for the trip. Round 337 to the nearest ten and 3 h 54 min to the nearest hour. 85 km/h

40. A motorcycle travels 447 km on a full tank of gasoline. If the tank holds 15 L, estimate how many litres per 100 km the motorcycle gets. Round 447 to the nearest ten. 3.3 L/100 km

41. A worker is paid $189 for working 38 h. Estimate the hourly wage. Round $189 to the nearest hundred and 38 to the nearest ten. $5/h

42. A lawn has an area of 76 m^2. The developer wants to resod 32 lawns. Estimate how many *square metres* of sod are needed. Round each number to the nearest ten. 2560 m^2

Photo Research International

10.9 Solving Problems by Estimating

Performance Objective: The student can make estimates that might involve any one of the four operations.

Now you have had practice in estimating sums, differences, products, and quotients. So the problems in this section involve all four operations. In some cases, you will have to decide which operation to use.

TRUCK SCALES

Example 1: 1 m^3 of aluminum has a mass of 2700 kg. Estimate the mass of 11 m^3 of aluminum.

ESTIMATE:

Think

2700 about 3000
11 about 10

$3000 \times 10 = 30\ 000$ kg

The mass is about 30 000 kg.

Example 2: At a record store, you buy three records. The total bill is \$11.94. You give the clerk \$15.00. Estimate how much change you should receive.

ESTIMATE:

Think

\$11.94 about \$12.00

$$\begin{array}{r} \$15.00 \\ -12.00 \\ \hline \$3.00 \end{array}$$

So you should get about \$3.00 change.

Example 3: A real-estate developer is subdividing a hectare field into building lots. Each lot has an area of 512 m^2. Estimate how many lots there would be (10 000 m^2 in 1 ha).

ESTIMATE:

Think

512 about 500
5445 about 5000

$$\begin{array}{r} 20 \\ 500\,\overline{)\,10\,000} \end{array} \quad \text{or} \quad \frac{10\,0\cancel{0}\cancel{0}}{500} = 20$$

By this estimate, there are 20 lots.

Exercises

1–12. Pick the *best estimate* for each problem.

1.	$8.47 + $3.62	**a.** $11.00	**b.** $12.00	**c.** $13.00
2.	28 × 71	**a.** 210	**b.** 2100	**c.** 1400
3.	$5.00 − $2.91	**a.** $2.00	**b.** $3.00	**c.** $3.10
4.	124 ÷ 31	**a.** 6	**b.** 5	**c.** 4
5.	17 + 42 + 87	**a.** 140	**b.** 150	**c.** 160
6.	$7.95 × 18	**a.** $140	**b.** $160	**c.** $200
7.	1721 − 898	**a.** 800	**b.** 900	**c.** 1000
8.	261 ÷ 13.1	**a.** 20	**b.** 200	**c.** 30
9.	178 + 121 + 391	**a.** 500	**b.** 600	**c.** 700
10.	473 x 21.5	a. 1000	**b.** 10 000	c. 8000
11.	6399 ÷ 81	**a.** 8	**b.** 80	**c.** 800
12.	$4.29 − $3.98	**a.** $.20	**b.** $.30	**c.** $1.00

13–18. Estimate each answer by rounding each number to the nearest ten. Then find the exact answer.

13. 27 + 42 70; 69

14. 27 × 42 1200; 1134

15. 42 − 27 10; 15

16. 117 ÷ 39 3; 3

17. 84 + 161 240; 245

18. 17 + 33 + 41 90; 91

19–24. Estimate each answer by rounding each number to the nearest hundred. Then find the exact answer.

19. 1728 − 471 1200; 1257

20. 492 × 183 100 000; 90 036

21. 1809 ÷ 201 9; 9

22. 1128 + 381 1500; 1509

23. 981 − 673 300; 308

24. 2135 ÷ 305 7; 7

25–30. Estimate each answer by rounding each dollar-and-cents amount to the nearest dollar. Then find the exact answer.

25. \$9.93 + \$8.26 \$18.00; \$18.19

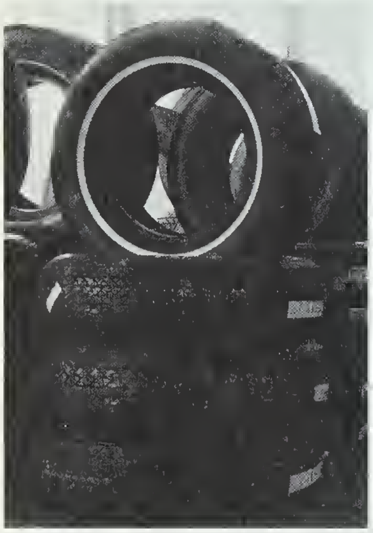
26. \$9.93 − \$8.26 \$2.00; \$1.67

27. \$9.27 × 6 \$54.00; \$55.62

28. \$.87 + \$.29 + \$.26 \$1.00; \$1.42

29. \$99.95 × 24 \$2400; \$2398.80

30. \$49.95 ÷ 5 \$10.00; \$9.99


William Means

31. A ranch has 27 sections of 92 ha each. Estimate how many hectares this is. Round each number to the nearest ten. 2700 ha

32. At a tire sale, four tires sell for \$164. Estimate the cost of one tire. Round \$164 to the nearest ten. \$40

33. A clothing store gives its employees a 25% discount on any purchase. An employee buys clothes that sell for \$86.96, and the discount is \$21.74. Estimate how much the employee has to pay. Round each amount to the nearest dollar. \$65

Johnson/DeWys, Inc.

34. A woman earned \$15,325 last year, but her income taxes were \$3125. Estimate her pay after taxes. Round each number to the nearest thousand. \$12,000

35. A packing crate is filled with 24 identical automobile parts that have a mass of 1.8 kg apiece. Estimate the mass of all the parts. Round 24 to the nearest ten and 1.8 to the nearest whole number. 40 kg.

36. The attendance at four meetings during a month was 182, 212, 197, and 221. Estimate the total attendance for the month. Round each number to the nearest hundred. 800

CAREER CORNER

SPORTS AND SALES

The mathematics in this chapter might be useful in jobs like the following:

1. *Baseball coach:* A pitcher and a certain player have faced each other 100 times in their careers. The pitcher has given up 20 hits to the player and has walked the player 5 times. Use this information to find the probability that the pitcher will give up a hit or a walk to the player the next time at bat. $\frac{1}{4}$

2. *Salesperson:* Out of the last 50 calls, a salesperson made 21 sales. Using this information, what is the probability of making a sale on the next call? What is the probability of making two sales in a row? (Assume sales calls are independent events.) $\frac{21}{50}$, $\frac{441}{2500}$

3. *Basketball coach:* A player has made 72 free throws out of 98 attempted. Estimate the player's free throw average. Round each number to the nearest ten. 70%

4. *Salesperson:* A customer needs 19 storm windows that sell for $21 apiece. The salesperson estimates the cost by rounding each number to the nearest ten. What is the estimated cost? $400

Wide World Photos

Test for Chapter 10

1. In an experiment, a single marble was drawn from a bag containing 20 marbles—12 black and 8 orange. After each draw, the marble was replaced. What is the experimental probability of drawing a black marble if black occurred 28 times out of 50? Express it as a fraction, a decimal, and a percent. $\frac{14}{25}$, 0.56, 56% (10.1)

2. In Exercise **1**, what is the mathematical probability of drawing a black marble? Express it as a fraction, a decimal, and a percent. $\frac{3}{5}$, 0.6, 60% (10.1)

3. In an experiment, a single coin is to be tossed 100 times. In how many tosses would you expect tails to occur? (10.2)
50

4. A bag contains 10 marbles—4 yellow, 3 blue, and 3 red. If a single marble is to be drawn, what is the mathematical probability of drawing a yellow or a blue marble? Express it as a fraction. $\frac{7}{10}$ (10.3)

5. A coin is to be flipped and a die tossed at the same time. What is the mathematical probability of getting heads on the coin and 3 on the die? Express it as a fraction. (10.4)
$\frac{1}{12}$

6–9. Estimate each answer by rounding each number to the nearest ten. Then find the exact answer.

6. 48 + 62 110; 110 **7.** 19 + 81 + 77 180; 177 (10.5)

8. 111 − 63 50; 48 **9.** 178 − 42 140; 136 (10.6)

10–13. Estimate each answer by rounding each number to the nearest hundred. Then find the exact answer.

10. 389 × 427 160 000; 166 103 **11.** 107 × 88 10 000; 9416 (10.7)

12. 1236 ÷ 309 4; 4 **13.** 780 ÷ 195 4; 4 (10.8)

14. If three tennis balls sell for $2.69, estimate the cost of each ball. Round $2.69 to the nearest tenth of a dollar. (10.9)
$0.90

RATE YOURSELF

Number Correct	
14	Superior
12–13	Excellent
10–11	Good
8–9	Fair
0–7	Poor

Chapter 10 Review

1. In a coin-tossing experiment, the following results were obtained out of 50 trials: (10.1)

Heads	Tails
22	28

What is ex pr (tails)? Express it as a decimal. 0.56

2. In Exercise 1, what is the mathematical probability of tossing tails? Express it as a percent. 50% (10.1)

3. When a die is to be tossed, how many equally likely outcomes are there? 6 (10.2)

4. A sack contains 8 marbles—5 red and 3 blue. Are "drawing red" and "drawing blue" equally likely outcomes if one marble is to be drawn from the sack? No (10.2)

5. Outcomes *A* and *B* are mutually exclusive events. If pr (*A*) is $\frac{1}{5}$ and pr (*B*) is $\frac{3}{5}$, what is pr (*A* or *B*) as a percent? 80% (10.3)

6. A sack contains 15 marbles—7 white, 5 yellow, and 3 green. If one marble is to be drawn, what is pr (white or green) as a fraction? $\frac{2}{3}$ (10.3)

7. Outcomes *C* and *D* are independent events. If pr (*C*) is $\frac{1}{3}$ and pr (*D*) is $\frac{1}{4}$, what is pr (*C* and *D*) as a fraction? $\frac{1}{12}$ (10.4)

8. A die is to be rolled twice in a row. What is pr (3 and then 5) as a fraction? $\frac{1}{36}$ (10.4)

9–12. Estimate each sum by rounding each addend to the nearest ten. Then find the exact sum. (10.5)

9. $39 + 29$ 70; 68

10. $43 + 61$ 100; 104

11. $68 + 31$ 100; 99

12. $34 + 16 + 23$ 70; 73

13–16. Estimate each difference by rounding each number to the nearest thousand. Then find the exact difference. (10.6)

13. 6314 − 5280 1000; 1034

14. 9615 − 1987 8000; 7628

15. 4376 − 1776 2000; 2600

16. 6281 − 4111 2000; 2170

17–20. Estimate each product by rounding each number to the nearest hundred. Then find the exact product. (10.7)

17. 142 × 233 20 000; 33 086

18. 687 × 129 70 000; 88 623

19. 389 × 487 200 000; 189 443

20. 592 × 602 360 000; 356 384

21–24. Estimate each quotient by rounding each number to the nearest hundred. Then find the exact quotient. (10.8)

21. 985 ÷ 197 5; 5

22. 1212 ÷ 202 6; 6

23. 1584 ÷ 396 4; 4

24. 2128 ÷ 304 7; 7

25. On a trip you spend the following amounts for gasoline: $7.85, $6.30, $8.15, and $6.78. Estimate the total cost of the gasoline by rounding each amount to the nearest dollar. $29.00 (10.9)

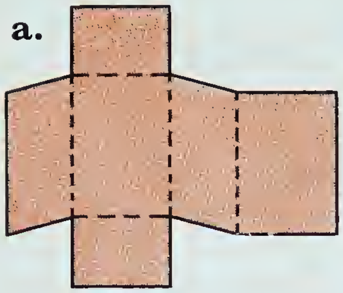
26. The list price for a new car is $4127, but the dealer offers a discount of $325. Estimate the cost of the car. Round each amount to the nearest hundred. $3800 (10.9)

27. A ball of twine has 34 m of twine on it. Estimate how many metres of twine there are in 36 balls. Round each number to the nearest ten. 1200 m (10.9)

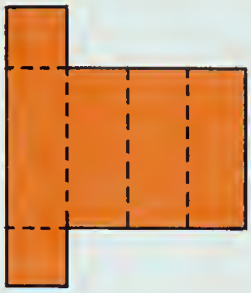
Another Look at Chapters 9 and 10

1. Which pattern below could be cut out and folded to form a rectangular solid? b (9.1)

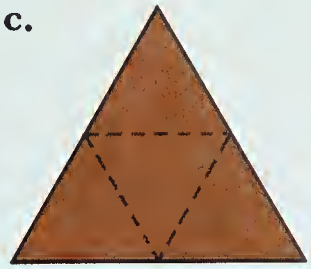
a.



b.

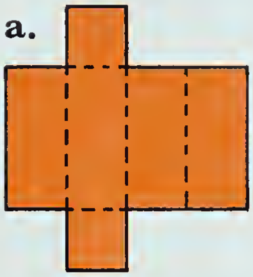


c.

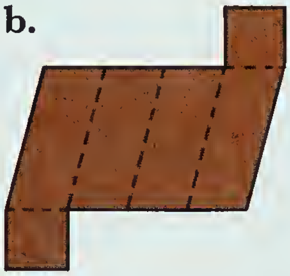


2. Which pattern below could be cut out and folded to form a right prism? a (9.2)

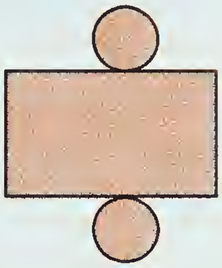
a.



b.

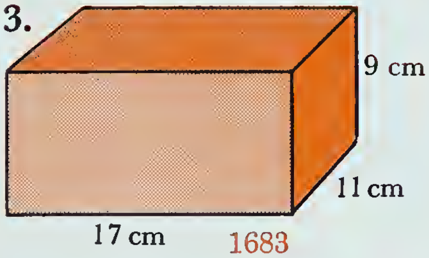


c.



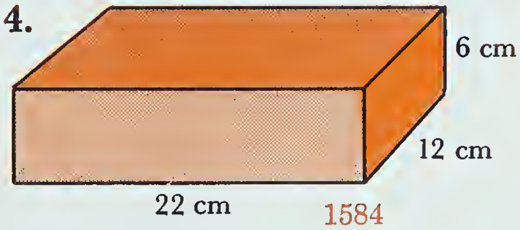
How many 1 cm cubes would each box hold? (9.3)

3.


9 cm
11 cm
17 cm

1683

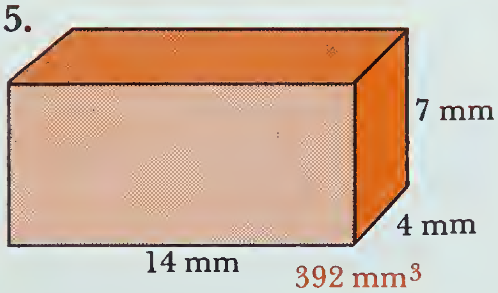
4.


6 cm
12 cm
22 cm

1584

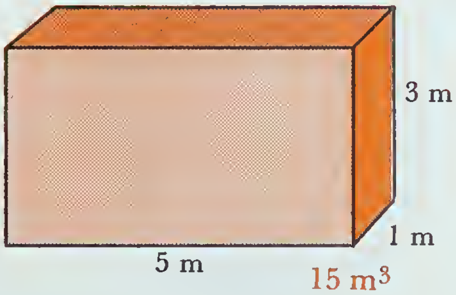
What is the volume of each rectangular solid? (9.4)

5.


7 mm
4 mm
14 mm

392 mm^3

6.



15 m^3

Change each volume. (9.5)

7. 0.085 m^3 to cm^3 85 000 cm^3

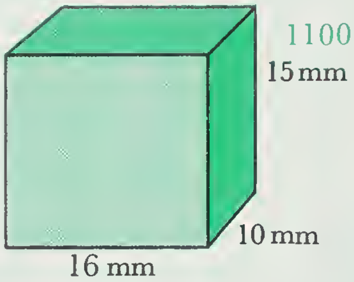
8. 32 m^3 to L 3200 L

9. 640 000 cm^3 to m^3 0.64 m^3

10. 0.4 cm^3 to mL 0.4 mL

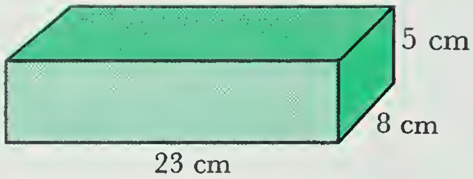
Find the surface area of each rectangular solid. (9.6)

11.


15 mm
10 mm
16 mm

1100 mm^2

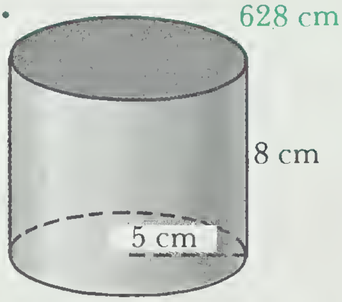
12.


5 cm
8 cm
23 cm

678 cm^2

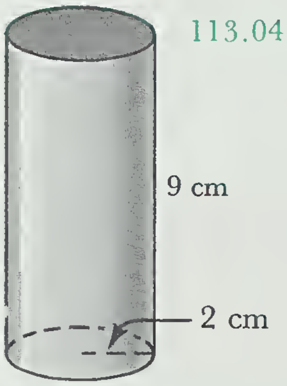
Find the volume of each cylinder. Use 3.14 for π. (9.7)

13.



628 cm^3

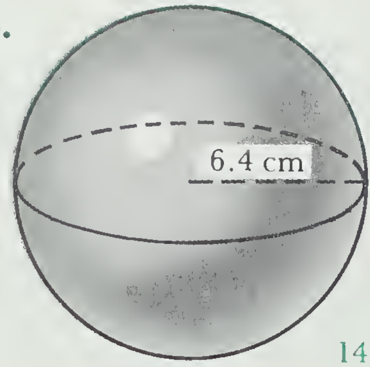
14.



113.04 cm^3

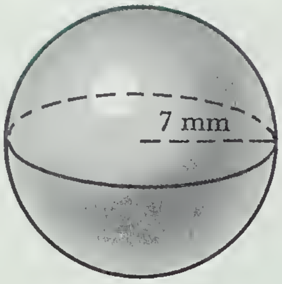
Find the volume of each sphere. Use $\frac{22}{7}$ for π. (9.8)

15.



1441.792 cm^3

16.



1437.33 mm^3

17. In a coin-tossing experiment, the following results were obtained out of 25 trials:

Heads	Tails
11	14

What is ex pr (heads)? Express it as a percent. (10.1)
44%

18. A die is to be tossed. What is pr (4) as a fraction? (10.1)
$\frac{1}{6}$

19. When a coin is to be tossed, how many equally likely outcomes are there? 2 (10.2)

20. A sack contains 12 marbles—8 white and 4 orange. Are "drawing white" and "drawing orange" equally likely outcomes if one marble is to be drawn from the sack? (10.2)
No

21. Events A and B are mutually exclusive events. If pr (A) is $\frac{1}{4}$ and pr (B) is $\frac{3}{4}$, what is pr (A or B) as a percent?
100% (10.3)

22. A sack contains 15 marbles—7 yellow, 3 red, and 5 blue. If one marble is to be drawn, what is pr (yellow or blue) as a fraction? $\frac{4}{5}$ (10.3)

23. Events C and D are independent. If pr (C) is $\frac{1}{8}$ and pr (D) is $\frac{3}{8}$, what is pr (C and D) as a fraction? $\frac{3}{64}$ (10.4)

24. A die is to be rolled twice in a row. What is pr (3 and then 5) as a fraction? $\frac{1}{36}$ (10.4)

25–28. Estimate each sum by rounding each addend to the nearest ten. Then find the exact sum. (10.5)

25. 76 + 53 130; 129

26. 89 + 19 110; 108

27. 47 + 22 70; 69

28. 31 + 64 + 77 170; 172

29–32. Estimate each difference by rounding each number to the nearest thousand. Then find the exact difference. (10.6)

29. $$\begin{array}{r} 12\,895 \\ -\ \ 5\,788 \\ \hline \end{array}$$
7000; 7107

30. $$\begin{array}{r} 8989 \\ -1178 \\ \hline \end{array}$$
8000; 7811

31. $$\begin{array}{r} 6143 \\ -4798 \\ \hline \end{array}$$
1000; 1345

32. $$\begin{array}{r} 1870 \\ -\ \ 998 \\ \hline \end{array}$$
1000, 872

33–36. Estimate each product by rounding each number to the nearest hundred. Then find the exact product. (10.7)

33. $$\begin{array}{r} 119 \\ \times 379 \\ \hline \end{array}$$
40 000; 45 101

34. $$\begin{array}{r} 764 \\ \times 208 \\ \hline \end{array}$$
160 000; 158 912

35. $$\begin{array}{r} 690 \\ \times 581 \\ \hline \end{array}$$
420 000; 400 890

36. $$\begin{array}{r} 442 \\ \times 885 \\ \hline \end{array}$$
360 000; 391 170

37–40. Estimate each quotient by rounding each number to the nearest hundred. Then find the exact quotient. (10.8)

37. 1545 ÷ 309 5; 5

38. 1379 ÷ 197 7; 7

39. 1806 ÷ 301 6; 6

40. 975 ÷ 195 5; 5

41. There are 231 marbles in 1 jar. Estimate the number of marbles in 18 jars. Round 231 to the nearest hundred and 18 to the nearest ten. (10.9)
4000 marbles

42. On a calculator you get the following result: 18 369 ÷ 2041 = 9. Check your result by rounding each number to the nearest thousand. 9 (10.9)

43. You buy four items that sell for \$4.98, \$3.19, \$6.79, and \$1.98. Estimate the total cost of the items. Round each amount to the nearest dollar. \$17.00 (10.9)

Computing Skills Refresher

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1

Reading and Writing Whole Numbers

Performance Objective: The student can
- tell the value of any digit of a whole number (to hundred billions).
- express a whole number in words (to hundred billions).

Billions			Millions			Thousands			Ones			
Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	place values
			5	4	3	5	4	3	5	4	3	digits

This 5 is in the hundred millions *place*. Its *value* is 5 hundred million. What is the value of each of the other 5's? What is the value of each 4? Of each 3?

543 543 543 is *five hundred forty-three million*
five hundred forty-three thousand
five hundred forty-three.

$543\ \underbrace{000}_{\text{Zeros are not read.}}\ 543$ is *five hundred forty-three million*
five hundred forty-three.

Exercises

1–6. What is the value of each underlined digit?

1. 3$\underline{4}$2 4 tens

2. $\underline{5}$781 5 thousands

3. $\underline{7}$8 604 7 ten thousands

4. $\underline{2}$34 765 2 hundred thousands

5. 8$\underline{6}$7 543 102 6 ten millions

6. $\underline{8}$5 067 341 290 8 ten billions

7–20. Express in words.

7. 34 Thirty-four

8. 427 Four hundred twenty-seven

9. 8942 Eight thousand nine hundred forty-two

10. 24 763 Twenty-four thousand seven hundred sixty-three

11. 942 231 Nine hundred forty-two thousand two hundred thirty-one

12. 1 249 657

13. 234 234 234

14. 42 635 123 741

15. 20 001 Twenty thousand one

16. 80 256 Eighty thousand two hundred fifty-six

17. 10 000 000 Ten million

18. 37 942 001 Thirty-seven million nine hundred forty-two thousand one

19. 702 000 005 Seven hundred two million five

20. 6 905 621 043 Six billion nine hundred five million six hundred twenty-one thousand forty-three

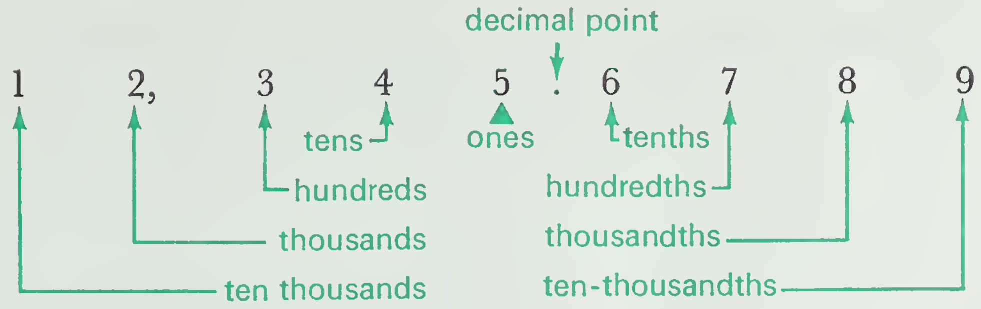
12. One million two hundred forty-nine thousand six hundred fifty-seven

13. Two hundred thirty-four million two hundred thirty-four thousand two hundred thirty-four

14. Forty-two billion six hundred thirty-five million one hundred twenty-three thousand seven hundred forty-one

2

Reading and Writing Decimals



Performance Objective:
The student can
- tell the value of any digit of a decimal (to ten thousandths).
- express a decimal in words.

These numbers	may be read this way	or this way.
27.8	*twenty-seven* and *eight-tenths*	*twenty-seven* point *eight*
16.08	*sixteen* and *eight-hundredths*	*sixteen* point *zero eight*
8.16	*eight* and *sixteen-hundredths*	*eight* point *one six*
0.0031	*thirty-one ten-thousandths*	point *zero zero three one*
1.6	*one* and *six-tenths*	*one* point *six*
1.60	*one* and *sixty-hundredths*	*one* point *six zero*
1.600	*one* and *six-hundred thousandths*	*one* point *six zero zero*

These are equal.

But 1.00006 is *one* and *six hundred-thousandths*.
So the reading using *point* is clearer.

Exercises

1–6. What is the value of each underlined digit?

1. $64\underline{2}.8$ 2 ones

2. $78.\underline{6}4$ 6 tenths

3. $123.4\underline{5}6$ 5 hundredths

4. $103.00\underline{4}$ 4 thousandths

5. $3.040\underline{2}$ 2 ten-thousandths

6. $5.4\underline{0}01$ 0 hundredths

7–20. Express in words using *and*. Then express using *point*.

Ex. 7–20. One answer is given.

7. 4.6 Four and six-tenths

8. 4.06 Four and six-hundredths

9. 4.006 Four and six-thousandths

10. 4.0006 Four and six ten-thousandths

11. 48.73 Forty-eight and seventy-three hundredths

12. 20.076 Twenty and seventy-six thousandths

13. 842.840 Eight hundred forty-two and eight hundred forty thousandths

14. 8614.06 Eight thousand six hundred fourteen and six-hundredths

15. 7.0421 Seven and four hundred twenty-one ten-thousandths

16. 30.13 Thirty and thirteen-hundredths

17. 0.5283 Five thousand two hundred eighty-three ten-thousandths

18. 176.0076 One hundred seventy-six and seventy-six ten-thousandths

19. 31.0420 Thirty-one and four hundred twenty ten-thousandths

20. 80.8200 Eighty and eight thousand two hundred ten-thousandths

3

Reading and Writing Special Numbers

Performance Objective: The student can read and write times, amounts of money, dates, and other useful numbers.

	These numbers	are usually read this way.
Time	3:22	*three twenty-two* or *three hours twenty-two minutes*
	6:00	*six o'clock* or *six hours*
Money	62¢ or $0.62	*sixty-two cents*
	$3.50	*three dollars and fifty cents* or *three and a half* (*dollars*) or *three-fifty* (if dollars is understood)
Order	1st, 2nd, 3rd, · · ·	*first*, *second*, *third*, and so on
Dates	22 Dec.	*twenty-second* of December or
	Dec. 22	December *twenty-second*
Years,	1800	*eighteen hundred*
House Numbers	1776	*seventeen seventy-six*

Many numbers, such as telephone numbers, zip codes, and social security numbers, are usually read one digit at a time.

60153 ⟷ *six zero one five three* or *six oh one five three*

Exercises

1–10. Express in words.

1. Nov. 12, 1843

2. 18 June 1900

3. $2.25

1. November twelfth, eighteen forty-three or Twelfth of November, eighteen forty-three
2. June eighteenth, nineteen hundred or Eighteenth of June, nineteen hundred
3. Two dollars and twenty-five cents or Two and a quarter (dollars)or Two twenty-five

4. $7.00
Seven dollars

5. 1742 8th Avenue
Seventeen forty-two Eighth Avenue

6. $0.19
Nineteen cents

7. 4:30 (time of day)
Four-thirty (or half past four)

8. 345-1952 (phone)
Three four five, one nine five two

9. 18 25 (time of day)
Eighteen twenty-five

10. 13:53 (length of time)
Thirteen hours fifty-three minutes

11–20. Write in symbols.

11. two hours fifty minutes 2:50

12. two-fifty (time of day) 2:50

13. nine zero one six zero zero
901600

14. nineteen thirty-six 1936

15. ten seventy, Fourth Street
1070 4th Street

16. nine-fifty (dollars) $9.50

17. six and a quarter dollars
$6.25

18. three cents $.03 or 3¢

19. thirty-two forty-one, One-hundred-twelfth Street
3241 112th Street

20. the tenth of November, sixteen hundred
Nov. 10, 1600 or 10 Nov. 1600

4 Rounding Numbers

When the digit to the *right* of the place being rounded to
- is less than 5, round DOWN.
- is 5 or more, round UP.

Examples:

Round to the nearest hundredth.

hundredths place

1. 7.036 7.04

5 or more; round up

2. 7.035 7.04

less than 5; round down

3. 7.0326 7.03

Look at only the *first* digit to the right of the place to which you are rounding.

Performance Objective: The student can round a decimal or a whole number to a specified place.

Round to the nearest hundred.

hundreds place

less than 5; round down

4. 70 347 70 300

5. 70 353 70 400

5 or more; round up

6. 70 378 70 400

7. 70 978 71 000

Exercises

1–6. Round to the nearest hundredth.

1. 2.044 2.04 **2.** 7.834 7.83 **3.** 8.365 8.37

4. 3.6281 3.63 **5.** 25.0509 25.05 **6.** 9.997 10.00

7–14. Round to the nearest hundred.

7. 4820 4800 **8.** 72 659 72 700 **9.** 3972 4000 **10.** 685 700

11. 1349 1300 **12.** 99 100 **13.** 24.7 0 **14.** 9006 9000

15–20. Round to the nearest thousandth.

15. 4.0074 4.007 **16.** 3.2828 3.283 **17.** 3152.0655 3152.066

18. 0.000 09 .000 **19.** 1.099 62 1.100 **20.** 12.001 06 12.001

21–26. Round each number in Exercises **1–6** to the nearest tenth.
21. 2.0 **22.** 7.8 **23.** 8.4 **24.** 3.6 **25.** 25.1 **26.** 10.0

27–34. Round each number in Exercises **7–14** to the nearest ten.
27. 4820 **28.** 72,660 **29.** 3970 **30.** 690 **31.** 1350 **32.** 100 **33.** 20 **34.** 9010

35–40. Round to the nearest cent.

35. $75.146 $75.15 **36.** $2.334 $2.33 **37.** $0.0172 $.02

38. $8.003 $8.00 **39.** $3.895 $3.90 **40.** $2.995 $3.00

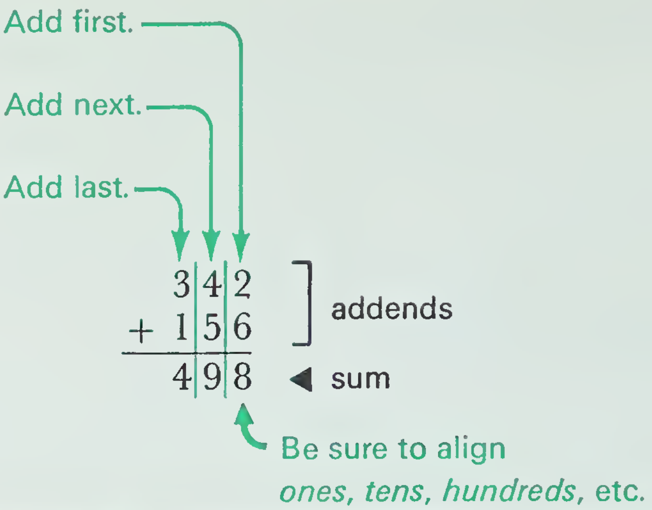
5

Adding Whole Numbers

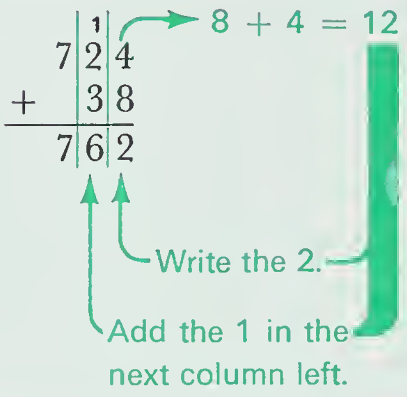
Performance Objective: The student can add two or more whole numbers.

Examples:

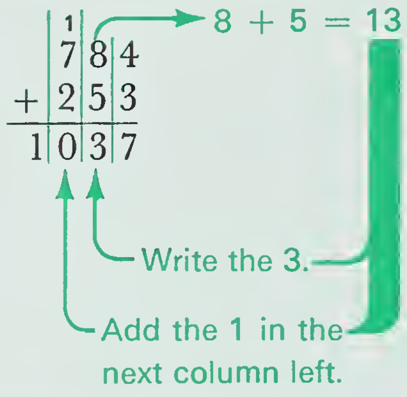
1. 342 + 156


Add first.
Add next.
Add last.
addends
sum
Be sure to align *ones*, *tens*, *hundreds*, etc.

2. a.

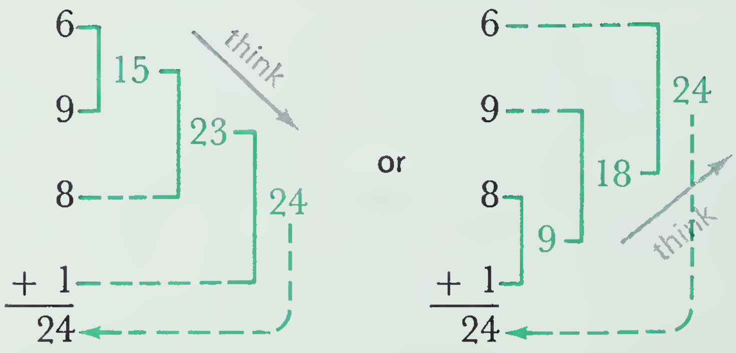

8 + 4 = 12
Write the 2.
Add the 1 in the next column left.

b.


8 + 5 = 13
Write the 3.
Add the 1 in the next column left.

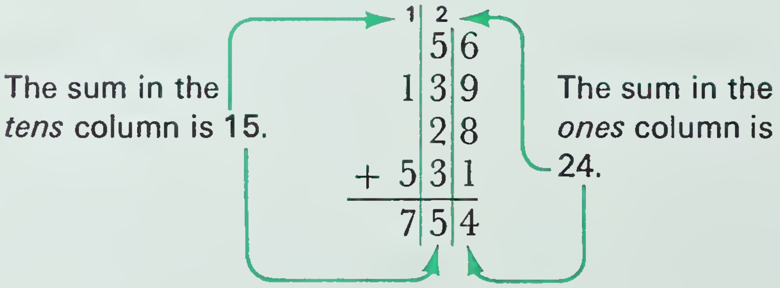
1
784
+ 253
1037

3. a. To add more than two numbers, add in any order.



6
9
8
+ 1
24
15
23
24
think
or
6
9
8
+ 1
24
9
18
24
think

b.


The sum in the *tens* column is 15.
The sum in the *ones* column is 24.

1 2
56
139
28
+ 531
754

Exercises

1–36. Add.

1. $\begin{array}{r} 32 \\ +26 \\ \hline 58 \end{array}$

2. $\begin{array}{r} 85 \\ +14 \\ \hline 99 \end{array}$

3. $\begin{array}{r} 50 \\ +27 \\ \hline 77 \end{array}$

4. $\begin{array}{r} 83 \\ +10 \\ \hline 93 \end{array}$

5. $\begin{array}{r} 256 \\ +523 \\ \hline 779 \end{array}$

6. $\begin{array}{r} 631 \\ +157 \\ \hline 788 \end{array}$

7. $\begin{array}{r} 934 \\ +\ \ 25 \\ \hline 959 \end{array}$

8. $\begin{array}{r} 16 \\ +482 \\ \hline 498 \end{array}$

9. $\begin{array}{r} 67 \\ +23 \\ \hline 90 \end{array}$

10. $\begin{array}{r} 37 \\ +15 \\ \hline 52 \end{array}$

11. $\begin{array}{r} 38 \\ +87 \\ \hline 125 \end{array}$

12. $\begin{array}{r} 97 \\ +88 \\ \hline 185 \end{array}$

13. $\begin{array}{r} 352 \\ +629 \\ \hline 981 \end{array}$

14. $\begin{array}{r} 587 \\ +932 \\ \hline 1519 \end{array}$

15. $\begin{array}{r} 747 \\ +163 \\ \hline 910 \end{array}$

16. $\begin{array}{r} 897 \\ +498 \\ \hline 1395 \end{array}$

17. $\begin{array}{r} 438 \\ 278 \\ +224 \\ \hline 940 \end{array}$

18. $\begin{array}{r} 289 \\ 632 \\ +523 \\ \hline 1444 \end{array}$

19. $\begin{array}{r} 823 \\ 432 \\ +746 \\ \hline 2001 \end{array}$

20. $\begin{array}{r} 727 \\ 105 \\ +367 \\ \hline 1199 \end{array}$

21. $\begin{array}{r} 8742 \\ 5631 \\ 9214 \\ +7580 \\ \hline 31\,167 \end{array}$

22. $\begin{array}{r} 9461 \\ 8394 \\ 2046 \\ +3774 \\ \hline 23\,675 \end{array}$

23. $\begin{array}{r} 5674 \\ 8936 \\ 6248 \\ +2995 \\ \hline 23\,853 \end{array}$

24. $\begin{array}{r} 6363 \\ 8437 \\ 9273 \\ +1307 \\ \hline 25\,380 \end{array}$

25. $\begin{array}{r} 5372 \\ 436 \\ 95 \\ +3724 \\ \hline 9627 \end{array}$

26. $\begin{array}{r} 378 \\ 4968 \\ 27 \\ +1201 \\ \hline 6574 \end{array}$

27. $\begin{array}{r} 56 \\ 325 \\ 5100 \\ +9519 \\ \hline 15\,000 \end{array}$

28. $\begin{array}{r} 466 \\ 8051 \\ 7989 \\ +\ \ \ 78 \\ \hline 16\,584 \end{array}$

29. 980 + 970 + 20 + 30
2000

30. 8526 + 387 + 143 + 97
9153

31. 151 + 152 + 153 + 1540
1996

32. 28 + 4388 + 192 + 650
5258

33. 10 + 20 + 30 + 40
100

34. 135 + 175 + 45 + 65
420

35. 7000 + 4 + 30 + 500
7534

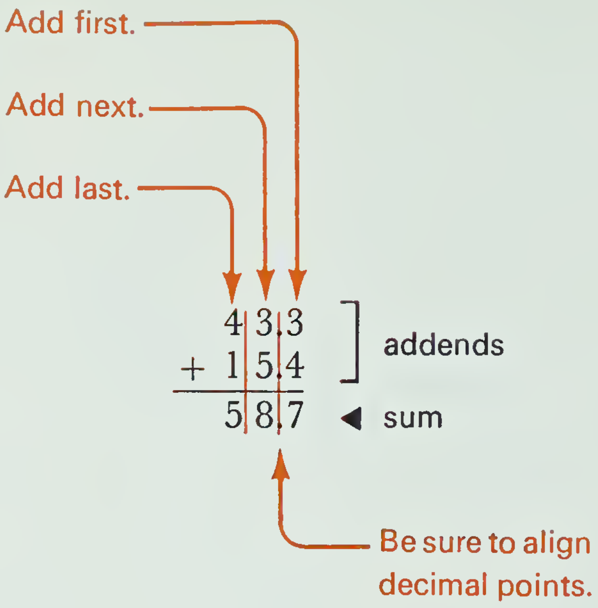
36. 48 + 49 + 51 + 52
200

6 Adding Decimals

Examples:

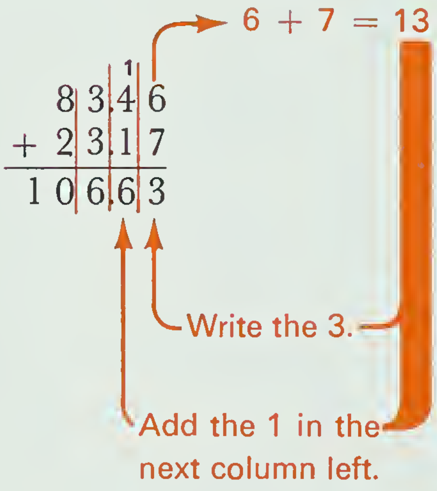
Performance Objective:
The student can
- add two or more decimals.
- add decimals and whole numbers.

1. 43.3 + 15.4



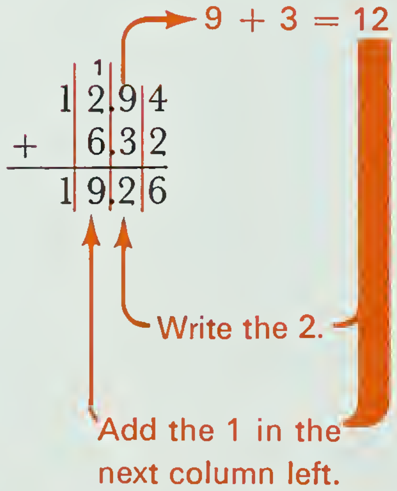
Add first.
Add next.
Add last.
4 3.3
+ 1 5.4
5 8.7
addends
sum
Be sure to align decimal points.

2. a.



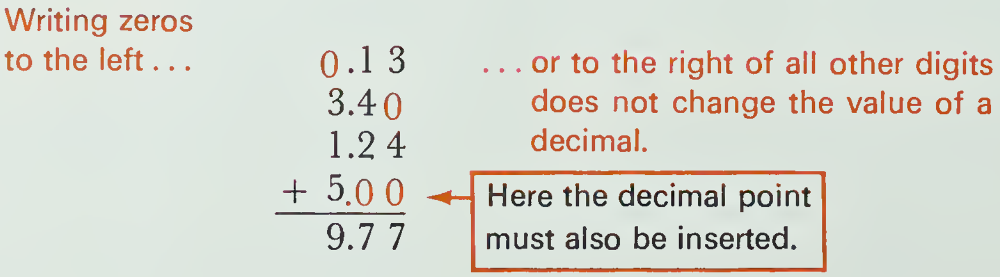
6 + 7 = 13
1
8 3.4 6
+ 2 3.1 7
1 0 6.6 3
Write the 3.
Add the 1 in the next column left.

b.



9 + 3 = 12
1
1 2.9 4
+ 6.3 2
1 9.2 6
Write the 2.
Add the 1 in the next column left.

3. .13 + 3.4 + 1.24 + 5



Writing zeros to the left . . .
0.1 3
3.4 0
1.2 4
+ 5.0 0
9.7 7
. . . or to the right of all other digits does not change the value of a decimal.
Here the decimal point must also be inserted.

Exercises

1–38. Add.

1. $\begin{array}{r} 4.8 \\ +5.1 \\ \hline \end{array}$ 9.9

2. $\begin{array}{r} 23.3 \\ +74.5 \\ \hline \end{array}$ 97.8

3. $\begin{array}{r} 71.26 \\ +27.43 \\ \hline \end{array}$ 98.69

4. $\begin{array}{r} 25.04 \\ +33.65 \\ \hline \end{array}$ 58.69

5. $\begin{array}{r} 3.264 \\ +2.735 \\ \hline \end{array}$ 5.999

6. $\begin{array}{r} 8.041 \\ +3.507 \\ \hline \end{array}$ 11.548

7. $\begin{array}{r} 25.001 \\ +\ \ 3.063 \\ \hline \end{array}$ 28.064

8. $\begin{array}{r} 7.42 \\ +32.43 \\ \hline \end{array}$ 39.85

9. $\begin{array}{r} 2.39 \\ +5.12 \\ \hline \end{array}$ 7.51

10. $\begin{array}{r} 3.58 \\ +9.24 \\ \hline \end{array}$ 12.82

11. $\begin{array}{r} 1.038 \\ +4.698 \\ \hline \end{array}$ 5.736

12. $\begin{array}{r} 0.6946 \\ +0.1398 \\ \hline \end{array}$ 0.8344

13. $\begin{array}{r} 5.7 \\ +3.6 \\ \hline \end{array}$ 9.3

14. $\begin{array}{r} 11.85 \\ +\ \ 4.93 \\ \hline \end{array}$ 16.78

15. $\begin{array}{r} 24.63 \\ +152.74 \\ \hline \end{array}$ 177.37

16. $\begin{array}{r} 13.95 \\ +\ \ 2.47 \\ \hline \end{array}$ 16.42

17. $\begin{array}{r} 39.48 \\ +65.75 \\ \hline \end{array}$ 105.23

18. $\begin{array}{r} 8.743 \\ +7.999 \\ \hline \end{array}$ 16.742

19. $\begin{array}{r} 0.5874 \\ +0.8357 \\ \hline \end{array}$ 1.4231

20. $\begin{array}{r} 14.937 \\ +\ \ 9.288 \\ \hline \end{array}$ 24.225

21. 7.38 + 8.42 15.79

22. 3.32 + 4.53 7.85

23. 7.603 + 1.309 8.912

24. 2.4 + 1.38 3.78

25. 12.601 + 0.13 12.731

26. 4 + 13.88 17.88

27. $\begin{array}{r} 1.3 \\ 2.1 \\ +6.5 \\ \hline \end{array}$ 9.9

28. $\begin{array}{r} 4.63 \\ 1.10 \\ +3.24 \\ \hline \end{array}$ 8.97

29. $\begin{array}{r} 28.081 \\ 7.315 \\ +\ \ 3.139 \\ \hline \end{array}$ 38.535

30. $\begin{array}{r} 0.3104 \\ 1.7092 \\ +4.1908 \\ \hline \end{array}$ 6.2104

31. $\begin{array}{r} 6.2 \\ 1.305 \\ +12.06 \\ \hline \end{array}$ 19.565

32. $\begin{array}{r} 8.47 \\ 9.1 \\ +0.053 \\ \hline \end{array}$ 17.623

33. $\begin{array}{r} 5.999 \\ 3 \\ +2.01 \\ \hline \end{array}$ 11.009

34. $\begin{array}{r} 6.805 \\ 0.373 \\ +12.006 \\ \hline \end{array}$ 19.184

35. 6.20 + 3.95 + 1.81 + 5.04 17.00 or 17

36. 21.3 + 8.9 + 112.6 + 13.5 156.3

37. 8.47 + 9 + 2.3 + 0.04 19.81

38. 7.234 + 0.096 + 12 + 13.2 32.53

7

Subtracting Whole Numbers

Performance Objective:
The student can
- subtract one whole number from another.
- check subtraction by adding.

Subtraction and addition are related operations. So addition facts are used to do subtraction, and addition is used to check subtraction.

$$\begin{array}{lrl} & 5 & \leftarrow \text{sum} \\ & -\,2 & \rbrack\ \text{addends} \\ \cline{2-2} \text{difference} \rightarrow & 3 & \rbrack \end{array}$$

Examples:

Subtract first.

Subtract next.

Subtract last.

1. 984 − 351

$$\begin{array}{r|c|c} 9 & 8 & 4 \\ -\ 3 & 5 & 1 \\ \hline 6 & 3 & 3 \end{array}$$

CHECK: Add; the sum is 984.

Be sure to align *ones*, *tens*, *hundreds*, etc.

2.

$$\begin{array}{r} 5\,4\,6 \\ -\quad 2\,3 \\ \hline \end{array}$$

Think

$$\begin{array}{r|c|c} 5 & 4 & 6 \\ -\ 0 & 2 & 3 \\ \hline 5 & 2 & 3 \end{array}$$

3.

$$\begin{array}{r} 8\,6\,4 \\ -\ 2\,3\,7 \\ \hline \end{array}$$

4 is smaller than 7, so rewrite 864 as

$$\begin{array}{r|c|c} 8 & \overset{5}{\cancel{6}} & \overset{14}{\cancel{4}} \\ -\ 2 & 3 & 7 \\ \hline 6 & 2 & 7 \end{array}$$

CHECK: Add; the sum is 864.

4.

1st rewrite

2nd rewrite

$$\begin{array}{r} 9\,2\,1 \\ -\ 3\,7\,5 \\ \hline \end{array} \qquad \begin{array}{r|c|c} 9 & \overset{1}{\cancel{2}} & \overset{11}{\cancel{1}} \\ -\ 3 & 7 & 5 \\ \hline & & 6 \end{array} \qquad \begin{array}{r|c|c} \overset{8}{\cancel{9}} & \overset{\overset{11}{\cancel{1}}}{\cancel{2}} & \overset{11}{\cancel{1}} \\ -\ 3 & 7 & 5 \\ \hline 5 & 4 & 6 \end{array}$$

CHECK: Add; the sum is 921.

5.

$$\begin{array}{r} 3\,0\,2 \\ -\ 1\,8\,7 \\ \hline \end{array} \qquad \begin{array}{r|c|c} \overset{2}{\cancel{3}} & \overset{10}{\cancel{0}} & 2 \\ -\ 1 & 8 & 7 \\ \hline & & \end{array} \qquad \begin{array}{r|c|c} \overset{2}{\cancel{3}} & \overset{\overset{9}{\cancel{10}}}{\cancel{0}} & \overset{12}{\cancel{2}} \\ -\ 1 & 8 & 7 \\ \hline 1 & 1 & 5 \end{array}$$

CHECK: Add; the sum is 302.

Exercises

1–46. Subtract. Check by adding.

1. $\begin{array}{r} 98 \\ -71 \\ \hline 27 \end{array}$ **2.** $\begin{array}{r} 54 \\ -14 \\ \hline 40 \end{array}$ **3.** $\begin{array}{r} 87 \\ -\ 5 \\ \hline 82 \end{array}$ **4.** $\begin{array}{r} 33 \\ -20 \\ \hline 13 \end{array}$

5. $\begin{array}{r} 536 \\ -\ 25 \\ \hline 511 \end{array}$ **6.** $\begin{array}{r} 974 \\ -574 \\ \hline 400 \end{array}$ **7.** $\begin{array}{r} 986 \\ -700 \\ \hline 286 \end{array}$ **8.** $\begin{array}{r} 575 \\ -302 \\ \hline 273 \end{array}$

9. $\begin{array}{r} 286 \\ -147 \\ \hline 139 \end{array}$ **10.** $\begin{array}{r} 836 \\ -627 \\ \hline 209 \end{array}$ **11.** $\begin{array}{r} 377 \\ -148 \\ \hline 229 \end{array}$ **12.** $\begin{array}{r} 957 \\ -328 \\ \hline 629 \end{array}$

13. $\begin{array}{r} 543 \\ -271 \\ \hline 272 \end{array}$ **14.** $\begin{array}{r} 921 \\ -730 \\ \hline 191 \end{array}$ **15.** $\begin{array}{r} 876 \\ -584 \\ \hline 292 \end{array}$ **16.** $\begin{array}{r} 317 \\ -125 \\ \hline 192 \end{array}$

17. $\begin{array}{r} 845 \\ -546 \\ \hline 299 \end{array}$ **18.** $\begin{array}{r} 217 \\ -138 \\ \hline 79 \end{array}$ **19.** $\begin{array}{r} 630 \\ -343 \\ \hline 287 \end{array}$ **20.** $\begin{array}{r} 809 \\ -198 \\ \hline 611 \end{array}$

21. $\begin{array}{r} 894 \\ -\ 68 \\ \hline 826 \end{array}$ **22.** $\begin{array}{r} 672 \\ -\ 69 \\ \hline 603 \end{array}$ **23.** $\begin{array}{r} 431 \\ -\ 89 \\ \hline 342 \end{array}$ **24.** $\begin{array}{r} 926 \\ -\ 70 \\ \hline 856 \end{array}$

25. $\begin{array}{r} 7985 \\ -6431 \\ \hline 1554 \end{array}$ **26.** $\begin{array}{r} 5864 \\ -3721 \\ \hline 2143 \end{array}$ **27.** $\begin{array}{r} 6479 \\ -2436 \\ \hline 4043 \end{array}$ **28.** $\begin{array}{r} 8391 \\ -6171 \\ \hline 2220 \end{array}$

29. $\begin{array}{r} 8721 \\ -5688 \\ \hline 3033 \end{array}$ **30.** $\begin{array}{r} 9437 \\ -6291 \\ \hline 3146 \end{array}$ **31.** $\begin{array}{r} 8341 \\ -6373 \\ \hline 1968 \end{array}$ **32.** $\begin{array}{r} 7431 \\ -3242 \\ \hline 4189 \end{array}$

33. $\begin{array}{r} 9498 \\ -3789 \\ \hline 5709 \end{array}$ **34.** $\begin{array}{r} 3347 \\ -2163 \\ \hline 1184 \end{array}$ **35.** $\begin{array}{r} 8936 \\ -3747 \\ \hline 5189 \end{array}$ **36.** $\begin{array}{r} 8193 \\ -2385 \\ \hline 5808 \end{array}$

37. $\begin{array}{r} 52\ 439 \\ -38\ 849 \\ \hline 13\ 590 \end{array}$ **38.** $\begin{array}{r} 89\ 732 \\ -61\ 423 \\ \hline 28\ 309 \end{array}$ **39.** $\begin{array}{r} 72\ 891 \\ -\ 3\ 493 \\ \hline 69\ 398 \end{array}$ **40.** $\begin{array}{r} 63\ 987 \\ -23\ 998 \\ \hline 39\ 989 \end{array}$

41. 6431 – 4926 1505 **42.** 501 – 368 133 **43.** 1304 – 725 579

44. 1793 – 241 1552 **45.** 4826 – 510 4316 **46.** 5800 – 1492 4308

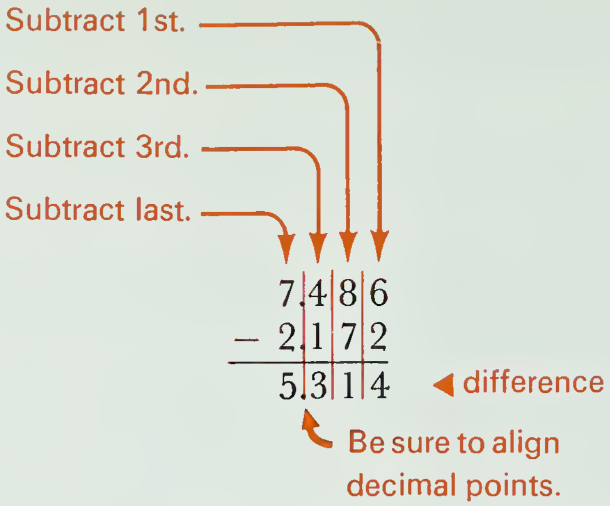
8

Subtracting Decimals

Performance Objective:
The student can
- subtract one decimal from another.
- subtract a whole number from a decimal and vice versa.

Examples:

1. 7.486 − 2.172



2.

$$\begin{array}{r} 87.83 \\ -\,14.57 \\ \hline \end{array}$$

3 is smaller than 7, so rewrite 87.83 as

$$\begin{array}{r} 8\,7.\overset{7}{\cancel{8}}\,\overset{13}{\cancel{3}} \\ -\;1\,4.5\,7 \\ \hline 7\,3.2\,6 \end{array}$$

3.

$$\begin{array}{r} 185.31 \\ -\,173.59 \\ \hline \end{array}$$

1st rewrite

$$\begin{array}{r} 1\,8\,5.\overset{2}{\cancel{3}}\,\overset{11}{\cancel{1}} \\ -\;1\,7\,3.5\,9 \\ \hline 2 \end{array}$$

2nd rewrite

$$\begin{array}{r} 1\,8\,\overset{4}{\cancel{5}}.\overset{12}{\cancel{\overset{2}{\cancel{3}}}}\,\overset{11}{\cancel{1}} \\ -\;1\,7\,3.5\,9 \\ \hline 1\,1.7\,2 \end{array}$$

4.

$$\begin{array}{r} 4.3\,6 \\ -\;2.0\,0 \\ \hline 2.3\,6 \end{array}$$

Writing zeros
to the right or to the left
of all other digits does
not change the value
of a decimal.

$$\begin{array}{r} 3\,1.0\,0 \\ -\;0\,0.2\,4 \\ \hline 3\,0.7\,6 \end{array}$$

Exercises

1–60. Subtract.

1. $\begin{array}{r} 6.95 \\ -\,3.21 \\ \hline 3.74 \end{array}$ **2.** $\begin{array}{r} 7.84 \\ -\,6.31 \\ \hline 1.53 \end{array}$ **3.** $\begin{array}{r} 6.652 \\ -\,5.431 \\ \hline 1.221 \end{array}$ **4.** $\begin{array}{r} 71.5 \\ -\,31.2 \\ \hline 40.3 \end{array}$

5. $\begin{array}{r} 8.649 \\ -\,6.517 \\ \hline 2.132 \end{array}$ **6.** $\begin{array}{r} 29.93 \\ -\,15.72 \\ \hline 14.21 \end{array}$ **7.** $\begin{array}{r} 375.8 \\ -\,251.7 \\ \hline 124.1 \end{array}$ **8.** $\begin{array}{r} 0.8463 \\ -\,0.2431 \\ \hline 0.6122 \end{array}$

9. $\begin{array}{r} 8.75 \\ -8.31 \\ \hline 0.44 \end{array}$

10. $\begin{array}{r} 3.64 \\ -3.61 \\ \hline 0.03 \end{array}$

11. $\begin{array}{r} 5.71 \\ -5.61 \\ \hline \text{0.10 or 0.1} \end{array}$

12. $\begin{array}{r} 6.23 \\ -5.23 \\ \hline \text{1.00 or 1} \end{array}$

13. $\begin{array}{r} 8.91 \\ -3.43 \\ \hline 5.48 \end{array}$

14. $\begin{array}{r} 98.74 \\ -37.25 \\ \hline 61.49 \end{array}$

15. $\begin{array}{r} 5.73 \\ -3.29 \\ \hline 2.44 \end{array}$

16. $\begin{array}{r} 28.6 \\ -26.9 \\ \hline 1.7 \end{array}$

17. $\begin{array}{r} 8.36 \\ -6.43 \\ \hline 1.93 \end{array}$

18. $\begin{array}{r} 36.53 \\ -12.63 \\ \hline \text{23.90 or 23.9} \end{array}$

19. $\begin{array}{r} 3.817 \\ -2.143 \\ \hline 1.674 \end{array}$

20. $\begin{array}{r} 894.7 \\ -736.5 \\ \hline 158.2 \end{array}$

21. $\begin{array}{r} 6.321 \\ -3.142 \\ \hline 3.179 \end{array}$

22. $\begin{array}{r} 35.437 \\ -21.358 \\ \hline 14.079 \end{array}$

23. $\begin{array}{r} 394.75 \\ -142.87 \\ \hline 251.88 \end{array}$

24. $\begin{array}{r} 15.32 \\ -12.97 \\ \hline 2.35 \end{array}$

25. $\begin{array}{r} 821.6 \\ -351.7 \\ \hline 469.9 \end{array}$

26. $\begin{array}{r} 286.78 \\ -175.99 \\ \hline 110.79 \end{array}$

27. $\begin{array}{r} 8.3654 \\ -6.5865 \\ \hline 1.7789 \end{array}$

28. $\begin{array}{r} 894.71 \\ -156.38 \\ \hline 738.33 \end{array}$

29. $\begin{array}{r} 17.412 \\ -13.646 \\ \hline 3.766 \end{array}$

30. $\begin{array}{r} 194.28 \\ -189.49 \\ \hline 4.79 \end{array}$

31. $\begin{array}{r} 34.731 \\ -33.845 \\ \hline 0.886 \end{array}$

32. $\begin{array}{r} 6.4331 \\ -6.4229 \\ \hline 0.0102 \end{array}$

33. $\begin{array}{r} 6.38 \\ -0.24 \\ \hline 6.14 \end{array}$

34. $\begin{array}{r} 27.35 \\ -\ \ 3.12 \\ \hline 24.23 \end{array}$

35. $\begin{array}{r} 134.81 \\ -\ \ \ 3.71 \\ \hline \text{131.10 or 131.1} \end{array}$

36. $\begin{array}{r} 253.6 \\ -\ \ 23.4 \\ \hline 230.2 \end{array}$

37. $\begin{array}{r} 4.273 \\ -0.026 \\ \hline 4.247 \end{array}$

38. $\begin{array}{r} 1.328 \\ -0.246 \\ \hline 1.082 \end{array}$

39. $\begin{array}{r} 29.4 \\ -\ \ 0.8 \\ \hline 28.6 \end{array}$

40. $\begin{array}{r} 84.72 \\ -\ \ 8.32 \\ \hline \text{76.40 or 76.4} \end{array}$

41. $\begin{array}{r} 76.31 \\ -\ \ 1.53 \\ \hline 74.78 \end{array}$

42. $\begin{array}{r} 9.215 \\ -0.037 \\ \hline 9.178 \end{array}$

43. $\begin{array}{r} 130.6 \\ -\ \ \ 0.8 \\ \hline 129.8 \end{array}$

44. $\begin{array}{r} 13.04 \\ -\ \ 7.81 \\ \hline 5.23 \end{array}$

45. $\begin{array}{r} 132.65 \\ -\ \ 11.68 \\ \hline 120.97 \end{array}$

46. $\begin{array}{r} 461.34 \\ -\ \ \ 1.34 \\ \hline \text{460.00 or 460} \end{array}$

47. $\begin{array}{r} 827.34 \\ -\ \ 98.56 \\ \hline 728.78 \end{array}$

48. $\begin{array}{r} 477.81 \\ -\ \ 87.95 \\ \hline 389.86 \end{array}$

49. 189.35 − 6.4
182.95

50. 67.3 − 0.592
66.708

51. 223 − 1.1
221.9

52. 27.32 − 0.21
27.11

53. 342.05 − 0.01
342.04

54. 200 − 199.9
0.1

55. 100 − 0.9
99.1

56. 60 − 0.01
59.99

57. 13.46 − 2
11.46

58. 136.2 − 9
127.2

59. 463.25 − 463
0.25

60. 1.1375 − 0.2
0.9375

9

Practice Adding and Subtracting (whole numbers and decimals)

Performance Objective: The student practices adding or subtracting whole numbers and decimals.

1–44. Add.

1. $\begin{array}{r} 34 \\ +55 \\ \hline 89 \end{array}$ **2.** $\begin{array}{r} 30 \\ +67 \\ \hline 97 \end{array}$ **3.** $\begin{array}{r} 8.4 \\ +1.3 \\ \hline 9.7 \end{array}$ **4.** $\begin{array}{r} 21 \\ +56 \\ \hline 77 \end{array}$

5. $\begin{array}{r} 182 \\ +417 \\ \hline 599 \end{array}$ **6.** $\begin{array}{r} 295 \\ +304 \\ \hline 599 \end{array}$ **7.** $\begin{array}{r} 26.1 \\ +42.3 \\ \hline 68.4 \end{array}$ **8.** $\begin{array}{r} 1.22 \\ +7.15 \\ \hline 8.37 \end{array}$

9. $\begin{array}{r} 541 \\ +45 \\ \hline 586 \end{array}$ **10.** $\begin{array}{r} 0.23 \\ +1.73 \\ \hline 1.96 \end{array}$ **11.** $\begin{array}{r} 40.2 \\ +8.2 \\ \hline 48.4 \end{array}$ **12.** $\begin{array}{r} 310 \\ +16 \\ \hline 326 \end{array}$

13. $\begin{array}{r} 31.9 \\ +3.8 \\ \hline 35.7 \end{array}$ **14.** $\begin{array}{r} 827 \\ +29 \\ \hline 856 \end{array}$ **15.** $\begin{array}{r} 5.66 \\ +0.27 \\ \hline 5.93 \end{array}$ **16.** $\begin{array}{r} 757 \\ +27 \\ \hline 784 \end{array}$

17. $\begin{array}{r} 9.94 \\ +0.64 \\ \hline 10.58 \end{array}$ **18.** $\begin{array}{r} 75.4 \\ +9.5 \\ \hline 84.9 \end{array}$ **19.** $\begin{array}{r} 86 \\ +580 \\ \hline 666 \end{array}$ **20.** $\begin{array}{r} 97 \\ +842 \\ \hline 939 \end{array}$

21. $\begin{array}{r} 89 \\ +53 \\ \hline 142 \end{array}$ **22.** $\begin{array}{r} 6.7 \\ +6.3 \\ \hline 13.0 \text{ or } 13 \end{array}$ **23.** $\begin{array}{r} 489 \\ +63 \\ \hline 552 \end{array}$ **24.** $\begin{array}{r} 97.3 \\ +5.3 \\ \hline 102.6 \end{array}$

25. $23 + 34 + 12$ 69

26. $5.2 + 0.29 + 1.3$ 6.79

27. $352 + 179 + 720$ 1251

28. $4.74 + 0.21 + 18.3$ 23.25

29. $\begin{array}{r} 13.7 \\ 21.2 \\ +98.3 \\ \hline 133.2 \end{array}$ **30.** $\begin{array}{r} 542 \\ 68 \\ +206 \\ \hline 816 \end{array}$ **31.** $\begin{array}{r} 12.14 \\ 35.21 \\ +17.58 \\ \hline 64.93 \end{array}$ **32.** $\begin{array}{r} 2617 \\ 54 \\ +319 \\ \hline 2990 \end{array}$

33. $9 + 7 + 6 + 8$ 30

34. $4 + 8 + 3 + 7$ 22

35. $0.5 + 0.13 + 7 + 2.9$ 10.53

36. $3.4 + 0.12 + 0.8 + 3$ 7.32

37. $\begin{array}{r} 13.7 \\ 70.4 \\ 32.1 \\ +91.6 \\ \hline 207.8 \end{array}$ **38.** $\begin{array}{r} 618 \\ 126 \\ 583 \\ +940 \\ \hline 2267 \end{array}$ **39.** $\begin{array}{r} 4.83 \\ 0.27 \\ 1.63 \\ +0.52 \\ \hline 7.25 \end{array}$ **40.** $\begin{array}{r} 36 \\ 152 \\ 877 \\ +91 \\ \hline 1156 \end{array}$

41. $\begin{array}{r} 15.62 \\ 38.04 \\ 92.61 \\ +15.34 \\ \hline 161.61 \end{array}$ **42.** $\begin{array}{r} 1826 \\ 8031 \\ 5100 \\ +3567 \\ \hline 18\,524 \end{array}$ **43.** $\begin{array}{r} 2830 \\ 194 \\ 6529 \\ +83 \\ \hline 9636 \end{array}$ **44.** $\begin{array}{r} 0.28 \\ 19.76 \\ 4.31 \\ +0.19 \\ \hline 24.54 \end{array}$

45–88. Subtract. Check by adding.

45. $\begin{array}{r} 95 \\ -71 \\ \hline 24 \end{array}$ **46.** $\begin{array}{r} 34 \\ -12 \\ \hline 22 \end{array}$ **47.** $\begin{array}{r} 86 \\ -34 \\ \hline 52 \end{array}$ **48.** $\begin{array}{r} 77 \\ -52 \\ \hline 25 \end{array}$

49. $\begin{array}{r} 758 \\ -513 \\ \hline 245 \end{array}$ **50.** $\begin{array}{r} 842 \\ -401 \\ \hline 441 \end{array}$ **51.** $\begin{array}{r} 39.6 \\ -17.5 \\ \hline 22.1 \end{array}$ **52.** $\begin{array}{r} 5.18 \\ -3.13 \\ \hline 2.05 \end{array}$

53. $\begin{array}{r} 832 \\ -\ 21 \\ \hline 811 \end{array}$ **54.** $\begin{array}{r} 478 \\ -\ 43 \\ \hline 435 \end{array}$ **55.** $\begin{array}{r} 9.59 \\ -0.37 \\ \hline 9.22 \end{array}$ **56.** $\begin{array}{r} 6.57 \\ -0.30 \\ \hline 6.27 \end{array}$

57. $\begin{array}{r} 363 \\ -\ 27 \\ \hline 336 \end{array}$ **58.** $\begin{array}{r} 465 \\ -239 \\ \hline 226 \end{array}$ **59.** $\begin{array}{r} 16.50 \\ -\ 4.38 \\ \hline 12.12 \end{array}$ **60.** $\begin{array}{r} 964.5 \\ -132.7 \\ \hline 831.8 \end{array}$

61. $\begin{array}{r} 427 \\ -153 \\ \hline 274 \end{array}$ **62.** $\begin{array}{r} 926 \\ -\ 74 \\ \hline 852 \end{array}$ **63.** $\begin{array}{r} 1.556 \\ -1.284 \\ \hline 0.272 \end{array}$ **64.** $\begin{array}{r} 27.32 \\ -\ 4.90 \\ \hline 22.42 \end{array}$

65. $\begin{array}{r} 534 \\ -178 \\ \hline 356 \end{array}$ **66.** $\begin{array}{r} 1325 \\ -\ 876 \\ \hline 449 \end{array}$ **67.** $\begin{array}{r} 95.0 \\ -68.3 \\ \hline 26.7 \end{array}$ **68.** $\begin{array}{r} 4.232 \\ -1.784 \\ \hline 2.448 \end{array}$

69. 9225 − 1382 7843

70. 16.41 − 8.53 7.88

71. 4820 − 835 3985

72. 292.3 − 97.6 194.7

73. $\begin{array}{r} 8.41 \\ -3 \\ \hline 5.41 \end{array}$ **74.** $\begin{array}{r} 18.24 \\ +0.63 \\ \hline 17.61 \end{array}$ **75.** $\begin{array}{r} 9.67 \\ -0.3 \\ \hline 9.37 \end{array}$ **76.** $\begin{array}{r} 14 \\ -\ 0.27 \\ \hline 13.73 \end{array}$

77. 32.7 − 4.683 28.017

78. 12.48 − 4 8.48

79. 623.58 − 0.7 622.88

80. 92 − 0.375 91.625

81. 4623 − 176 4447

82. 843.7 − 21.5 822.2

83. 2964 − 34.61 2929.39

84. 9800 − 963 8837

85. 6841 − 916 5925

86. 400 − 0.725 399.275

87. 2.432 − 2 0.432

88. 29.6832 − 0.99 28.6932

10

Multiplication Symbols and Facts

Performance Objective:
The student can
- recognize several common ways of indicating multiplication.
- recall the basic multiplication facts for whole numbers from 0 through 9.

These all mean *multiply 3 and 4.*

3 × 4   3(4) $\begin{array}{r}4\\ \times 3\\ \hline\end{array}$ $\begin{array}{r}3\\ \times 4\\ \hline\end{array}$

4 × 3   (4)(3)

3 and 4 are **factors**. Their **product** is 12.

Exercises

1–64. Multiply.

1. 5 × 2 10   **2.** 1 × 8 8   **3.** 2 × 4 8   **4.** 6 × 1 6

5. 3(6) 18   **6.** 9(4) 36   **7.** 4(6) 24   **8.** 5(8) 40

9. (2)(3) 6   **10.** (5)(7) 35   **11.** (4)(5) 20   **12.** (2)(2) 4

13. $\begin{array}{r}6\\ \times 5\\ \hline\end{array}$ 30   **14.** $\begin{array}{r}8\\ \times 9\\ \hline\end{array}$ 72   **15.** $\begin{array}{r}9\\ \times 6\\ \hline\end{array}$ 54   **16.** $\begin{array}{r}8\\ \times 7\\ \hline\end{array}$ 56

17. 9 × 0 0   **18.** 7 × 4 28   **19.** 9 × 9 81   **20.** 5 × 9 45

21. 9(7) 63   **22.** 6(7) 42   **23.** 7(7) 49   **24.** 3(8) 24

25. 9(3) 27   **26.** 1(7) 7   **27.** (7)(3) 21   **28.** 2(6) 12

29. $\begin{array}{r}8\\ \times 6\\ \hline\end{array}$ 48   **30.** $\begin{array}{r}7\\ \times 5\\ \hline\end{array}$ 35   **31.** $\begin{array}{r}3\\ \times 8\\ \hline\end{array}$ 24   **32.** $\begin{array}{r}9\\ \times 8\\ \hline\end{array}$ 72

33. 9 × 5 45   **34.** 6 × 8 48   **35.** 4 × 9 36   **36.** 7 × 6 42

37. 8(8) 64   **38.** 8(4) 32   **39.** 3(5) 15   **40.** 5(6) 30

41. $\begin{array}{r}1\\ \times 7\\ \hline\end{array}$ 7   **42.** $\begin{array}{r}7\\ \times 9\\ \hline\end{array}$ 63   **43.** $\begin{array}{r}6\\ \times 3\\ \hline\end{array}$ 18   **44.** $\begin{array}{r}3\\ \times 9\\ \hline\end{array}$ 27

45. $\begin{array}{r}4\\ \times 8\\ \hline\end{array}$ 32   **46.** $\begin{array}{r}4\\ \times 7\\ \hline\end{array}$ 28   **47.** $\begin{array}{r}7\\ \times 3\\ \hline\end{array}$ 21   **48.** $\begin{array}{r}2\\ \times 0\\ \hline\end{array}$ 0

49. 6 × 9 54   **50.** 8 × 5 40   **51.** 4 × 3 12   **52.** 7 × 8 56

53. 6(6) 36   **54.** 8(0) 0   **55.** (4)(4) 16   **56.** (6)(4) 24

57. 5 × 4 20   **58.** 3 × 3 9   **59.** (7)(2) 14   **60.** 5(5) 25

61. 5 × 3 15   **62.** 3(4) 12   **63.** (2)(9) 18   **64.** 2 × 8 16

11

Multiplying by Multiples of 10

Performance Objective: The student can multiply whole numbers by powers of ten or by multiples of ten.

Examples:

1.
$10 \times 234 = 2340$
$100 \times 234 = 23400$
$1000 \times 234 = 234000$

Write this factor followed by the same number of zeros as in this factor.

2.
$10 \times 240 = 2400$
$100 \times 240 = 24000$
$1000 \times 240 = 240000$

Write this factor followed by the same number of zeros as in this factor.

3.
$$\begin{array}{r} 900 \\ \times \quad 3 \\ \hline 2700 \end{array}$$
◀ 2 zeros
◀ +0 zeros
◀ 2 zeros

Multiply 9 × 3; then write the zeros.

4.
$$\begin{array}{r} 900 \\ \times \ 30 \\ \hline 27000 \end{array}$$
◀ 2 zeros
◀ +1 zero
◀ 3 zeros

Multiply 9 × 3; then write the zeros.

Exercises

1–48. Multiply.

1. 10 × 29 290 **2.** 10 × 87 870 **3.** 63 × 10 630 **4.** 93 × 10 930

5. 100 × 38 3800 **6.** 100 × 47 4700 **7.** 67 × 100 6700 **8.** 83 × 100 8300

9. 1000 × 91 91 000 **10.** 1000 × 74 74 000 **11.** 87 × 1000 87 000 **12.** 17 × 1000 17 000

13. 10 × 830 8300 **14.** 100 × 270 27 000 **15.** 1000 × 480 480 000 **16.** 370 × 100 37 000

17. 210 × 10 2100 **18.** 10 × 3800 38 000 **19.** 320 × 1000 320 000 **20.** 9100 × 100 910 000

21. 136 × 10 1360 **22.** 142 × 1000 142 000 **23.** 100 × 197 19 700 **24.** 1000 × 307 307 000

25. 4634 × 10 46 340 **26.** 1000 × 8245 8 245 000 **27.** 8960 × 100 896 000 **28.** 1000 × 3240 3 240 000

29. 30 × 5 150 **30.** 300 × 5 1500 **31.** 3000 × 5 15 000 **32.** 5000 × 9 45 000

33. 9 × 50 450 **34.** 8 × 400 3200 **35.** 6 × 700 4200 **36.** 7 × 7000 49 000

37. 20 × 4 80 **38.** 600 × 3 1800 **39.** 6000 × 5 30 000 **40.** 800 × 7 5600

41. 80 × 60 4800 **42.** 800 × 60 48 000 **43.** 80 × 600 48 000 **44.** 8000 × 600 4 800 000

45. 50 × 500 25 000 **46.** 700 × 40 28 000 **47.** 600 × 600 360 000 **48.** 5000 × 40 200 000

12

Multiplying Whole Numbers

Performance Objective: The student can multiply two or more whole numbers.

Examples:

1.

$$\begin{array}{r} 32 \\ \times\ 4 \\ \hline 128 \end{array}$$

4 × 30

4 × 2

2.

$$\begin{array}{rl} 24 & \\ \times\ 12 & \\ \hline 48 & 2 \times 24 \\ 240 & 10 \times 24 \\ \hline 288 & 240 + 48 \end{array}$$

3.

$$\begin{array}{rl} 32 & \\ \times\ 6 & \\ \hline 12 & 6 \times 2 \\ 180 & 6 \times 30 \\ \hline 192 & 180 + 12 \end{array}$$

or

$$\begin{array}{r} {}^{1} \\ 32 \\ \times\ 6 \\ \hline 192 \end{array}$$

← The 1 of 12 may be written here to remind you to add it to the next product.

← 6 × 2 = 12; write the 2; add the 1 to 6 × 3.

4.

$$\begin{array}{rl} 32 & \\ \times\ 46 & \\ \hline 12 & 6 \times 2 \\ 180 & 6 \times 30 \\ 80 & 40 \times 2 \\ 1200 & 40 \times 30 \\ \hline 1472 & \text{Add.} \end{array}$$

or

$$\begin{array}{rl} {}^{1} & \\ 32 & \\ \times\ 46 & \\ \hline 192 & 6 \times 32 \\ 1280 & 40 \times 32 \\ \hline 1472 & \text{Add.} \end{array}$$

5.

$$\begin{array}{rl} 234 & \\ \times\ 201 & \\ \hline 234 & 1 \times 234 \\ 0 & 0 \times 234 \\ 46800 & 200 \times 234 \\ \hline 47034 & \text{Add.} \end{array}$$

or

$$\begin{array}{rl} 234 & \\ \times\ 201 & \\ \hline 234 & 1 \times 234 \\ 46800 & 200 \times 234 \\ \hline 47034 & \text{Add.} \end{array}$$

With more than two factors, multiply in any order or grouping.

6. $5 \times 3 \times 2$ $\qquad$ $5 \times 3 \times 2$ $\qquad$ $5 \times 3 \times 2$

Think 15×2 or 5×6 or $10 \times 3 = 30$

7. $3 \times 4 \times 5 \times 6 = 360$

$$\left.\begin{array}{l} (3 \times 4) \times (5 \times 6) \\ [3 \times (4 \times 5)] \times 6 \\ [(3 \times 4) \times 5] \times 6 \\ 3 \times [4 \times (5 \times 6)] \end{array}\right\}$$

These are only a few of the ways to group the factors to multiply them. The *order* of the factors can also be changed. But the result will always be 360.

Exercises

1–52. Multiply.

1. 42 × 3 126

2. 84 × 2 168

3. 70 × 8 560

4. 21 × 9 189

5. 410 × 6 2460

6. 824 × 2 1648

7. 721 × 4 2884

8. 932 × 3 2796

9. 134 × 22 2948

10. 943 × 12 11 316

11. 834 × 21 17 514

12. 701 × 19 13 319

13. 623 × 32 19 936

14. 503 × 23 11 569

15. 311 × 89 27 679

16. 412 × 34 14 008

17. 28 × 3 84

18. 34 × 4 136

19. 48 × 3 144

20. 36 × 7 252

21. 54 × 8 432

22. 93 × 7 651

23. 89 × 8 712

24. 73 × 7 511

25. 46 × 24 1104

26. 38 × 14 532

27. 74 × 26 1924

28. 84 × 32 2688

29. 83 × 41 3403

30. 37 × 91 3367

31. 85 × 31 2635

32. 78 × 35 2730

33. 321 × 63 20 223

34. 821 × 37 30 377

35. 621 × 87 54 027

36. 531 × 98 52 038

37. 248 × 72 17 856

38. 576 × 83 47 808

39. 138 × 35 4830

40. 537 × 65 34 905

41. 823 × 87 71 601

42. 935 × 65 60 775

43. 825 × 78 64 350

44. 345 × 65 22 425

45. 876 × 302 264 552

46. 623 × 304 189 392

47. 965 × 806 777 790

48. 480 × 701 336 480

49. 5 × 12 × 8 480

50. 4 × 17 × 5 340

51. 7 × 5 × 2 × 5 350

52. 3 × 15 × 4 × 11 1980

13

Multiplying Whole Numbers and Decimals

If only one factor has digits to the right of the decimal point, the product has that same number of digits to the right of the decimal point.

Performance Objective: The student can

- multiply whole numbers and decimals, correctly placing the decimal point in the product.
- multiply a decimal by a power of ten by relocating the decimal point.

Examples:

1.

$$\begin{array}{rl} 32 & \\ \times 0.1 & \blacktriangleleft \text{tenths} \\ \cline{1-1} 3.2 & \blacktriangleleft \text{tenths} \end{array}$$

2.

$$\begin{array}{rl} 7\,2\,1 & \\ \times 0.3\,4 & \blacktriangleleft \text{hundredths} \\ \cline{1-1} 2\,8\,8\,4 & \text{no decimal} \\ 2\,1\,6\,3 & \text{points here} \\ \cline{1-1} 2\,4\,5.1\,4 & \blacktriangleleft \text{hundredths} \end{array}$$

3.

$$\begin{array}{rl} 6\,3.2 & \blacktriangleleft \text{tenths} \\ \times\ 2\,3 & \\ \cline{1-1} 1\,8\,9\,6 & \\ 1\,2\,6\,4 & \\ \cline{1-1} 1\,4\,5\,3.6 & \blacktriangleleft \text{tenths} \end{array}$$

4. $10 \times 0.\underbrace{7}5 = 7.5$

1 zero ← 1 digit

$100 \times 24.\underbrace{63} = 2463$

2 zeros ← 2 digits

$1000 \times 1.\underbrace{200} = 1200$

3 zeros ← 3 digits

Use this shortcut to multiply by 10, 100, 1000, and so on. If the product is a whole number, no decimal point is needed.

Exercises

1–6. Place the decimal point in each product.

1. $\begin{array}{r} 42 \\ \times 0.6 \\ \hline 25_{\wedge}2 \end{array}$ **2.** $\begin{array}{r} 306 \\ \times 0.08 \\ \hline 24_{\wedge}48 \end{array}$ **3.** $\begin{array}{r} 5.063 \\ \times \quad 9 \\ \hline 45_{\wedge}567 \end{array}$ **4.** $\begin{array}{r} 0.814 \\ \times \quad 40 \\ \hline 32_{\wedge}560 \end{array}$

5. $10 \times 5.41 = 54_{\wedge}1$

6. $963. \times 1000 = 963{,}000$

7–24. Multiply.

7. 579×0.05 28.95

8. 8324×0.5 4162

9. 73.49×3 220.47

10. 6×9.03 54.18

11. 0.1×1246 124.6

12. $450(0.01)$ 4.5

13. 1628×2.3 3744.4

14. 843×0.61 514.23

15. $16.3(12)$ 195.6

16. $594(10.3)$ 6118.2

17. 6.34×123 779.82

18. 6.864×5 34.32

19. 10×7.412 74.12

20. 100×0.80 80

21. 9.8×100 980

22. 6.9×100 690

23. 50.24×20 1004.8

24. 0.001×90 0.09

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Multiplying Decimals by Decimals

Performance Objective: The student can multiply decimals, correctly placing the decimal point in the product.

Examples:

1.

$$\begin{array}{r l}
4.\overbrace{3} & \blacktriangleleft\quad 1\text{ digit}\\
\times\ 0.\overbrace{1} & \blacktriangleleft\quad +1\text{ digit}\\
\hline
0.\underbrace{43} & \blacktriangleleft\quad 2\text{ digits}
\end{array}$$

no decimal points used here

2.

$$\begin{array}{r l}
36.\overbrace{82} & \blacktriangleleft\quad 2\text{ digits}\\
\times\ 0.\overbrace{111} & \blacktriangleleft\quad +3\text{ digits}\\
\hline
3682 & \\
3682 & \\
3682 & \\
\hline
4.\underbrace{08702} & \blacktriangleleft\quad 5\text{ digits}
\end{array}$$

Count the total number of digits to the right of the decimal points in both factors. The product has this many digits to the right of the decimal point.

Exercises

1–8. Place the decimal point in each product. Write additional zeros if necessary.

1. $\begin{array}{r} 92.3\\ \times\ 6.2\\ \hline 572_{\wedge}26 \end{array}$ **2.** $\begin{array}{r} 6.25\\ \times\ 0.04\\ \hline {}_{\wedge}2500 \end{array}$ **3.** $\begin{array}{r} 10.003\\ \times\ 0.086\\ \hline {}_{\wedge}860258 \end{array}$ **4.** $\begin{array}{r} 0.942\\ \times\ 0.103\\ \hline {}_{\wedge}097026 \end{array}$

5. $\begin{array}{r} 3.786\\ \times\ 4.1\\ \hline 15_{\wedge}5226 \end{array}$ **6.** $\begin{array}{r} 38.14\\ \times\ 0.002\\ \hline {}_{\wedge}07628 \end{array}$ **7.** $\begin{array}{r} 0.0004\\ \times\ 275.1\\ \hline {}_{\wedge}11004 \end{array}$ **8.** $\begin{array}{r} 1.626\\ \times\ 0.05\\ \hline {}_{\wedge}08130 \end{array}$

9–26. Multiply.

9. $\begin{array}{r} 10.5\\ \times\ 7.6\\ \hline 79.8 \end{array}$ **10.** $\begin{array}{r} 8.37\\ \times\ 0.42\\ \hline 3.5154 \end{array}$ **11.** $\begin{array}{r} 4.17\\ \times\ 2.8\\ \hline 11.676 \end{array}$ **12.** $\begin{array}{r} 2.831\\ \times\ 4.2\\ \hline 11.8902 \end{array}$

13. $\begin{array}{r} 0.731\\ \times\ 0.86\\ \hline 0.628\ 66 \end{array}$ **14.** $\begin{array}{r} 0.4321\\ \times\ 0.8\\ \hline 0.345\ 68 \end{array}$ **15.** $\begin{array}{r} 18.73\\ \times\ 9.20\\ \hline 172.3160 \end{array}$ **16.** $\begin{array}{r} 0.1563\\ \times\ 1.606\\ \hline 0.251\ 017\ 8 \end{array}$

17 $\begin{array}{r} 32.83\\ \times\ 0.009\\ \hline 0.295\ 47 \end{array}$ **18.** $\begin{array}{r} 3.016\\ \times\ 0.07\\ \hline 0.211\ 12 \end{array}$ **19.** $\begin{array}{r} 88.01\\ \times\ 0.005\\ \hline 0.44\ 05 \end{array}$ **20.** $\begin{array}{r} 147.6\\ \times\ 0.0004\\ \hline 0.059\ 04 \end{array}$

21. 12.65(3.11)
39.3415

22. 2.8×0.0093
0.026 04

23. 153.6×6.04
927.744

24. (4.021)(2)(9.6)
77.2032

25. (3)(0.0008)(52.36
0.125 664

26. (99.9)(5)(1.11)
554.445

15

Division Symbols and Facts

Performance Objective:
The student can
- recognize several common ways of indicating division.
- recall the basic division facts for whole numbers from 0 through 9.
- check division by multiplying.

These all mean *divide 12 by 3.* $3\overline{)12}$ $12 \div 3$ $\frac{12}{3}$

Division and multiplication are related operations. So multiplication facts are used to do division, and multiplication is used to check division.

4 ◀ quotient
divisor ▶ $3\overline{)12}$ ◀ dividend

4] factors
$\times 3$
12 ◀ product

Exercises

1–60. Divide.

1. $5\overline{)10}$ 2 **2.** $6\overline{)48}$ 8 **3.** $9\overline{)45}$ 5 **4.** $8\overline{)24}$ 3
5. $21 \div 3$ 7 **6.** $72 \div 9$ 8 **7.** $36 \div 4$ 9 **8.** $8 \div 4$ 2
9. $\frac{14}{2}$ 7 **10.** $\frac{30}{5}$ 6 **11.** $\frac{28}{4}$ 7 **12.** $\frac{35}{7}$ 5
13. $8\overline{)40}$ 5 **14.** $3\overline{)27}$ 9 **15.** $6\overline{)24}$ 4 **16.** $5\overline{)20}$ 4
17. $18 \div 6$ 3 **18.** $64 \div 8$ 8 **19.** $36 \div 9$ 4 **20.** $42 \div 6$ 7
21. $\frac{32}{4}$ 8 **22.** $\frac{18}{3}$ 6 **23.** $\frac{24}{4}$ 6 **24.** $\frac{18}{2}$ 9
25. $5\overline{)15}$ 3 **26.** $3\overline{)12}$ 4 **27.** $4\overline{)16}$ 4 **28.** $3\overline{)24}$ 8
29. $\frac{25}{5}$ 5 **30.** $\frac{14}{7}$ 2 **31.** $\frac{16}{8}$ 2 **32.** $\frac{5}{5}$ 1
33. $9 \div 3$ 3 **34.** $12 \div 4$ 3 **35.** $28 \div 7$ 4 **36.** $63 \div 9$ 7
37. $6\overline{)36}$ 6 **38.** $9\overline{)27}$ 3 **39.** $6\overline{)30}$ 5 **40.** $7\overline{)56}$ 8
41. $\frac{9}{1}$ 9 **42.** $\frac{45}{5}$ 9 **43.** $\frac{32}{8}$ 4 **44.** $\frac{15}{3}$ 5
45. $12 \div 6$ 2 **46.** $48 \div 8$ 6 **47.** $54 \div 9$ 6 **48.** $7 \div 1$ 7
49. $\frac{63}{7}$ 9 **50.** $\frac{21}{7}$ 3 **51.** $\frac{40}{5}$ 8 **52.** $\frac{81}{9}$ 9
53. $7\overline{)49}$ 7 **54.** $6\overline{)54}$ 9 **55.** $7\overline{)42}$ 6 **56.** $8\overline{)72}$ 9
57. $18 \div 9$ 2 **58.** $35 \div 5$ 7 **59.** $56 \div 8$ 7 **60.** $20 \div 4$ 5

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Remainders

Performance Objective: The student can
- find the quotient and remainder for whole-number divisors and quotients from 0 through 9.
- check division by multiplying and then adding the remainder.

Example: $6\overline{)51}$ ◀ Since $6 \times 8 = 48$ and $6 \times 9 = 54$, $6\overline{)51}$ is between 8 and 9.

$$\begin{array}{rrl} \text{Write the 8.} \blacktriangleright & 8 & \\ & 6\overline{)51} & \\ \text{Multiply } 6 \times 8. \blacktriangleright & \underline{48} & \\ \text{Subtract } 51 - 48. \blacktriangleright & 3 & \blacktriangleleft \text{Remainder} \end{array}$$

That is,

$$\begin{array}{r} 8\ \text{R3} \\ 6\overline{)51} \end{array}$$

Note that the remainder is *always* less than the divisor.

CHECK:

$$\begin{array}{rl} 6 & \triangleleft \text{ divisor} \\ \underline{\times 8} & \triangleleft \text{ quotient} \\ 48 & \\ \underline{+\ \ 3} & \triangleleft \text{ remainder} \\ 51 & \triangleleft \text{ dividend} \end{array}$$

Exercises

1–52. Divide. Check your answers.

1. $3\overline{)29}$ 9 R2 **2.** $5\overline{)38}$ 7 R3 **3.** $6\overline{)43}$ 7 R1 **4.** $3\overline{)11}$ 3 R2

5. $8\overline{)60}$ 7 R4 **6.** $4\overline{)39}$ 9 R3 **7.** $3\overline{)20}$ 6 R2 **8.** $4\overline{)25}$ 6 R1

9. $27 \div 6$ 4 R3 **10.** $10 \div 4$ 2 R2 **11.** $15 \div 4$ 3 R3 **12.** $23 \div 5$ 4 R3

13. $\frac{32}{7}$ 4 R4 **14.** $\frac{17}{5}$ 3 R2 **15.** $\frac{19}{7}$ 2 R5 **16.** $\frac{34}{6}$ 5 R4

17. $8\overline{)19}$ 2 R3 **18.** $2\overline{)15}$ 7 R1 **19.** $7\overline{)24}$ 3 R3 **20.** $5\overline{)49}$ 9 R4

21. $11 \div 5$ 2 R1 **22.** $29 \div 4$ 7 R1 **23.** $17 \div 2$ 8 R1 **24.** $16 \div 6$ 2 R4

25. $\frac{20}{9}$ 2 R2 **26.** $\frac{37}{7}$ 5 R2 **27.** $\frac{35}{8}$ 4 R3 **28.** $\frac{23}{6}$ 3 R5

29. $5\overline{)41}$ 8 R1 **30.** $8\overline{)50}$ 6 R2 **31.** $9\overline{)31}$ 3 R4 **32.** $6\overline{)59}$ 9 R5

33. $32 \div 5$ 6 R2 **34.** $16 \div 3$ 5 R1 **35.** $19 \div 4$ 4 R3 **36.** $41 \div 8$ 5 R1

37. $\frac{30}{8}$ 3 R6 **38.** $\frac{22}{4}$ 5 R2 **39.** $\frac{26}{3}$ 8 R2 **40.** $\frac{13}{3}$ 4 R1

41. $7\overline{)47}$ 6 R5 **42.** $6\overline{)38}$ 6 R2 **43.** $7\overline{)62}$ 8 R6 **44.** $9\overline{)43}$ 4 R7

45. $19 \div 2$ 9 R1 **46.** $70 \div 8$ 8 R6 **47.** $34 \div 4$ 8 R2 **48.** $50 \div 9$ 5 R5

49. $\frac{22}{3}$ 7 R1 **50.** $\frac{57}{9}$ 6 R3 **51.** $\frac{29}{5}$ 5 R4 **52.** $\frac{51}{6}$ 8 R3

17

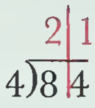
Dividing Whole Numbers (1-digit divisors)

Performance Objective: The student can divide a whole number by a one-digit divisor (remainders possible).

Divide each digit of the dividend in turn by the divisor, starting with the place with the largest value.

Examples:

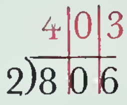
1.



CHECK:

$$\begin{array}{r} 21 \\ \times\ 4 \\ \hline 84 \end{array}$$

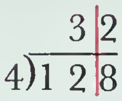
2.



CHECK:

$$\begin{array}{r} 403 \\ \times\ 2 \\ \hline 806 \end{array}$$

3.



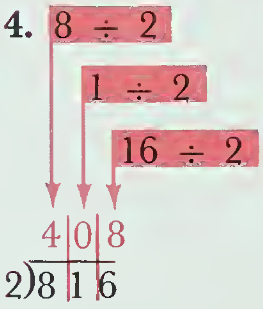
1 is smaller than 4, so start with 12 ÷ 4.

CHECK:

$$\begin{array}{r} 32 \\ \times\ 4 \\ \hline 128 \end{array}$$

If there is a remainder from dividing the digit in any place but the *ones*, the remainder must be grouped with the next digit to the right before dividing again.

4.

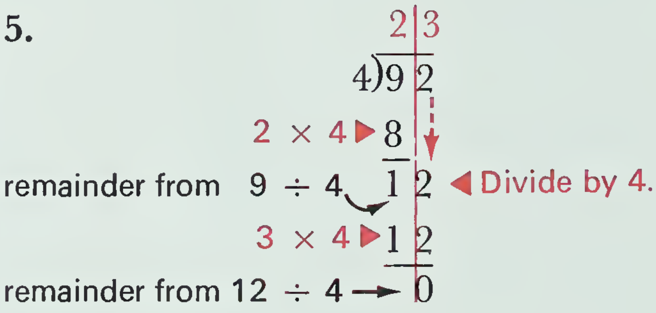


8 ÷ 2
1 ÷ 2
16 ÷ 2
4 0 8
2)8 1 6

CHECK:

$$\begin{array}{r} 408 \\ \times\ 2 \\ \hline 816 \end{array}$$

5.

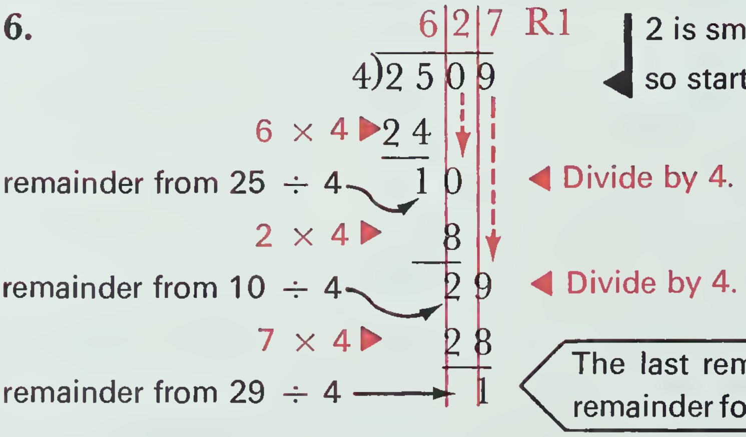


2 3
4)9 2
2 × 4 ▶ 8
remainder from 9 ÷ 4
1 2
◀ Divide by 4.
3 × 4 ▶ 1 2
remainder from 12 ÷ 4 → 0

CHECK:

$$\begin{array}{r} 23 \\ \times\ 4 \\ \hline 92 \end{array}$$

6.



6 2 7 R1
4)2 5 0 9
6 × 4 ▶ 2 4
remainder from 25 ÷ 4
1 0
◀ Divide by 4.
2 × 4 ▶ 8
remainder from 10 ÷ 4
2 9
◀ Divide by 4.
7 × 4 ▶ 2 8
remainder from 29 ÷ 4 → 1

2 is smaller than 4, so start with 25 ÷ 4.

The last remainder is the remainder for the problem.

CHECK:

$$\begin{array}{r} 627 \\ \times\ 4 \\ \hline 2508 \\ +\ 1 \\ \hline 2509 \end{array}$$

Exercises

1–60. Divide. Check your answers.

1. $\begin{array}{r} 24 \\ 2\overline{)48} \end{array}$
2. $\begin{array}{r} 32 \\ 3\overline{)96} \end{array}$
3. $\begin{array}{r} 21 \\ 4\overline{)84} \end{array}$
4. $\begin{array}{r} 11 \\ 5\overline{)55} \end{array}$
5. $\begin{array}{r} 122 \\ 4\overline{)488} \end{array}$
6. $\begin{array}{r} 213 \\ 3\overline{)639} \end{array}$
7. $\begin{array}{r} 324 \\ 2\overline{)648} \end{array}$
8. $\begin{array}{r} 2112 \\ 4\overline{)8448} \end{array}$
9. $\begin{array}{r} 301 \\ 3\overline{)903} \end{array}$
10. $\begin{array}{r} 110 \\ 5\overline{)550} \end{array}$
11. $\begin{array}{r} 2012 \\ 4\overline{)8048} \end{array}$
12. $\begin{array}{r} 3104 \\ 2\overline{)6208} \end{array}$
13. $\begin{array}{r} 51 \\ 5\overline{)255} \end{array}$
14. $\begin{array}{r} 52 \\ 3\overline{)156} \end{array}$
15. $\begin{array}{r} 211 \\ 8\overline{)1688} \end{array}$
16. $\begin{array}{r} 41 \\ 7\overline{)287} \end{array}$
17. $\begin{array}{r} 91 \\ 6\overline{)546} \end{array}$
18. $\begin{array}{r} 82 \\ 4\overline{)328} \end{array}$
19. $\begin{array}{r} 923 \\ 2\overline{)1846} \end{array}$
20. $\begin{array}{r} 832 \\ 3\overline{)2496} \end{array}$
21. $\begin{array}{r} 701 \\ 5\overline{)3505} \end{array}$
22. $\begin{array}{r} 903 \\ 3\overline{)2709} \end{array}$
23. $\begin{array}{r} 701 \\ 4\overline{)2804} \end{array}$
24. $\begin{array}{r} 903 \\ 2\overline{)1806} \end{array}$
25. $\begin{array}{r} 309 \\ 3\overline{)927} \end{array}$
26. $\begin{array}{r} 105 \\ 5\overline{)525} \end{array}$
27. $\begin{array}{r} 108 \\ 7\overline{)756} \end{array}$
28. $\begin{array}{r} 207 \\ 4\overline{)828} \end{array}$
29. $\begin{array}{r} 108 \\ 6\overline{)648} \end{array}$
30. $\begin{array}{r} 2093 \\ 2\overline{)4186} \end{array}$
31. $\begin{array}{r} 3206 \\ 3\overline{)9618} \end{array}$
32. $\begin{array}{r} 2106 \\ 4\overline{)8424} \end{array}$
33. $\begin{array}{r} 208 \\ 6\overline{)1248} \end{array}$
34. $\begin{array}{r} 408 \\ 8\overline{)3264} \end{array}$
35. $\begin{array}{r} 509 \\ 7\overline{)3563} \end{array}$
36. $\begin{array}{r} 609 \\ 5\overline{)3045} \end{array}$
37. $\begin{array}{r} 24 \\ 3\overline{)72} \end{array}$
38. $\begin{array}{r} 17 \\ 4\overline{)68} \end{array}$
39. $\begin{array}{r} 16 \\ 6\overline{)96} \end{array}$
40. $\begin{array}{r} 151 \\ 5\overline{)755} \end{array}$
41. $\begin{array}{r} 467 \\ 2\overline{)934} \end{array}$
42. $\begin{array}{r} 284 \\ 3\overline{)852} \end{array}$
43. $\begin{array}{r} 136 \\ 7\overline{)952} \end{array}$
44. $\begin{array}{r} 146 \\ 4\overline{)584} \end{array}$
45. $\begin{array}{r} 442 \\ 9\overline{)3978} \end{array}$
46. $\begin{array}{r} 847 \\ 8\overline{)6776} \end{array}$
47. $\begin{array}{r} 441 \\ 6\overline{)2646} \end{array}$
48. $\begin{array}{r} 7303 \\ 5\overline{)36\,515} \end{array}$
49. $\begin{array}{r} 542 \text{ R5} \\ 7\overline{)3799} \end{array}$
50. $\begin{array}{r} 562 \text{ R3} \\ 4\overline{)2251} \end{array}$
51. $\begin{array}{r} 663 \text{ R7} \\ 9\overline{)5974} \end{array}$
52. $\begin{array}{r} 387 \text{ R2} \\ 5\overline{)1937} \end{array}$
53. $\begin{array}{r} 762 \text{ R6} \\ 8\overline{)6102} \end{array}$
54. $\begin{array}{r} 2753 \text{ R2} \\ 6\overline{)16\,520} \end{array}$
55. $\begin{array}{r} 4832 \text{ R2} \\ 3\overline{)14\,498} \end{array}$
56. $\begin{array}{r} 865 \text{ R1} \\ 2\overline{)1731} \end{array}$
57. $\begin{array}{r} 864 \text{ R1} \\ 7\overline{)6049} \end{array}$
58. $\begin{array}{r} 461 \text{ R2} \\ 5\overline{)2307} \end{array}$
59. $\begin{array}{r} 1837 \text{ R2} \\ 4\overline{)7350} \end{array}$
60. $\begin{array}{r} 575 \text{ R2} \\ 8\overline{)4602} \end{array}$

18

Dividing Whole Numbers (2- and 3-digit divisors)

Performance Objective: The student can divide a whole number by a two- or three-digit divisor (remainders possible).

Examples:

1. $23\overline{)72}$ ◀ 7 is smaller than 23, so start with 72 ÷ 23.

Think [72 ÷ 23 is about the same as 7 ÷ 2; 7 ÷ 2 is between 3 and 4.

$$\begin{array}{rr} & 3 \\ & 23\overline{)72} \\ 3 \times 23 \blacktriangleright & \underline{69} \\ & 3 \end{array}$$

◀ Try 3.

◀ remainder

$$\begin{array}{r} 3\ \text{R3} \\ 23\overline{)72} \end{array}$$

CHECK:

$$\begin{array}{r} 23 \\ \underline{\times\ 3} \\ 69 \\ \underline{+\ 3} \\ 72 \end{array}$$

2. $25\overline{)72}$ ◀ 7 is smaller than 25, so start with 72 ÷ 25.

Think [72 ÷ 25 is about the same as 7 ÷ 2; 7 ÷ 2 is between 3 and 4.

$$\begin{array}{rr} & 3 \\ & 25\overline{)72} \\ 3 \times 25 \blacktriangleright & 75 \end{array}$$

◀ Try 3.

◀ 72 is smaller than 75, so 3 is too large.

$$\begin{array}{rr} & 2 \\ & 25\overline{)72} \\ 2 \times 25 \blacktriangleright & \underline{50} \\ & 22 \end{array}$$

◀ Try 2.

◀ remainder

$$\begin{array}{r} 2\ \text{R22} \\ 25\overline{)72} \end{array}$$

CHECK:

$$\begin{array}{r} 25 \\ \underline{\times\ 2} \\ 50 \\ \underline{+22} \\ 72 \end{array}$$

3.

$$\begin{array}{rr} & 2\,0\,4 \\ & 231\overline{)47{,}1\,2\,4} \\ 2 \times 231 \blacktriangleright & \underline{46\,2} \\ \text{remainder from } 471 \div 231 & 9\,2 \\ \text{remainder from } 92 \div 231 & 9\,2\,4 \\ 4 \times 231 \blacktriangleright & \underline{9\,2\,4} \\ & 0 \end{array}$$

◀ 4 and 47 are smaller than 231, so start with 471 ÷ 231. But 471 ÷ 231 is about the same as 4 ÷ 2. Try 2.

◀ 92 is smaller than 231; put 0 in the quotient, then bring down the 4 and find 924 ÷ 231.

Check by multiplying.

Exercises

1–50. Divide. Check your answers.

1. $\begin{array}{l}\quad\ \ 2\\38\overline{)76}\end{array}$

2. $\begin{array}{l}\quad\ \ 1\text{ R31}\\52\overline{)83}\end{array}$

3. $\begin{array}{l}\quad\ \ 3\\26\overline{)78}\end{array}$

4. $\begin{array}{l}\quad\ \ 2\text{ R19}\\34\overline{)87}\end{array}$

5. $\begin{array}{l}\quad\ \ 11\text{ R5}\\59\overline{)654}\end{array}$

6. $\begin{array}{l}\quad\ \ 23\text{ R19}\\32\overline{)755}\end{array}$

7. $\begin{array}{l}\quad\ \ 32\text{ R4}\\23\overline{)740}\end{array}$

8. $\begin{array}{l}\quad\ \ 22\text{ R11}\\37\overline{)825}\end{array}$

9. $\begin{array}{l}\quad\ \ 126\text{ R24}\\42\overline{)5316}\end{array}$

10. $\begin{array}{l}\quad\ \ 126\text{ R4}\\56\overline{)7060}\end{array}$

11. $\begin{array}{l}\quad\ \ 221\\35\overline{)7735}\end{array}$

12. $\begin{array}{l}\quad\ \ 133\\24\overline{)3192}\end{array}$

13. $\begin{array}{l}\quad\ \ 2\text{ R25}\\34\overline{)93}\end{array}$

14. $\begin{array}{l}\quad\ \ 2\text{ R9}\\28\overline{)65}\end{array}$

15. $\begin{array}{l}\quad\ \ 1\text{ R35}\\37\overline{)72}\end{array}$

16. $\begin{array}{l}\quad\ \ 1\text{ R42}\\43\overline{)85}\end{array}$

17. $\begin{array}{l}\quad\ \ 36\\27\overline{)972}\end{array}$

18. $\begin{array}{l}\quad\ \ 19\text{ R16}\\31\overline{)605}\end{array}$

19. $\begin{array}{l}\quad\ \ 18\text{ R17}\\46\overline{)845}\end{array}$

20. $\begin{array}{l}\quad\ \ 39\\21\overline{)819}\end{array}$

21. $\begin{array}{l}\quad\ \ 22\text{ R28}\\33\overline{)754}\end{array}$

22. $\begin{array}{l}\quad\ \ 12\text{ R46}\\54\overline{)694}\end{array}$

23. $\begin{array}{l}\quad\ \ 32\text{ R18}\\24\overline{)786}\end{array}$

24. $\begin{array}{l}\quad\ \ 22\text{ R24}\\36\overline{)816}\end{array}$

25. $\begin{array}{l}\quad\ \ 28\text{ R2}\\23\overline{)646}\end{array}$

26. $\begin{array}{l}\quad\ \ 23\text{ R31}\\39\overline{)928}\end{array}$

27. $\begin{array}{l}\quad\ \ 17\text{ R32}\\47\overline{)831}\end{array}$

28. $\begin{array}{l}\quad\ \ 22\text{ R21}\\29\overline{)659}\end{array}$

29. $\begin{array}{l}\quad\ \ 51\\75\overline{)3825}\end{array}$

30. $\begin{array}{l}\quad\ \ 61\text{ R7}\\43\overline{)2630}\end{array}$

31. $\begin{array}{l}\quad\ \ 14\text{ R6}\\87\overline{)1224}\end{array}$

32. $\begin{array}{l}\quad\ \ 83\text{ R41}\\54\overline{)4523}\end{array}$

33. $\begin{array}{l}\quad\ \ 77\text{ R20}\\38\overline{)2946}\end{array}$

34. $\begin{array}{l}\quad\ \ 1031\\82\overline{)84\,542}\end{array}$

35. $\begin{array}{l}\quad\ \ 3019\text{ R26}\\32\overline{)96\,634}\end{array}$

36. $\begin{array}{l}\quad\ \ 3079\text{ R15}\\25\overline{)76\,990}\end{array}$

37. $\begin{array}{l}\quad\ \ 14\text{ R56}\\682\overline{)9604}\end{array}$

38. $\begin{array}{l}\quad\ \ 11\text{ R76}\\157\overline{)1803}\end{array}$

39. $\begin{array}{l}\quad\ \ 14\\538\overline{)7532}\end{array}$

40. $\begin{array}{l}\quad\ \ 233\text{ R192}\\324\overline{)75\,684}\end{array}$

41. $\begin{array}{l}\quad\ \ 410\text{ R126}\\241\overline{)98\,936}\end{array}$

42. $\begin{array}{l}\quad\ \ 221\text{ R408}\\423\overline{)93\,891}\end{array}$

43. $\begin{array}{l}\quad\ \ 1074\\583\overline{)626\,142}\end{array}$

44. $\begin{array}{l}\quad\ \ 1530\text{ R72}\\364\overline{)556\,992}\end{array}$

45. $\begin{array}{l}\quad\ \ 4523\\142\overline{)642\,266}\end{array}$

46. $\begin{array}{l}\quad\ \ 64\text{ R112}\\471\overline{)30\,256}\end{array}$

47. $\begin{array}{l}\quad\ \ 49\text{ R440}\\506\overline{)25\,234}\end{array}$

48. $\begin{array}{l}\quad\ \ 37\\283\overline{)10\,471}\end{array}$

49. $\begin{array}{l}\quad\ \ 566\\327\overline{)185\,082}\end{array}$

50. $\begin{array}{l}\quad\ \ 791\\252\overline{)199\,332}\end{array}$

19 Dividing Decimals by Whole Numbers

Performance Objective:
The student can
- divide a decimal by a whole number, correctly placing the decimal point in the quotient.
- express the quotient of two whole numbers as a decimal.

Examples:

1.
$$\begin{array}{r} 0.3 \\ 2\,\overline{)\,0.6} \end{array} \qquad \text{CHECK:} \quad \begin{array}{r} 0.3 \\ \times\ 2 \\ \hline 0.6 \end{array}$$

Align decimal points.

2.
$$\begin{array}{r} 0.05 \\ 7\,\overline{)\,0.35} \end{array} \qquad \text{CHECK:} \quad \begin{array}{r} 0.05 \\ \times\ \ 7 \\ \hline 0.35 \end{array}$$

3.
$$\begin{array}{r} 0.802 \\ 8\overline{)6.416} \end{array}$$

4.
$$\begin{array}{r} 4.10 \\ 6\overline{)24.60} \end{array}$$

(Check by multiplying as before.)

5. $43.4 \div 35$

Step 1 Divide as for whole numbers, and place decimal point as shown above.

$$\begin{array}{r} 1.2 \\ 35\overline{)43.4} \\ 35 \\ \hline 8\ 4 \\ 7\ 0 \\ \hline 1\ 4 \end{array}$$

14 is smaller than 35.

Step 2

$$\begin{array}{r} 1.2\,4 \\ 35\overline{)4\,3.4\,0} \\ 3\,5 \\ \hline 8\,4 \\ 7\,0 \\ \hline 1\,4\,0 \\ 1\,4\,0 \\ \hline 0 \end{array}$$

As many zeros as necessary may be written here.

Now divide 140 by 35.

$$\text{CHECK:} \quad \begin{array}{r} 1.24 \\ \times\ \ 35 \\ \hline 6\ 20 \\ 37\ 2 \\ \hline 43.40 \end{array}$$

This is equal to 43.4.

6.
$$\begin{array}{r} 6\ \text{R3} \\ 6\overline{)39} \\ 36 \\ \hline 3 \end{array} \qquad \text{CHECK:} \quad \begin{array}{r} 6 \\ \times 6 \\ \hline 36 \\ +3 \\ \hline 39 \end{array}$$

or

$$\begin{array}{r} 6.5 \\ 6\overline{)39.0} \\ 36 \\ \hline 3\ 0 \\ 3\ 0 \\ \hline 0 \end{array} \qquad \text{CHECK:} \quad \begin{array}{r} 6.5 \\ \times\ \ 6 \\ \hline 39.0 \end{array}$$

or 39

Exercises

1–28. Divide. Check your answers.

1. $\begin{array}{r}0.3\\3\overline{)0.9}\end{array}$
2. $\begin{array}{r}0.4\\2\overline{)0.8}\end{array}$
3. $\begin{array}{r}0.1\\5\overline{)0.5}\end{array}$
4. $\begin{array}{r}0.2\\4\overline{)0.8}\end{array}$
5. $\begin{array}{r}0.08\\2\overline{)0.16}\end{array}$
6. $\begin{array}{r}0.09\\3\overline{)0.27}\end{array}$
7. $\begin{array}{r}0.08\\6\overline{)0.48}\end{array}$
8. $\begin{array}{r}0.07\\9\overline{)0.63}\end{array}$
9. $\begin{array}{r}0.08\\7\overline{)0.56}\end{array}$
10. $\begin{array}{r}0.09\\5\overline{)0.45}\end{array}$
11. $\begin{array}{r}0.08\\4\overline{)0.32}\end{array}$
12. $\begin{array}{r}0.09\\8\overline{)0.72}\end{array}$
13. $\begin{array}{r}0.41\\6\overline{)2.46}\end{array}$
14. $\begin{array}{r}0.602\\7\overline{)4.214}\end{array}$
15. $\begin{array}{r}0.403\\5\overline{)2.015}\end{array}$
16. $\begin{array}{r}0.61\\9\overline{)5.49}\end{array}$
17. $\begin{array}{r}3.11\\7\overline{)21.77}\end{array}$
18. $\begin{array}{r}6.32\\3\overline{)18.96}\end{array}$
19. $\begin{array}{r}7.21\\4\overline{)28.84}\end{array}$
20. $\begin{array}{r}6.02\\6\overline{)36.12}\end{array}$
21. $\begin{array}{r}3.15\\27\overline{)85.05}\end{array}$
22. $\begin{array}{r}2.68\\35\overline{)93.80}\end{array}$
23. $\begin{array}{r}6.24\\12\overline{)74.88}\end{array}$
24. $\begin{array}{r}0.532\\40\overline{)21.280}\end{array}$
25. $\begin{array}{r}6.27\\34\overline{)213.18}\end{array}$
26. $\begin{array}{r}3.68\\52\overline{)191.36}\end{array}$
27. $\begin{array}{r}5.28\\43\overline{)227.04}\end{array}$
28. $\begin{array}{r}5.24\\68\overline{)356.32}\end{array}$

29–56. Divide. Do not use remainder notation. Check.

29. $\begin{array}{r}0.505\\92\overline{)46.46}\end{array}$
30. $\begin{array}{r}0.654\\25\overline{)16.35}\end{array}$
31. $\begin{array}{r}8.65\\34\overline{)294.1}\end{array}$
32. $\begin{array}{r}41.66\\15\overline{)624.9}\end{array}$
33. $\begin{array}{r}0.007\,35\\46\overline{)0.3381}\end{array}$
34. $\begin{array}{r}0.055\\18\overline{)0.99}\end{array}$
35. $\begin{array}{r}0.915\\64\overline{)58.56}\end{array}$
36. $\begin{array}{r}9.35\\88\overline{)822.8}\end{array}$
37. $\begin{array}{r}0.0325\\68\overline{)2.21}\end{array}$
38. $\begin{array}{r}0.0825\\76\overline{)6.27}\end{array}$
39. $\begin{array}{r}0.248\\25\overline{)6.2}\end{array}$
40. $\begin{array}{r}0.003\,25\\16\overline{)0.052}\end{array}$
41. $\begin{array}{r}0.3375\\28\overline{)9.45}\end{array}$
42. $\begin{array}{r}0.002\\300\overline{)0.6}\end{array}$
43. $\begin{array}{r}0.528\\25\overline{)13.2}\end{array}$
44. $\begin{array}{r}0.06\,25\\12\overline{)0.75}\end{array}$
45. $\begin{array}{r}0.0266\\500\overline{)13.3}\end{array}$
46. $\begin{array}{r}0.0625\\8\overline{)0.5}\end{array}$
47. $\begin{array}{r}0.0375\\40\overline{)1.5}\end{array}$
48. $\begin{array}{r}0.806\,25\\8\overline{)6.45}\end{array}$
49. $\begin{array}{r}39.5\\2\overline{)79}\end{array}$
50. $\begin{array}{r}7.6\\5\overline{)38}\end{array}$
51. $\begin{array}{r}31.5\\6\overline{)189}\end{array}$
52. $\begin{array}{r}14.75\\4\overline{)59}\end{array}$
53. $\begin{array}{r}23.375\\8\overline{)187}\end{array}$
54. $\begin{array}{r}0.0875\\80\overline{)7}\end{array}$
55. $\begin{array}{r}32.875\\16\overline{)526}\end{array}$
56. $\begin{array}{r}53.625\\8\overline{)429}\end{array}$

20

Dividing by 10, 100, 1000, · · ·

Performance Objective: The student can divide by powers of ten by relocating the decimal point.

In **multiplying** by 10, 100, 1000, · · · ⇨ for each zero, move the decimal point *right* one place.

In **dividing** by 10, 100, 1000, · · · ⇨ for each zero, move the decimal point *left* one place.

Examples:

1 zero

1. $7\underbrace{3}_{\text{1 digit}}. \div 10 = 7.\underbrace{3}_{\text{1 digit}}$

2 zeros

2. $0\underbrace{3}_{\text{2 digits}}.2 \div 100 = 0.\underbrace{032}_{\text{2 digits}}$

3 zeros

3. $5\underbrace{463}_{\text{3 digits}}.2 \div 1000 = 5.\underbrace{4632}_{\text{3 digits}}$

Exercises

1–6. Place the decimal point in each quotient. Write additional zeros where necessary.

1. 740 ÷ 10 = 74‸0 or 74

2. 250 ÷ 1000 = ‸250 or 0.25

3. 96 ÷ 1000 = 96 0.096

4. 823.34 ÷ 100 = 8‸2334

5. 64.8 ÷ 100 = ‸648

6. 1.4 ÷ 1000 = 14 0.0014

7–24. Divide.

7. 752 ÷ 10 75.2

8. 7 ÷ 10 0.7

9. 0.9 ÷ 10 0.09

10. 80.4 ÷ 10 8.04

11. 2000 ÷ 10 200

12. 0.003 ÷ 10 0.0003

13. 327 ÷ 100 3.27

14. 42 ÷ 100 0.42

15. 7.46 ÷ 100 0.0746

16. 0.653 ÷ 100 0.006 53

17. 0.0094 ÷ 100 0.000 094

18. 8000 ÷ 100 80

19. 29 342 ÷ 1000 29.342

20. 12 ÷ 1000 0.012

21. 0.56 ÷ 1000 0.000 56

22. 4200 ÷ 1000 4.2

23. 0.007 ÷ 1000 0.000 007

24. 34.6 ÷ 10 000 0.003 46

21

Dividing by Decimals

Performance Objective: The student can rewrite a division problem with a decimal divisor as an equivalent problem with a whole-number divisor.

Examples:

1. Step 1 $0.02\overline{)48.6}$

Multiply divisor by 100 to make it a whole number.

Multiply dividend by same amount as divisor.

$2\overline{)4860.}$

Step 2

Divide. ▶ $\begin{array}{r} 2430 \\ 2\overline{)4860} \end{array}$

Be sure to check the *original* problem.

CHECK: $\begin{array}{r} 2430 \\ \times\ 0.02 \\ \hline 48.60 \end{array}$

2. $9.142\overline{)3.6584}$ ⇨ $9142\overline{)3658.4}$

Multiply by 1000.

Divide and check.

Exercises

1–8. What is the smallest multiplier (10, 100, 1000, · · ·) that would make each number a whole number?

1. 3.46 100
2. 0.002 1000
3. 6.4 10
4. 530.2 10
5. 0.0241 10 000
6. 0.78 100
7. 8.004 1000
8. 0.9 10

9–32. Divide.

9. 18.7 ÷ 0.22 85
10. 1717.2 ÷ 54 31.8
11. 413.6 ÷ 0.44 940
12. $0.046\overline{)1.748}$ 38
13. $0.008\overline{)2.8624}$ 357.8
14. $0.048\overline{)0.12144}$ 2.53
15. $0.087\overline{)2.1663}$ 24.9
16. $0.68\overline{)0.021\ 828}$ .0321
17. $0.27\overline{)0.098\ 28}$ 0.364
18. 4865 ÷ 0.7 6950
19. 3124 ÷ 0.5 6248
20. 7608 ÷ 0.8 9510
21. $0.8\overline{)6.344}$ 7.93
22. $0.74\overline{)4.8248}$ 6.52
23. $0.64\overline{)0.7872}$ 1.23
24. $0.32\overline{)10.944}$ 34.2
25. $0.85\overline{)38.845}$ 45.7
26. $0.34\overline{)163.2}$ 480
27. 6.1664 ÷ 1.6 3.854
28. 21.2888 ÷ 0.23 92.56
29. 294.8 ÷ 8.8 33.5
30. $0.031\overline{)1736}$ 56 0000
31. $2.4\overline{)164.4}$ 68.5
32. $0.47\overline{)40.044}$ 85.2

22

Rounding Quotients

Performance Objective: The student can round a decimal quotient to a specified place.

Examples:

1. Find 4.868 ÷ 2 to the nearest tenth.

To round to this place, this digit is needed.

$$\begin{array}{r} 2.43 \\ 2\overline{)4.868} \end{array}$$

→ Stop dividing. Round to tenths.

→ Check quotient before rounding:

$$\begin{array}{r} 2.43 \\ \times\quad 2 \\ \hline 4.86 \end{array}$$

less than 5; round down

2.43

2.4 ◀ quotient to nearest tenth

2. Find 0.036 18 ÷ 0.03 to the nearest hundredth.

To round to this place, this digit is needed.

$$0.03\overline{)0.036\ 18} \rightarrow 3\overline{)3.618}$$

Check before rounding.

$$\begin{array}{r} 1.206 \\ 3\overline{)3.618} \end{array}$$

◀ Round this to hundredths.

more than 5; round up

1.206

1.21 ◀ quotient to nearest hundredth

3. Find 64.84 ÷ 0.8 to the nearest tenth.

To round to this place, this digit is needed.

$$0.8\overline{)64.84} \rightarrow 8\overline{)648.40}$$

Write this zero.

Check before rounding.

$$\begin{array}{r} 81.05 \\ 8\overline{)648.40} \end{array}$$

◀ Stop dividing; round to tenths.

81.1 ◀ quotient to nearest tenth

4. Find to the nearest tenth.

a. 30 ÷ 5

$$\begin{array}{r} 6.0 \\ 5\overline{)30.0} \\ \underline{30} \\ 0 \end{array}$$

b. 31 ÷ 4

$$\begin{array}{r} 7.75 \\ 4\overline{)31.00} \\ \underline{28} \\ 3\;0 \\ \underline{2\;8} \\ 20 \end{array}$$

◀ Round to 7.8.

Exercises

1–9. Find each quotient to the nearest tenth.

1. $\begin{array}{r} 1.5 \\ 6\overline{)8.82} \end{array}$ **2.** $\begin{array}{r} 5.1 \\ 5\overline{)25.30} \end{array}$ **3.** $\begin{array}{r} 32.1 \\ 3\overline{)96.318} \end{array}$

4. $\begin{array}{r} 6.0 \\ 23\overline{)138.92} \end{array}$ **5.** $\begin{array}{r} 1.7 \\ 22\overline{)37.82} \end{array}$ **6.** $\begin{array}{r} 6.3 \\ 3.4\overline{)21.532} \end{array}$

7. $\begin{array}{r} 8.3 \\ 0.47\overline{)3.8822} \end{array}$ **8.** $\begin{array}{r} 13.4 \\ 16.2\overline{)216.755} \end{array}$ **9.** $\begin{array}{r} 6.5 \\ 41.8\overline{)272.953} \end{array}$

10–18. Find each quotient to the nearest hundredth.

10. $\begin{array}{r} 0.63 \\ 4\overline{)2.536} \end{array}$ **11.** $\begin{array}{r} 0.83 \\ 6\overline{)4.962} \end{array}$ **12.** $\begin{array}{r} 5.34 \\ 9\overline{)48.06} \end{array}$

13. $\begin{array}{r} 3.26 \\ 75\overline{)244.55} \end{array}$ **14.** $\begin{array}{r} 3.63 \\ 17\overline{)61.676} \end{array}$ **15.** $\begin{array}{r} 0.2 \\ 0.221\overline{)0.0442} \end{array}$

16. $\begin{array}{r} 0.63 \\ 3.4\overline{)2.1532} \end{array}$ **17.** $\begin{array}{r} 1.34 \\ 16.2\overline{)21.6755} \end{array}$ **18.** $\begin{array}{r} 0.65 \\ 41.8\overline{)27.2953} \end{array}$

19–30. Find each quotient to the nearest tenth.

19. $\begin{array}{r} 69.6 \\ 1.4\overline{)97.38} \end{array}$ **20.** $\begin{array}{r} 624.8 \\ 0.5\overline{)312.4} \end{array}$ **21.** $\begin{array}{r} 953.5 \\ 0.4\overline{)381.4} \end{array}$

22. 12.09 ÷ 0.024 503.8 **23.** 4.3328 ÷ 0.087 49.8 **24.** 3.504 ÷ 0.023 152.3

25. 49.99 ÷ 0.5 100.0 **26.** 26.1 ÷ 2.6 10.0 **27.** 0.3006 ÷ 0.01 30.1

28. 81 ÷ 5 16.2 **29.** 65 ÷ 4 16.3 **30.** 17 ÷ 8 2.1

31–39. Find each quotient to the nearest thousandth.

31. $\begin{array}{r} 0.182 \\ 0.54\overline{)0.098\ 46} \end{array}$ **32.** $\begin{array}{r} 62.632 \\ 0.25\overline{)15.658} \end{array}$ **33.** $\begin{array}{r} 128.889 \\ 0.063\overline{)8.12} \end{array}$

34. $\begin{array}{r} 13.129 \\ 0.062\overline{)0.814} \end{array}$ **35.** $\begin{array}{r} 0.426 \\ 0.47\overline{)0.2} \end{array}$ **36.** $\begin{array}{r} 0.137 \\ 72\overline{)9.84} \end{array}$

37. 8.6 ÷ 0.24 35.833 **38.** 436.66 ÷ 0.35 1247.600 **39.** 82.1 ÷ 0.03 2736.667

23

Practice Multiplying and Dividing (whole numbers and decimals)

Performance Objective: The student practices multiplying or dividing whole numbers and decimals.

1–52. Multiply.

1. 383 × 10 3830
2. 726 × 100 72 600
3. 21 × 1000 21 000

4. 10 × 2983 29 830
5. 100 × 45 4500
6. 1000 × 684 684 000

7. 42 × 3 126
8. 31 × 8 248
9. 63 × 2 126
10. 13 × 4 52

11. 34 × 12 408
12. 97 × 11 1067
13. 21 × 67 1407
14. 43 × 22 946

15. 73 × 9 657
16. 85 × 6 510
17. 38 × 7 266
18. 56 × 4 224

19. 63 × 27 1701
20. 59 × 83 4897
21. 47 × 53 2491
22. 68 × 35 2380

23. 422 × 304 128 288
24. 542 × 103 55 826
25. 723 × 309 223 407
26. 586 × 24 14 064

27. 368 × 2.1 772.8
28. 465 × 8.6 3999
29. 78.4 × 23 1803.2
30. 1.27 × 91 115.57

31. 691 × 0.6 414.6
32. (8.34)(7) 58.38
33. 59.6 × 0.01 0.596

34. 73.4(100) 7340
35. 10 × 3.87 38.7
36. 0.68 × 1000 680

37. 6.9 × 0.2 1.38
38. 3.27 × 0.6 1.962
39. 51.4 × 0.32 16.448
40. 96.23 × 1.26 121.2498

41. 0.681 × 2.14 1.457 34
42. 0.962 × 0.103 0.099 086
43. 6.003 × 28.6 171.6858
44. 820.4 × 0.0006 0.492 24

45. 8008 × 0.003 24.024
46. 0.1111 × 4.4 0.488 84
47. 9823 × 0.2232 2192.4936
48. 426 × 0.999 425.574

49. 7 × 2 × 4 × 11 616
50. 6 × 3.2 × 20 384

51. 2 × 4.2 × 5 × 3.1 130.2
52. 10.3 × 3 × 2.5 77.25

53–90. Divide. Use remainder notation where necessary. Check by multiplying.

53. 72 ÷ 8 9 **54.** 54 ÷ 7 7 R5 **55.** 84 ÷ 4 21 **56.** 65 ÷ 9 7 R2

57. $\overset{321}{3\overline{)963}}$ **58.** $\overset{210}{4\overline{)840}}$ **59.** $\overset{804}{2\overline{)1608}}$ **60.** $\overset{211}{6\overline{)1266}}$

61. $\overset{203}{7\overline{)1421}}$ **62.** $\overset{105}{5\overline{)525}}$ **63.** $\overset{202}{9\overline{)1818}}$ **64.** $\overset{307}{3\overline{)921}}$

65. $\overset{52}{6\overline{)312}}$ **66.** $\overset{23}{8\overline{)184}}$ **67.** $\overset{57}{4\overline{)228}}$ **68.** $\overset{82}{7\overline{)574}}$

69. $\overset{538}{6\overline{)3228}}$ **70.** $\overset{552}{8\overline{)4416}}$ **71.** $\overset{841}{9\overline{)7569}}$ **72.** $\overset{697}{4\overline{)2788}}$

73. 698 ÷ 5 139 R3 **74.** 397 ÷ 4 99 R1 **75.** 489 ÷ 6 81 R3 **76.** 658 ÷ 3 219 R1

77. $\frac{4499}{8}$ 562 R3 **78.** $\frac{4128}{7}$ 589 R5 **79.** $\frac{3913}{5}$ 782 R3 **80.** $\frac{2188}{6}$ 364 R4

81. $\overset{326}{24\overline{)7824}}$ **82.** $\overset{241}{32\overline{)7712}}$ **83.** $\overset{124\text{ R7}}{43\overline{)5339}}$ **84.** $\overset{123\text{ R14}}{53\overline{)6533}}$

85. $\overset{256}{37\overline{)9472}}$ **86.** $\overset{636\text{ R46}}{58\overline{)36\,934}}$ **87.** $\overset{623\text{ R5}}{29\overline{)18\,072}}$

88. $\overset{204}{322\overline{)65\,688}}$ **89.** $\overset{652}{213\overline{)138\,876}}$ **90.** $\overset{332\text{ R94}}{431\overline{)143\,186}}$

91–110. Divide. Do not use remainder notation. Check.

91. $\overset{7.02}{9\overline{)63.18}}$ **92.** $\overset{0.311}{7\overline{)2.177}}$ **93.** $\overset{3.6}{24\overline{)86.4}}$ **94.** $\overset{5.7}{31\overline{)176.7}}$

95. $\overset{3.5}{12\overline{)42}}$ **96.** $\overset{2.32}{65\overline{)150.8}}$ **97.** $\overset{3.275}{28\overline{)91.7}}$ **98.** $\overset{20.75}{48\overline{)996}}$

99. 84 ÷ 10 8.4 **100.** 37.2 ÷ 100 0.372 **101.** 0.08 ÷ 100 0.0008

102. 2 ÷ 1000 0.002 **103.** $\overset{3.26}{0.25\overline{)0.815}}$ **104.** $\overset{2.05}{5.7\overline{)11.685}}$

105. $\overset{8.14}{0.03\overline{)0.2442}}$ **106.** $\overset{6.5}{0.011\overline{)0.0715}}$ **107.** $\overset{0.65}{42\overline{)27.3}}$

108. $\overset{2.25}{0.85\overline{)1.9125}}$ **109.** $\overset{1.25}{0.008\overline{)0.01}}$ **110.** $\overset{0.464}{0.75\overline{)0.348}}$

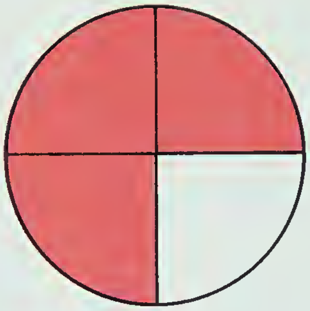
24

Meaning of Fractions

Performance Objective: The student can read proper fractions and interpret them using models.

Examples:

1.

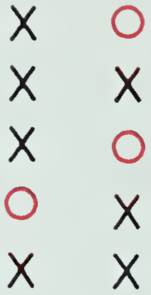


There are 4 parts the same size.

3 of the parts are shaded red.

$\frac{3}{4}$ of the figure is shaded red.

2.



There are 10 objects.

7 of the objects are X's.

$\frac{7}{10}$ of the set are X's.

numerator ▶ $\frac{3}{4}$ ◁ These fractions ▷ $\frac{7}{10}$ ◀ numerator

denominator ▶ ◀ denominator

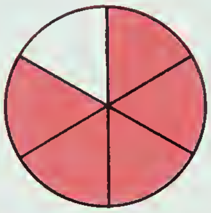
are read

three-fourths ◁ this way. ▷ *seven-tenths*

Exercises

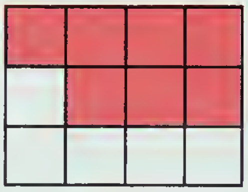
1–18. Use a fraction to tell how much is in red.

1.



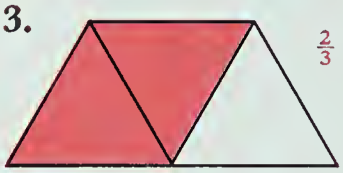
$\frac{5}{6}$

2.



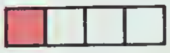
$\frac{7}{12}$

3.



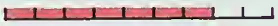
$\frac{2}{3}$

4.



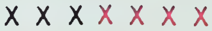
$\frac{1}{4}$

5.



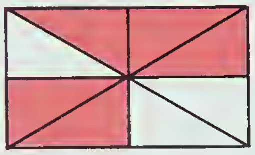
$\frac{7}{9}$

6.



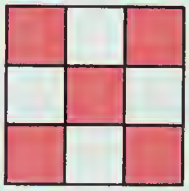
$\frac{4}{7}$

7.



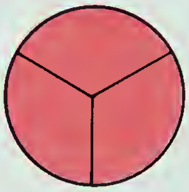
$\frac{5}{8}$

8.



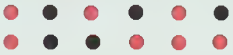
$\frac{5}{9}$

9.



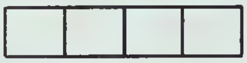
$\frac{3}{3}$

10.



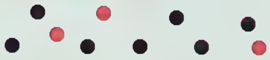
$\frac{7}{12}$

11.



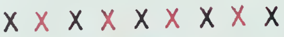
$\frac{0}{5}$

12.



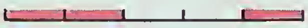
$\frac{3}{10}$

13.



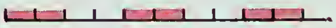
$\frac{4}{9}$

14.



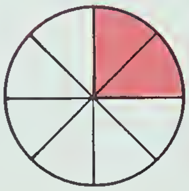
$\frac{3}{5}$

15.



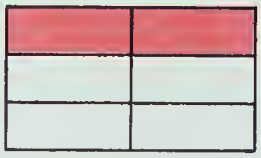
$\frac{6}{11}$

16.



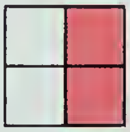
$\frac{2}{8}$ or $\frac{1}{4}$

17.



$\frac{2}{6}$ or $\frac{1}{3}$

18.



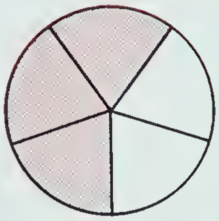
$\frac{2}{4}$ or $\frac{1}{2}$

19–38. Express in words.

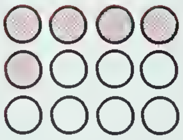
19. $\frac{1}{2}$ One-half
20. $\frac{1}{3}$ One-third
21. $\frac{1}{4}$ One-fourth
22. $\frac{3}{8}$ Three-eighths
23. $\frac{1}{16}$ One-sixteenth
24. $\frac{2}{5}$ Two-fifths
25. $\frac{2}{3}$ Two-thirds
26. $\frac{6}{8}$ Six-eighths
27. $\frac{7}{12}$ Seven-twelfths
28. $\frac{3}{20}$ Three-twentieths
29. $\frac{6}{50}$ Six-fiftieths
30. $\frac{4}{100}$ Four-hundredths
31. $\frac{5}{9}$ Five-ninths
32. $\frac{9}{17}$ Nine-seventeenths
33. $\frac{3}{31}$ Three thirty-firsts
34. $\frac{15}{16}$ Fifteen-sixteenths
35. $\frac{3}{3}$ Three-thirds
36. $\frac{17}{22}$ Seventeen twenty-seconds
37. $\frac{0}{5}$ Zero-fifths
38. $\frac{8}{1000}$ Eight-thousandths

39–50. Draw a figure like the one shown (but any convenient size). Then shade or color the amount indicated by the fraction.

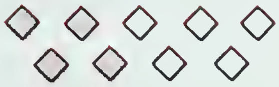
39. $\frac{3}{5}$



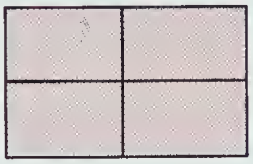
40. $\frac{5}{12}$



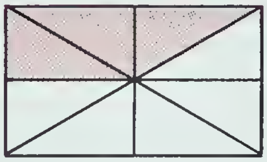
41. $\frac{4}{9}$



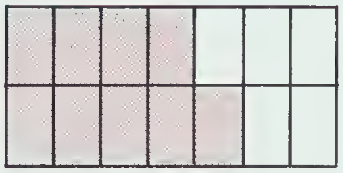
42. $\frac{4}{4}$



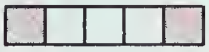
43. $\frac{3}{8}$



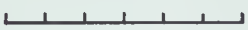
44. $\frac{9}{14}$



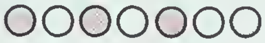
45. $\frac{2}{5}$



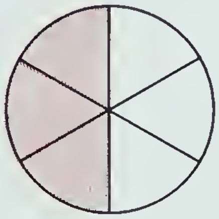
46. $\frac{0}{6}$



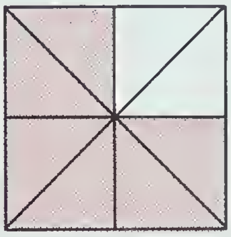
47. $\frac{3}{7}$



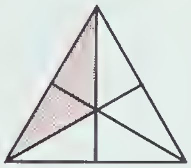
48. $\frac{1}{2}$



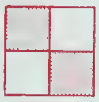
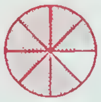
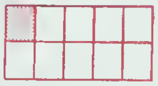
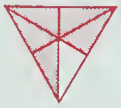
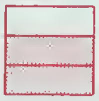
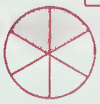
49. $\frac{3}{4}$



50. $\frac{1}{3}$



51–56. Draw the figure described. Separate it into parts and shade or color the amount indicated by the fraction.

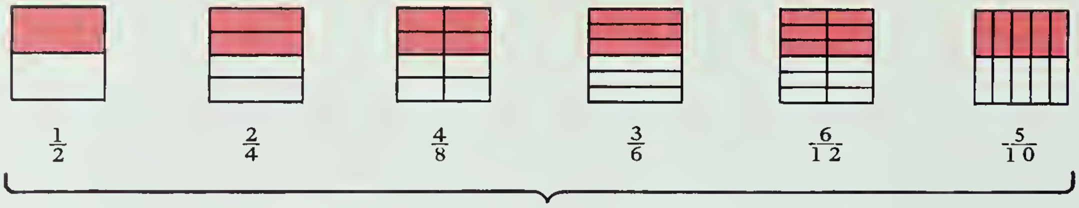
51. square; $\frac{2}{4}$ 
52. circle; $\frac{7}{8}$ 
53. rectangle; $\frac{1}{10}$ 
54. triangle; $\frac{5}{6}$ 
55. square; $\frac{2}{3}$ 
56. circle; $\frac{2}{6}$ 

25

Equivalent Fractions

Performance Objective:
The student can
- tell whether or not fractions are equivalent.
- write a fraction equivalent to a given fraction.

The same amount is shaded in each square,



so these are **equivalent fractions.**

Equivalent fractions have equal *cross-products.*

Examples:

1. $16 \longleftarrow \dfrac{2}{4} \times \dfrac{4}{8} \longrightarrow 16$

$16 = 16$, so $\frac{2}{4}$ and $\frac{4}{8}$ are equivalent. That is, $\frac{2}{4} = \frac{4}{8}$.

2. $28 \longleftarrow \dfrac{7}{8} \times \dfrac{3}{4} \longrightarrow 24$

28 does not equal 24, so $\frac{7}{8}$ and $\frac{3}{4}$ are **not** equivalent.

To find a fraction equivalent to a given fraction, multiply or divide both numerator and denominator by the same number.

3. $\dfrac{1}{2} = \dfrac{1 \times 5}{2 \times 5} = \dfrac{5}{10}$

4. $\dfrac{6}{12} = \dfrac{6 \div 3}{12 \div 3} = \dfrac{2}{4}$

5. $\dfrac{2}{5} = \dfrac{\boxed{?}}{15}$

Think $5 \times 3 = 15$ So $\dfrac{2 \times 3}{5 \times 3} = \dfrac{6}{15}$

That is, $\dfrac{2}{5} = \dfrac{\boxed{6}}{15}$

6. $\dfrac{2}{4} = \dfrac{\boxed{?}}{10}$ ◀ Since 10 is not a multiple of 4, the method above doesn't work.

Think $20 \longleftarrow \dfrac{2}{4} \times \dfrac{?}{10} \longrightarrow 4 \times \boxed{?}$ ◀ Find the cross-products.

So $20 = 4 \times \boxed{?}$; but $20 = 4 \times \boxed{5}$, so $\dfrac{2}{4} = \dfrac{\boxed{5}}{10}$.

Exercises

1–12. Tell whether or not the fractions are equivalent.

1. $\frac{2}{3}, \frac{8}{12}$ Yes

2. $\frac{3}{4}, \frac{9}{12}$ Yes

3. $\frac{4}{5}, \frac{8}{11}$ No

4. $\frac{3}{8}, \frac{5}{11}$ No

5. $\frac{12}{18}, \frac{2}{3}$ Yes

6. $\frac{5}{4}, \frac{8}{10}$ No

7. $\frac{4}{5}, \frac{12}{15}$ Yes

8. $\frac{9}{16}, \frac{19}{32}$ No

9. $\frac{0}{2}, \frac{0}{9}$ Yes

10. $\frac{5}{8}, \frac{15}{24}$ Yes

11. $\frac{3}{3}, \frac{5}{5}$ Yes

12. $\frac{45}{100}, \frac{23}{50}$ No

13–24. Use the given multiplier or divisor to find an equivalent fraction.

13. $\frac{2}{3}$; $\times$ 2 $\frac{4}{6}$

14. $\frac{18}{24}$; $\div$ 6 $\frac{3}{4}$

15. $\frac{3}{4}$; $\times$ 4 $\frac{12}{16}$

16. $\frac{7}{8}$; $\times$ 3 $\frac{21}{24}$

17. $\frac{25}{75}$; $\div$ 5 $\frac{5}{15}$

18. $\frac{3}{5}$; $\times$ 7 $\frac{21}{35}$

19. $\frac{1}{6}$; $\times$ 12 $\frac{12}{72}$

20. $\frac{42}{56}$; $\div$ 7 $\frac{6}{8}$

21. $\frac{20}{50}$; $\div$ 10 $\frac{2}{5}$

22. $\frac{0}{3}$; $\times$ 7 $\frac{0}{21}$

23. $\frac{5}{3}$; $\times$ 2 $\frac{10}{6}$

24. $\frac{4}{4}$; $\div$ 2 $\frac{2}{2}$

25–42. Tell what number should replace the ? to make the fractions equivalent.

25. $\frac{2}{3} = \frac{?}{9}$ 6

26. $\frac{3}{5} = \frac{?}{10}$ 6

27. $\frac{3}{4} = \frac{15}{?}$ 20

28. $\frac{5}{8} = \frac{?}{16}$ 10

29. $\frac{1}{12} = \frac{7}{?}$ 84

30. $\frac{?}{36} = \frac{5}{12}$ 15

31. $\frac{7}{10} = \frac{?}{100}$ 70

32. $\frac{90}{?} = \frac{9}{10}$ 100

33. $\frac{?}{100} = \frac{7}{20}$ 35

34. $\frac{2}{8} = \frac{?}{12}$ 3

35. $\frac{2}{10} = \frac{5}{?}$ 25

36. $\frac{4}{?} = \frac{6}{21}$ 14

37. $\frac{10}{12} = \frac{?}{18}$ 15

38. $\frac{?}{9} = \frac{0}{13}$ 0

39. $\frac{6}{9} = \frac{?}{12}$ 8

40. $\frac{?}{16} = \frac{6}{6}$ 16

41. $\frac{4}{14} = \frac{6}{?}$ 21

42. $\frac{?}{16} = \frac{15}{24}$ 10

26

Proper Fractions as Decimals

Performance Objective:
The student can
- change a proper fraction to a decimal.
- recognize that zero can be a numerator but cannot be a denominator.

$\frac{3}{4}$ means both $\left\{ \begin{array}{l} \textit{three-fourths} \\ 3 \div 4 \end{array} \right.$

Divide numerator by denominator to change the fraction to a decimal.

Examples:

1. $\frac{3}{4}$ ▶ $$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$ So $\frac{3}{4} = 0.75$

2. $\frac{3}{8}$ ▶ $$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{2\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$ So $\frac{3}{8} = 0.375$

3. $\frac{2}{11}$ ▶ $$\begin{array}{r} 0.1818 \\ 11\overline{)2.0000} \\ \underline{1\,1} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

Remainders repeat.

The division goes on and on, so you cannot find a simple decimal for $\frac{2}{11}$. Instead, you could write:

$$\frac{2}{11} = 0.\overline{18}$$

(The bar over 18 shows that these digits repeat over and over.)

A fraction whose numerator is less than its denominator is less than 1; such a fraction is a **proper fraction.**

A fraction with a 0 numerator equals 0.

4. $\frac{0}{3} = 0 \rightarrow \begin{array}{r} 0 \\ 3\overline{)0} \end{array}$

CHECK: $$\begin{array}{r} 0 \\ \underline{\times 3} \\ 0 \end{array}$$

A fraction cannot have 0 as a denominator.

5. a. $\frac{3}{0} = ? \longrightarrow 0\overset{?}{\overline{)3}}$

CHECK: $\begin{array}{r} ? \\ \times 0 \\ \hline 3 \end{array}$ ◀No number works, so there is no answer.

b. $\frac{0}{0} = ? \longrightarrow 0\overset{?}{\overline{)0}}$

CHECK: $\begin{array}{r} ? \\ \times 0 \\ \hline 0 \end{array}$ ◀Any number works, so there is no answer.

Exercises

1–36. Change to a decimal.

1. $\frac{1}{2}$ 0.5	2. $\frac{1}{4}$ 0.25	3. $\frac{1}{5}$ 0.2	4. $\frac{1}{8}$ 0.125
5. $\frac{1}{10}$ 0.1	6. $\frac{5}{8}$ 0.625	7. $\frac{7}{10}$ 0.7	8. $\frac{3}{16}$ 0.1875
9. $\frac{7}{8}$ 0.875	10. $\frac{3}{5}$ 0.6	11. $\frac{1}{16}$ 0.0625	12. $\frac{4}{5}$ 0.8
13. $\frac{5}{16}$ 0.3125	14. $\frac{9}{10}$ 0.9	15. $\frac{2}{5}$ 0.4	16. $\frac{0}{8}$ 0
17. $\frac{39}{50}$ 0.78	18. $\frac{23}{25}$ 0.92	19. $\frac{27}{100}$ 0.27	20. $\frac{3}{1000}$ 0.003
21. $\frac{13}{20}$ 0.65	22. $\frac{23}{40}$ 0.575	23. $\frac{9}{16}$ 0.5625	24. $\frac{33}{80}$ 0.4125
25. $\frac{7}{25}$ 0.28	26. $\frac{0}{20}$ 0	27. $\frac{9}{100}$ 0.09	28. $\frac{7}{16}$ 0.4375
29. $\frac{20}{50}$ 0.4	30. $\frac{47}{1000}$ 0.047	31. $\frac{3}{50}$ 0.06	32. $\frac{83}{100}$ 0.83
33. $\frac{427}{1000}$ 0.427	34. $\frac{99}{100}$ 0.99	35. $\frac{999}{1000}$ 0.999	36. $\frac{7}{20}$ 0.35

37–44. Change to a decimal. If necessary, use a bar over the digits that repeat.

37. $\frac{2}{3}$ $0.\overline{6}$	38. $\frac{5}{9}$ $0.\overline{5}$	39. $\frac{31}{32}$ 0.96875	40. $\frac{3}{7}$ $0.\overline{428\,571}$
41. $\frac{15}{16}$ 0.9375	42. $\frac{5}{12}$ $0.41\overline{6}$	43. $\frac{8}{13}$ $0.\overline{615\,384}$	44. $\frac{47}{64}$ 0.734 375

45–53. True or False?

45. $\frac{9}{25} = 0.36$ T	46. $\frac{11}{20} = 0.55$ T	47. $\frac{5}{0} = 0$ F
48. $\frac{13}{16} = 0.8125$ T	49. $\frac{5}{11} = 0.\overline{45}$ F	50. $\frac{15}{32} = 0.4687$ F
51. $\frac{7}{40} = 0.0175$ F	52. $\frac{1}{3} = 0.\overline{3}$ T	53. $\frac{19}{64} = 0.296\,875$ T

27

Improper Fractions as Decimals

Performance Objective:
The student can
- identify improper fractions.
- change an improper fraction to a decimal or a whole number.

$\frac{12}{3}$ means both $\left\{\begin{array}{l} \textit{twelve-thirds} \\ 12 \div 3 \end{array}\right.$

Examples:

1. $\frac{12}{3}$ ▶ $\begin{array}{r} 4 \\ 3\overline{)12} \end{array}$ So $\frac{12}{3} = 4$

2. $\frac{7}{4}$ ▶ $\begin{array}{r} 1.75 \\ 4\overline{)7.00} \\ \underline{4} \\ 3\,0 \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$ So $\frac{7}{4} = 1.75$

If the numerator is greater than the denominator, the fraction is greater than 1.

3. $\frac{6}{6} = 1$

4. $\frac{17}{17} = 1$

If the numerator and denominator are equal, the fraction equals 1.

A fraction that is greater than or equal to 1 is an **improper fraction.**

Exercises

1–4. Which fractions are improper fractions?

1. $\frac{1}{2}, \underline{\frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}}$

2. $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \underline{\frac{5}{5}, \frac{6}{5}}$

3. $\frac{2}{3}, \underline{\frac{3}{2}}, \frac{4}{7}, \underline{\frac{7}{4}, \frac{7}{7}, \frac{4}{4}}$

4. $\frac{2}{9}, \frac{7}{9}, \underline{\frac{9}{9}, \frac{11}{9}, \frac{21}{9}}$

5–24. Change to a whole number or a decimal.

5. $\frac{9}{2}$ 4.5 **6.** $\frac{11}{4}$ 2.75 **7.** $\frac{9}{8}$ 1.125 **8.** $\frac{26}{4}$ 6.5

9. $\frac{37}{10}$ 3.7 **10.** $\frac{19}{16}$ 1.1875 **11.** $\frac{7}{7}$ 1 **12.** $\frac{12}{10}$ 1.2

13. $\frac{48}{6}$ 8 **14.** $\frac{21}{8}$ 2.625 **15.** $\frac{31}{20}$ 1.55 **16.** $\frac{93}{4}$ 23.25

17. $\frac{17}{2}$ 8.5 **18.** $\frac{609}{30}$ 20.3 **19.** $\frac{300}{20}$ 15 **20.** $\frac{77}{50}$ 1.54

21. $\frac{273}{100}$ 2.73 **22.** $\frac{330}{100}$ 3.3 **23.** $\frac{1381}{1000}$ 1.381 **24.** $\frac{5200}{1000}$ 5.2

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Fractions in Lowest Terms

Performance Objective: The student can reduce a fraction to lowest terms.

A fraction is reduced to lowest terms when the only whole number that divides into both the numerator and the denominator is 1.

Examples:

1. $\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$ ◀ in lowest terms (The only whole number that divides both 3 and 4 is 1.)

CHECK: $60 \leftarrow \frac{15}{20} = \frac{3}{4} \rightarrow 60$

2. $\frac{8}{20} = \frac{8 \div 2}{20 \div 2} = \frac{4}{10}$ ◀ **not** in lowest terms (4 and 10 can both be divided by 2.)

$\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$ ◀ in lowest terms

3. Another way to reduce $\frac{8}{20}$ to lowest terms:

common factors of 8 and 20

all the factors of 8: 1, 2, 4, 8

all the factors of 20: 1, 2, 4, 5, 10, 20

greatest common factor of 8 and 20

$\frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \frac{2}{5}$

CHECK: $40 \leftarrow \frac{8}{20} = \frac{2}{5} \rightarrow 40$

Exercises

1–8. Reduce to lowest terms.

1. $\frac{6}{9}$ $\frac{2}{3}$ **2.** $\frac{4}{10}$ $\frac{2}{5}$ **3.** $\frac{6}{18}$ $\frac{1}{3}$ **4.** $\frac{10}{12}$ $\frac{5}{6}$

5. $\frac{10}{6}$ $\frac{5}{3}$ **6.** $\frac{15}{10}$ $\frac{3}{2}$ **7.** $\frac{18}{24}$ $\frac{3}{4}$ **8.** $\frac{5}{9}$ $\frac{5}{9}$

9–16. Find the greatest common factor of the two numbers.

9. 21, 24 3 **10.** 12, 16 4 **11.** 24, 36 12 **12.** 30, 36 6

13. 70, 100 10 **14.** 30, 70 10 **15.** 40, 60 20 **16.** 120, 100 20

17–24. Find the greatest common factor of the numerator and the denominator. Then reduce the fraction to lowest terms.

17. $\frac{36}{24}$ $\frac{36 \div 12}{24 \div 12} = \frac{3}{2}$ **18.** $\frac{30}{36}$ $\frac{30 \div 6}{36 \div 6} = \frac{5}{6}$ **19.** $\frac{50}{80}$ $\frac{50 \div 10}{80 \div 10} = \frac{5}{8}$ **20.** $\frac{40}{100}$ $\frac{40 \div 20}{100 \div 20} = \frac{2}{5}$

21. $\frac{15}{60}$ $\frac{15 \div 15}{60 \div 15} = \frac{1}{4}$ **22.** $\frac{12}{9}$ $\frac{12 \div 3}{9 \div 3} = \frac{4}{3}$ **23.** $\frac{12}{28}$ $\frac{12 \div 4}{28 \div 4} = \frac{3}{7}$ **24.** $\frac{9}{45}$ $\frac{9 \div 9}{45 \div 9} = \frac{1}{5}$

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Decimals as Fractions

Performance Objective: The student can change a decimal to a fraction in lowest terms.

Examples:

	In words	Decimal	Fraction
1.	*seven-tenths*	0.7	$\frac{7}{10}$
2.	*forty-three hundredths*	0.43	$\frac{43}{100}$
3.	*nine-hundredths*	0.09	$\frac{9}{100}$
4.	*twenty-nine thousandths*	0.029	$\frac{29}{1000}$
5.	*twenty-five hundredths*	0.25	$\frac{25}{100}$ ◀ not in lowest terms

$$\boxed{\frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4} \qquad \text{So } 0.25 = \frac{1}{4}}$$

Exercises

1–30. Change to a fraction in lowest terms.

1. 0.9 $\frac{9}{10}$

2. 0.99 $\frac{99}{100}$

3. 0.999 $\frac{999}{1000}$

4. 0.3 $\frac{3}{10}$

5. 0.03 $\frac{3}{100}$

6. 0.003 $\frac{3}{1000}$

7. 0.71 $\frac{71}{100}$

8. 0.023 $\frac{23}{1000}$

9. 0.149 $\frac{149}{1000}$

10. 0.5 $\frac{1}{2}$

11. 0.75 $\frac{3}{4}$

12. 0.025 $\frac{25}{1000}$

13. 0.125 $\frac{1}{8}$

14. 0.625 $\frac{5}{8}$

15. 0.6 $\frac{3}{5}$

16. 0.84 $\frac{21}{25}$

17. 0.12 $\frac{3}{25}$

18. 0.072 $\frac{9}{125}$

19. 0.004 $\frac{1}{250}$

20. 0.124 $\frac{31}{250}$

21. 0.02 $\frac{1}{50}$

22. 0.109 $\frac{109}{1000}$

23. 0.704 $\frac{88}{125}$

24. 0.16 $\frac{4}{25}$

25. 0.8 $\frac{8}{10}$

26. 0.08 $\frac{2}{25}$

27. 0.11 $\frac{11}{100}$

28. 0.27 $\frac{27}{100}$

29. 0.333 $\frac{333}{1000}$

30. 0.666 $\frac{333}{500}$

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Practice with Fractions

Performance Objective: The student practices changing fractions to equivalent fractions or to whole numbers or decimals.

1–9. Tell whether or not the fractions are equivalent.

1. $\frac{2}{4}, \frac{13}{26}$ Yes **2.** $\frac{5}{8}, \frac{3}{5}$ No Yes **3.** $\frac{7}{3}, \frac{28}{12}$

4. $\frac{3}{21}, \frac{5}{35}$ Yes **5.** $\frac{21}{9}, \frac{8}{3}$ No Yes **6.** $\frac{4}{18}, \frac{6}{27}$

7. $\frac{15}{12}, \frac{12}{9}$ No **8.** $\frac{2}{16}, \frac{3}{26}$ No Yes **9.** $\frac{27}{12}, \frac{9}{4}$

10–24. Tell what number should replace the ? to make the fractions equivalent.

10. $\frac{6}{7} = \frac{?}{42}$ 36 **11.** $\frac{7}{8} = \frac{?}{32}$ 28 **12.** $\frac{6}{5} = \frac{24}{?}$ 20

13. $\frac{7}{?} = \frac{3}{9}$ 21 **14.** $\frac{0}{7} = \frac{?}{9}$ 0 **15.** $\frac{?}{9} = \frac{4}{3}$ 12

16. $\frac{5}{2} = \frac{30}{?}$ 12 **17.** $\frac{21}{?} = \frac{7}{5}$ 15 **18.** $\frac{3}{2} = \frac{?}{12}$ 18

19. $\frac{3}{3} = \frac{7}{?}$ 7 **20.** $\frac{?}{35} = \frac{6}{30}$ 7 **21.** $\frac{4}{15} = \frac{24}{?}$ 90

22. $\frac{21}{?} = \frac{14}{8}$ 12 **23.** $\frac{4}{12} = \frac{?}{3}$ 1 **24.** $\frac{?}{28} = \frac{18}{42}$ 12

25–40. Change to a whole number or to a decimal.

25. $\frac{3}{10}$ 0.3 **26.** $\frac{9}{4}$ 2.25 **27.** $\frac{11}{16}$ 0.6875 **28.** $\frac{9}{20}$ 0.45

29. $\frac{11}{8}$ 1.375 **30.** $\frac{13}{2}$ 6.5 **31.** $\frac{1}{6}$ $0.\overline{16}$ **32.** $\frac{35}{7}$ 5

33. $\frac{13}{25}$ 0.52 **34.** $\frac{5}{5}$ 1 **35.** $\frac{2}{15}$ $0.\overline{13}$ **36.** $\frac{47}{50}$ 0.94

37. $\frac{10}{3}$ $3.\overline{3}$ **38.** $\frac{23}{100}$ 0.23 **39.** $\frac{0}{6}$ 0 **40.** $\frac{32}{1000}$ 0.032

41–56. Change to a fraction in lowest terms.

41. $\frac{6}{24}$ $\frac{1}{4}$ **42.** $\frac{21}{28}$ $\frac{3}{4}$ **43.** $\frac{18}{30}$ $\frac{3}{5}$ **44.** $\frac{21}{6}$ $\frac{7}{2}$

45. $\frac{6}{16}$ $\frac{3}{8}$ **46.** $\frac{20}{35}$ $\frac{4}{7}$ **47.** $\frac{36}{8}$ $\frac{9}{2}$ **48.** $\frac{20}{28}$ $\frac{5}{7}$

49. 0.07 $\frac{7}{100}$ **50.** 0.4 $\frac{2}{5}$ **51.** 0.24 $\frac{6}{25}$ **52.** 0.057 $\frac{57}{1000}$

53. 0.009 $\frac{9}{1000}$ **54.** 0.375 $\frac{3}{8}$ **55.** 0.53 $\frac{53}{100}$ **56.** 0.175 $\frac{7}{40}$

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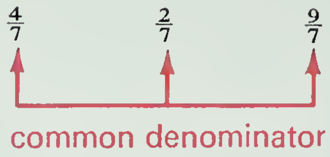
Adding Like Fractions

Like fractions have equal denominators.



Performance Objective: The student can add fractions having like denominators and reduce the sum to the lowest terms.

To add like fractions:

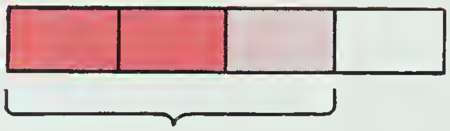
① Add the numerators.

② Repeat the common denominator.

③ Write the sum in lowest terms.

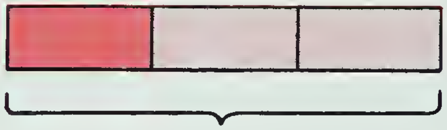
Examples:

1. $\frac{2}{4} + \frac{1}{4}$



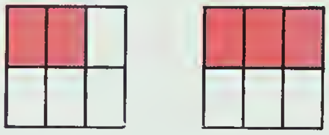
$= \frac{3}{4}$

2. $\frac{1}{3} + \frac{2}{3}$

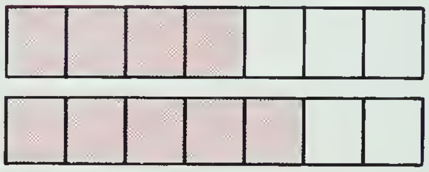


$= \frac{3}{3} = 1$

3. $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$



4.



$$\begin{array}{r} \frac{4}{7} \\ +\frac{5}{7} \\ \hline \frac{9}{7} \end{array}$$

5. $\frac{2}{9} + \frac{4}{9} + \frac{10}{9}$

$= \frac{16}{9}$

Think: $2 + 4 + 10 = 16$

6. $\frac{3}{16} + \frac{5}{16} + \frac{1}{16} + \frac{3}{16}$

Think: $3 + 5 + 1 + 3 = 12$

$= \frac{12}{16}$ ◀ not in lowest terms

$= \frac{3}{4}$

Exercises

1–34. Add. Write each sum as a whole number or as a fraction in lowest terms.

1. $\frac{1}{3} + \frac{1}{3}$ $\frac{2}{3}$ **2.** $\frac{3}{5} + \frac{1}{5}$ $\frac{4}{5}$ **3.** $\frac{5}{8} + \frac{2}{8}$ $\frac{7}{8}$

4. $\frac{3}{4} + \frac{2}{4}$ $\frac{5}{4}$ **5.** $\frac{3}{5} + \frac{4}{5}$ $\frac{7}{5}$ **6.** $\frac{3}{10} + \frac{0}{10}$ $\frac{3}{10}$

7. $\begin{array}{r} \frac{5}{12} \\ +\frac{6}{12} \\ \hline \end{array}$ $\frac{11}{12}$ **8.** $\begin{array}{r} \frac{2}{9} \\ +\frac{8}{9} \\ \hline \end{array}$ $\frac{10}{9}$ **9.** $\begin{array}{r} \frac{2}{7} \\ +\frac{6}{7} \\ \hline \end{array}$ $\frac{8}{7}$

10. $\frac{1}{4} + \frac{1}{4}$ $\frac{1}{2}$ **11.** $\frac{1}{8} + \frac{5}{8}$ $\frac{3}{4}$ **12.** $\frac{2}{10} + \frac{3}{10}$ $\frac{1}{2}$

13. $\frac{9}{12} + \frac{6}{12}$ $\frac{5}{4}$ **14.** $\frac{1}{6} + \frac{5}{6}$ 1 **15.** $\frac{8}{15} + \frac{12}{15}$ $\frac{4}{3}$

16. $\frac{14}{9} + \frac{4}{9}$ 2 **17.** $\frac{6}{20} + \frac{18}{20}$ $\frac{6}{5}$ **18.** $\frac{6}{6} + \frac{5}{6}$ $\frac{11}{6}$

19. $\frac{4}{13} + \frac{7}{13}$ $\frac{11}{13}$ **20.** $\frac{2}{5} + \frac{7}{5}$ $\frac{9}{5}$ **21.** $\frac{3}{8} + \frac{3}{8}$ $\frac{3}{4}$

22. $\frac{3}{10} + \frac{6}{10}$ $\frac{9}{10}$ **23.** $\frac{7}{15} + \frac{6}{15}$ $\frac{13}{15}$ **24.** $\frac{2}{12} + \frac{9}{12}$ $\frac{11}{12}$

25. $\frac{4}{15} + \frac{3}{15} + \frac{7}{15}$ $\frac{14}{15}$ **26.** $\frac{9}{10} + \frac{2}{10} + \frac{7}{10}$ $\frac{9}{5}$

27. $\frac{5}{11} + \frac{0}{11} + \frac{3}{11}$ $\frac{8}{11}$ **28.** $\frac{4}{9} + \frac{9}{9} + \frac{7}{9}$ $\frac{20}{9}$

29. $\frac{1}{10} + \frac{3}{10} + \frac{3}{10}$ $\frac{7}{10}$ **30.** $\frac{6}{7} + \frac{4}{7} + \frac{3}{7}$ $\frac{13}{7}$

31. $\frac{8}{9} + \frac{3}{9} + \frac{11}{9} + \frac{5}{9}$ 3 **32.** $\frac{4}{19} + \frac{8}{19} + \frac{5}{19}$ $\frac{17}{19}$

33. $\frac{6}{16} + \frac{3}{16} + \frac{7}{16}$ 1 **34.** $\frac{12}{7} + \frac{4}{7} + \frac{3}{7} + \frac{5}{7}$ $\frac{24}{7}$

35–40. Tell what number the ? stands for.

35. $\frac{5}{12} + \frac{?}{12} = \frac{7}{12}$ 2 **36.** $\frac{?}{11} + \frac{4}{11} = \frac{8}{11}$ 4 **37.** $\frac{4}{5} + \frac{?}{5} = \frac{12}{5}$ 8

38. $\frac{?}{8} + \frac{7}{8} = \frac{12}{8}$ 5 **39.** $\frac{10}{7} + \frac{?}{7} = \frac{12}{7}$ 2 **40.** $\frac{?}{5} + \frac{6}{5} = \frac{14}{5}$ 8

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Subtracting Like Fractions

Performance Objective:
The student can
- subtract fractions having like denominators and reduce the difference to lowest terms.
- check subtraction of fractions by adding.

To subtract like fractions:

① Subtract the numerators.
② Repeat the common denominator.
③ Check by adding.
④ Write the difference in lowest terms.

Examples:

1. $\frac{13}{10} - \frac{4}{10}$

$= \frac{9}{10}$

Think: $13 - 4 = 9$

CHECK: $\underset{\text{sum}}{\frac{13}{10}} - \underbrace{\frac{4}{10} = \frac{9}{10}}_{\text{addends}}$

2. $\frac{7}{8} - \frac{3}{8}$

$= \frac{4}{8}$

$= \frac{1}{2}$

Think: $7 - 3 = 4$

Check before writing in lowest terms.

Exercises

1–30. Subtract. Check by adding; then write the difference as a whole number or as a fraction in lowest terms.

1. $\frac{2}{3} - \frac{1}{3}$ $\frac{1}{3}$
2. $\frac{4}{5} - \frac{2}{5}$ $\frac{2}{5}$
3. $\frac{6}{7} - \frac{2}{7}$ $\frac{4}{7}$
4. $\frac{3}{4} - \frac{2}{4}$ $\frac{1}{4}$
5. $\frac{3}{2} - \frac{2}{2}$ $\frac{1}{2}$
6. $\frac{8}{9} - \frac{6}{9}$ $\frac{2}{9}$
7. $\frac{9}{5} - \frac{3}{5}$ $\frac{6}{5}$
8. $\frac{7}{3} - \frac{5}{3}$ $\frac{2}{3}$
9. $\frac{7}{8} - \frac{7}{8}$ 0
10. $\frac{4}{5} - \frac{3}{5}$ $\frac{1}{5}$
11. $\frac{5}{6} - \frac{0}{6}$ $\frac{5}{6}$
12. $\frac{7}{10} - \frac{4}{10}$ $\frac{3}{10}$
13. $\frac{3}{4} - \frac{1}{4}$ $\frac{1}{2}$
14. $\frac{6}{10} - \frac{1}{10}$ $\frac{1}{2}$
15. $\frac{7}{8} - \frac{3}{8}$ $\frac{1}{2}$
16. $\frac{5}{6} - \frac{3}{6}$ $\frac{1}{3}$
17. $\frac{9}{8} - \frac{7}{8}$ $\frac{1}{4}$
18. $\frac{9}{10} - \frac{7}{10}$ $\frac{1}{5}$
19. $\frac{4}{9} - \frac{1}{9}$ $\frac{1}{3}$
20. $\frac{5}{7} - \frac{2}{7}$ $\frac{3}{7}$
21. $\frac{9}{2} - \frac{5}{2}$ 2
22. $\frac{8}{13} - \frac{3}{13}$ $\frac{5}{13}$
23. $\frac{14}{25} - \frac{7}{25}$ $\frac{7}{25}$
24. $\frac{11}{14} - \frac{4}{14}$ $\frac{1}{2}$
25. $\frac{12}{7} - \frac{5}{7}$ 1
26. $\frac{12}{11} - \frac{6}{11}$ $\frac{6}{11}$
27. $\frac{11}{12} - \frac{5}{12}$ $\frac{1}{2}$
28. $\frac{37}{100} - \frac{13}{100}$ $\frac{6}{25}$
29. $\frac{13}{20} - \frac{7}{20}$ $\frac{3}{10}$
30. $\frac{49}{50} - \frac{48}{50}$ $\frac{1}{50}$

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Unlike Fractions

unlike fractions ▶ $\frac{1}{2}, \frac{1}{3}$ $\frac{3}{6}, \frac{2}{6}$ ◀ like fractions ▶ $\frac{6}{12}, \frac{4}{12}$

Unlike fractions have unequal denominators.

Performance Objective:
The student can
- identify unlike fractions.
- change two or more unlike fractions to like fractions with the least common denominator.

To change unlike fractions to like fractions with the least common denominator:

① Multiply each given denominator by 1, 2, 3, and so on; list.

② Pick the smallest number common to all the lists.

③ Rewrite each fraction, using the number from Step 2 as the new denominator.

Example: Change $\frac{2}{5}$, $\frac{1}{6}$, and $\frac{3}{10}$ to like fractions.

Step 1

denominators

×1	×2	×3	×4	×5	×6	. . .
5	10	15	20	25	30	. . .
6	12	18	24	30	36	. . .
10	20	30	40	50	60	. . .

Step 2 smallest number common to all three lists

Step 3 $\frac{2}{5} = \frac{2 \times 6}{5 \times 6} = \frac{12}{30}$ $\frac{1}{6} = \frac{1 \times 5}{6 \times 5} = \frac{5}{30}$ $\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$

Exercises

1–9. Use the lists to find the least common denominator.

1. $\frac{1}{3}, \frac{1}{4}$ 12 2. $\frac{1}{3}, \frac{1}{5}$ 15 3. $\frac{1}{4}, \frac{1}{5}$ 20
4. $\frac{1}{3}, \frac{1}{6}$ 6 5. $\frac{1}{4}, \frac{1}{6}$ 12 6. $\frac{1}{5}, \frac{1}{6}$ 30
7. $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ 12 8. $\frac{2}{3}, \frac{4}{5}$ 15 9. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ 12

×1	×2	×3	×4	×5	×6	×7	. . .
3	6	9	12	15	18	21	. . .
4	8	12	16	20	24	28	. . .
5	10	15	20	25	30	35	. . .
6	12	18	24	30	36	42	. . .

10–25. Change to like fractions with the least common denominator.

10. $\frac{2}{3}, \frac{3}{4}$ $\frac{8}{12}, \frac{9}{12}$ 11. $\frac{2}{3}, \frac{4}{5}$ $\frac{10}{15}, \frac{12}{15}$ 12. $\frac{3}{4}, \frac{3}{5}$ $\frac{15}{20}, \frac{12}{20}$ 13. $\frac{2}{3}, \frac{5}{6}$ $\frac{4}{6}, \frac{5}{6}$
14. $\frac{3}{4}, \frac{1}{6}$ $\frac{9}{12}, \frac{2}{12}$ 15. $\frac{4}{5}, \frac{5}{6}$ $\frac{24}{30}, \frac{25}{30}$ 16. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}$ 17. $\frac{2}{3}, \frac{5}{9}$ $\frac{6}{9}, \frac{5}{9}$
18. $\frac{7}{15}, \frac{3}{10}$ $\frac{14}{30}, \frac{9}{30}$ 19. $\frac{5}{8}, \frac{5}{7}$ $\frac{35}{56}, \frac{40}{56}$ 20. $\frac{1}{2}, \frac{5}{6}, \frac{3}{8}$ $\frac{12}{24}, \frac{20}{24}, \frac{9}{24}$ 21. $\frac{9}{8}, \frac{5}{32}$ $\frac{36}{32}, \frac{5}{32}$
22. $\frac{3}{7}, \frac{5}{4}$ $\frac{12}{28}, \frac{35}{28}$ 23. $\frac{5}{8}, \frac{2}{5}$ $\frac{25}{40}, \frac{16}{40}$ 24. $\frac{2}{3}, \frac{1}{4}, \frac{4}{5}$ $\frac{40}{60}, \frac{15}{60}, \frac{48}{60}$ 25. $\frac{8}{7}, \frac{7}{8}$ $\frac{64}{56}, \frac{49}{56}$

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Adding and Subtracting Unlike Fractions

Performance Objective: The student can add or subtract unlike fractions by first changing them to like fractions.

To find the sum (or the difference) of unlike fractions:

① Change to like fractions.

② Add (or subtract), giving the result in lowest terms.

Examples:

1. $\frac{1}{2} + \frac{2}{3}$

Change to like fractions.

$$\left[\begin{array}{lllll} 2 & 4 & \textcircled{6} & 8 & \cdots \\ 3 & \textcircled{6} & 9 & 12 & \cdots \end{array}\right. \quad \begin{array}{l} \rightarrow \quad \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \\ \rightarrow \quad \frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6} \end{array}$$

Add. $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6}$ Think: $3 + 4 = 7$

$= \frac{7}{6}$

2. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

Change to like fractions.

$$\left[\begin{array}{lllllll} 2 & 4 & 6 & 8 & 10 & \textcircled{12} & \cdots \\ 3 & 6 & 9 & \textcircled{12} & 15 & \cdots & \\ 4 & 8 & \textcircled{12} & 16 & \cdots & & \end{array}\right. \quad \begin{array}{l} \rightarrow \quad \frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12} \\ \rightarrow \quad \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \\ \rightarrow \quad \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \end{array}$$

Add. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{6}{12} + \frac{8}{12} + \frac{9}{12}$ Think: $6 + 8 + 9 = 23$

$= \frac{23}{12}$

3. $\frac{3}{5} - \frac{1}{2}$

Change to like fractions.

$$\left[\begin{array}{llllll} 2 & 4 & 6 & 8 & \textcircled{10} & \cdots \\ 5 & \textcircled{10} & 15 & \cdots & & \end{array}\right. \quad \begin{array}{l} \rightarrow \quad \frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} \\ \rightarrow \quad \frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} \end{array}$$

Subtract. $\frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10}$ Think: $6 - 5 = 1$

$= \frac{1}{10}$

4. $\frac{6}{8} - \frac{3}{4}$

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

$$\frac{6}{8} - \frac{3}{4} = \frac{6}{8} - \frac{6}{8}$$

Think: $6 - 6 = 0$

$$= \frac{0}{8}$$

$$= 0$$

Subtracting equal fractions gives 0.

Exercises

1–15. Add. Write the sum as a whole number or as a fraction in lowest terms.

1. $\frac{2}{3} + \frac{3}{4}$ $\frac{17}{12}$
2. $\frac{3}{8} + \frac{5}{12}$ $\frac{19}{24}$
3. $\frac{2}{3} + \frac{5}{9}$ $\frac{11}{9}$
4. $\frac{7}{15} + \frac{3}{10}$ $\frac{23}{30}$
5. $\frac{3}{8} + \frac{5}{7}$ $\frac{61}{56}$
6. $\frac{9}{8} + \frac{5}{32}$ $\frac{41}{32}$
7. $\frac{1}{2} + \frac{5}{6} + \frac{3}{8}$ $\frac{41}{24}$
8. $\frac{2}{3} + \frac{1}{4} + \frac{4}{5}$ $\frac{103}{60}$
9. $\frac{0}{3} + \frac{5}{4}$ $\frac{5}{4}$
10. $\frac{5}{8} + \frac{12}{12}$ $\frac{13}{8}$
11. $\frac{1}{30} + \frac{1}{40}$ $\frac{7}{120}$
12. $\frac{8}{7} + \frac{7}{8}$ $\frac{113}{56}$
13. $\frac{5}{4} + \frac{1}{10} + \frac{2}{5}$ $\frac{7}{4}$
14. $\frac{1}{16} + \frac{5}{8} + \frac{7}{2}$ $\frac{67}{16}$
15. $\frac{7}{3} + \frac{3}{4} + \frac{5}{6}$ $\frac{47}{12}$

16–30. Subtract. Check by adding; then write the difference as a whole number or as a fraction in lowest terms.

16. $\frac{3}{4} - \frac{2}{3}$ $\frac{1}{12}$
17. $\frac{5}{12} - \frac{3}{8}$ $\frac{1}{24}$
18. $\frac{2}{3} - \frac{5}{9}$ $\frac{1}{9}$
19. $\frac{7}{15} - \frac{3}{10}$ $\frac{1}{6}$
20. $\frac{5}{7} - \frac{3}{8}$ $\frac{19}{56}$
21. $\frac{9}{8} - \frac{5}{32}$ $\frac{31}{32}$
22. $\frac{5}{4} - \frac{0}{3}$ $\frac{5}{4}$
23. $\frac{12}{12} - \frac{5}{8}$ $\frac{3}{8}$
24. $\frac{1}{30} - \frac{1}{40}$ $\frac{1}{120}$
25. $\frac{8}{7} - \frac{7}{8}$ $\frac{15}{56}$
26. $\frac{2}{3} - \frac{1}{4}$ $\frac{5}{12}$
27. $\frac{4}{5} - \frac{2}{4}$ $\frac{3}{10}$
28. $\frac{6}{5} - \frac{2}{3}$ $\frac{8}{15}$
29. $\frac{5}{2} - \frac{1}{3}$ $\frac{13}{6}$
30. $\frac{3}{5} - \frac{9}{15}$ 0

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Whole Numbers and Fractions

Performance Objective: The student can

- use the properties of zero and one to simplify working with fractions.
- change a whole number, including zero or one, to a fraction.
- add or subtract whole numbers and fractions.

ZERO

- Every fraction with numerator 0 equals 0.
- Adding 0 to a number does not change its value.
- Subtracting 0 from a number does not change its value.
- Subtracting equal fractions gives 0.
- No fraction can have 0 as its denominator.

Examples:

1. $0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3} = \frac{0}{4} = \cdots$

2. **a.** $\frac{3}{4} + 0 = \frac{3}{4}$ **b.** $\frac{3}{4} + \frac{0}{4} = \frac{3}{4}$

c. $\frac{3}{4} + \frac{0}{5} = \frac{3}{4}$

3. **a.** $\frac{7}{8} - 0 = \frac{7}{8}$ **b.** $\frac{7}{8} - \frac{0}{8} = \frac{7}{8}$

c. $\frac{7}{8} - \frac{0}{3} = \frac{7}{8}$

4. **a.** $\frac{3}{5} - \frac{3}{5} = 0$ **b.** $\frac{1}{4} - \frac{2}{8} = 0$

ONE

- Every fraction with equal numerator and denominator equals 1.
- Every whole number can be written as a fraction with any given number as the denominator.
- To add (or to subtract) whole numbers and fractions, none of which equal 0:

  ① Change the fractions and whole numbers to like fractions.

  ② Add (or subtract).

5. **a.** $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \cdots$

b. $\frac{2}{3} + \frac{1}{3} = 1$ **c.** $\frac{7}{4} - \frac{3}{4} = 1$

d. $1 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{5}{3}$

e. $\frac{9}{4} - 1 = \frac{9}{4} - \frac{4}{4} = \frac{5}{4}$

6.
$$\begin{aligned} 3 &= \frac{3}{1} \\ &= \frac{3 \times 2}{1 \times 2} = \frac{6}{2} \\ &= \frac{3 \times 3}{1 \times 3} = \frac{9}{3} \\ &= \frac{3 \times 4}{1 \times 4} = \frac{12}{4} \end{aligned}$$

7. **a.** $\frac{3}{5} + 2 + \frac{1}{2}$ **b.** $5 - \frac{3}{7}$

$= \frac{6}{10} + \frac{20}{10} + \frac{5}{10}$ $= \frac{35}{7} - \frac{3}{7}$

$= \frac{31}{10}$ $= \frac{32}{7}$

Exercises

1–50. Add or subtract. Write the answer as a whole number or as a fraction in lowest terms. (Use shortcuts when possible.)

1. $\frac{5}{9} + 0$ $\frac{5}{9}$

2. $\frac{0}{7} + \frac{4}{7}$ $\frac{4}{7}$

3. $\frac{7}{9} + \frac{0}{3}$ $\frac{7}{9}$

4. $\frac{0}{2} + \frac{0}{5}$ 0

5. $\frac{1}{4} - 0$ $\frac{1}{4}$

6. $\frac{3}{10} - \frac{0}{10}$ $\frac{3}{10}$

7. $\frac{4}{5} - \frac{0}{8}$ $\frac{4}{5}$

8. $\frac{7}{11} - \frac{7}{11}$ 0

9. $\frac{7}{10} + \frac{3}{10}$ 1

10. $\frac{9}{5} - \frac{4}{5}$ 1

11. $\frac{6}{6} + \frac{6}{6}$ 2

12. $\frac{0}{12} + \frac{8}{3}$ $\frac{8}{3}$

13. $\frac{3}{5} + \frac{0}{5} + \frac{2}{5}$ 1

14. $\frac{3}{4} + \frac{5}{4} + \frac{0}{7}$ 2

15. $\frac{3}{9} + \frac{4}{9} + \frac{2}{9}$ 1

16. $\frac{2}{3} - \frac{6}{9}$ 0

17. $1 + \frac{4}{9}$ $\frac{13}{9}$

18. $\frac{7}{2} - 1$ $\frac{5}{2}$

19. $\frac{3}{7} + 1$ $\frac{10}{7}$

20. $\frac{6}{6} + \frac{5}{12}$ $\frac{17}{12}$

21. $\frac{12}{5} - \frac{3}{3}$ $\frac{7}{5}$

22. $1 - \frac{7}{11}$ $\frac{4}{11}$

23. $\frac{4}{4} - \frac{4}{7}$ $\frac{3}{7}$

24. $5 + \frac{2}{3}$ $\frac{17}{3}$

25. $\frac{4}{5} + 3$ $\frac{19}{5}$

26. $2 - \frac{4}{3}$ $\frac{2}{3}$

27. $\frac{9}{2} - 3$ $\frac{3}{2}$

28. $3 - \frac{0}{4}$ 3

29. $\frac{5}{2} + \frac{3}{2} + 2$ 6

30. $\frac{7}{3} + 1 + \frac{0}{2}$ $\frac{10}{3}$

31. $\frac{12}{2} + \frac{1}{5} + \frac{3}{5}$ $\frac{34}{5}$

32. $\frac{9}{4} + \frac{3}{3} + \frac{3}{8}$ $\frac{29}{8}$

33. $4 + \frac{3}{3} + \frac{0}{5}$ 5

34. $\frac{4}{3} + 0 + \frac{7}{7}$ $\frac{7}{3}$

35. $\frac{0}{8} - 0$ 0

36. $1 + \frac{5}{9} + \frac{4}{9}$ 2

37. $\frac{3}{15} - \frac{1}{5}$ 0

38. $\frac{12}{5} - 2$ $\frac{2}{5}$

39. $4 - \frac{5}{6}$ $\frac{19}{6}$

40. $\frac{3}{5} + \frac{0}{6} + \frac{7}{10}$ $\frac{13}{10}$

41. $\frac{2}{3} + \frac{4}{4} + \frac{1}{5}$ $\frac{28}{15}$

42. $2 + \frac{7}{8} + 3$ $\frac{47}{8}$

43.
$$\begin{array}{r} \frac{7}{5} \\ +\frac{3}{5} \\ \hline 2 \end{array}$$

44.
$$\begin{array}{r} \frac{4}{4} \\ +\frac{4}{9} \\ \hline \frac{13}{9} \end{array}$$

45.
$$\begin{array}{r} \frac{5}{7} \\ -\frac{0}{3} \\ \hline \frac{5}{7} \end{array}$$

46.
$$\begin{array}{r} 6 \\ -\ \frac{8}{3} \\ \hline \frac{10}{3} \end{array}$$

47.
$$\begin{array}{r} \frac{1}{4} \\ \frac{3}{3} \\ +\frac{0}{5} \\ \hline \frac{5}{4} \end{array}$$

48.
$$\begin{array}{r} \frac{4}{2} \\ \frac{1}{7} \\ +\frac{3}{7} \\ \hline \frac{18}{7} \end{array}$$

49.
$$\begin{array}{r} \frac{3}{3} \\ \frac{7}{7} \\ +\frac{6}{6} \\ \hline 3 \end{array}$$

50.
$$\begin{array}{r} \frac{0}{12} \\ 2 \\ +\ \frac{5}{9} \\ \hline \frac{23}{9} \end{array}$$

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Practice Adding and Subtracting Fractions

Performance Objective: The student practices adding and subtracting like fractions, unlike fractions, and whole numbers and fractions.

1–50. Add. Write each result as a whole number or as a fraction in lowest terms.

1. $\frac{3}{8} + \frac{2}{8}$ $\frac{5}{8}$

2. $\frac{4}{3} + \frac{1}{3}$ $\frac{5}{3}$

3. $\frac{6}{11} + \frac{1}{11}$ $\frac{7}{11}$

4. $\frac{1}{3} + \frac{3}{4}$ $\frac{13}{12}$

5. $\frac{2}{3} + \frac{3}{5}$ $\frac{19}{15}$

6. $\frac{5}{8} + \frac{3}{2}$ $\frac{17}{8}$

7. $\frac{1}{9} + \frac{7}{9}$ $\frac{8}{9}$

8. $\frac{2}{5} + \frac{4}{4}$ $\frac{7}{5}$

9. $7 + \frac{3}{4}$ $\frac{31}{4}$

10. $1 + \frac{7}{8}$ $\frac{15}{8}$

11. $\frac{1}{9} + \frac{1}{10}$ $\frac{19}{90}$

12. $\frac{5}{12} + \frac{1}{12}$ $\frac{1}{2}$

13. $\frac{7}{10} + 0$ $\frac{7}{10}$

14. $\frac{5}{9} + \frac{8}{9}$ $\frac{13}{9}$

15. $\frac{8}{3} + \frac{2}{3} + \frac{5}{3}$ 5

16. $\frac{2}{5} + \frac{2}{5}$ $\frac{4}{5}$

17. $\frac{1}{2} + 2 + \frac{2}{3}$ $\frac{19}{6}$

18. $\frac{7}{10} + \frac{4}{5} + \frac{1}{2}$ 2

19. $\frac{8}{5} + \frac{2}{5}$ 2

20. $\frac{4}{7} + \frac{2}{3} + \frac{5}{21}$ $\frac{31}{21}$

21. $\frac{5}{7} + \frac{5}{7}$ $\frac{10}{7}$

22. $\frac{3}{7} + \frac{0}{2} + \frac{5}{2}$ $\frac{41}{14}$

23. $\frac{3}{6} + \frac{1}{6}$ $\frac{2}{3}$

24. $\frac{2}{5} + \frac{1}{5} + \frac{2}{5}$ 1

25. $\frac{3}{5} + \frac{3}{3} + \frac{1}{2}$ $\frac{21}{10}$

26. $\frac{5}{3} + \frac{1}{6} + \frac{5}{9}$ $\frac{43}{18}$

27. $\frac{7}{15} + \frac{3}{15}$ $\frac{2}{3}$

28. $\frac{5}{6} + \frac{1}{2} + \frac{3}{4}$ $\frac{25}{12}$

29. $\frac{5}{6} + \frac{4}{6}$ $\frac{3}{2}$

30. $\frac{1}{3} + \frac{3}{5} + \frac{1}{4}$ $\frac{71}{60}$

31. $\begin{array}{r} \frac{7}{12} \\ +\frac{7}{12} \\ \hline \frac{7}{6} \end{array}$

32. $\begin{array}{r} \frac{7}{10} \\ +\frac{3}{10} \\ \hline 1 \end{array}$

33. $\begin{array}{r} \frac{5}{6} \\ +\frac{0}{8} \\ \hline \frac{5}{6} \end{array}$

34. $\begin{array}{r} \frac{9}{5} \\ +\frac{2}{7} \\ \hline \frac{73}{35} \end{array}$

35. $\begin{array}{r} \frac{7}{4} \\ +\frac{4}{7} \\ \hline \frac{65}{28} \end{array}$

36. $\begin{array}{r} \frac{3}{4} \\ +\frac{0}{4} \\ \hline \frac{3}{4} \end{array}$

37. $\begin{array}{r} 3 \\ +\frac{2}{5} \\ \hline \frac{17}{5} \end{array}$

38. $\begin{array}{r} \frac{7}{4} \\ +\frac{3}{4} \\ \hline \frac{5}{2} \end{array}$

39. $\begin{array}{r} \frac{1}{10} \\ +\frac{7}{10} \\ \hline \frac{4}{5} \end{array}$

40. $\begin{array}{r} \frac{7}{20} \\ +\frac{3}{5} \\ \hline \frac{19}{20} \end{array}$

41. $\begin{array}{r} \frac{4}{3} \\ +\frac{2}{5} \\ \hline \frac{26}{15} \end{array}$

42. $\begin{array}{r} \frac{11}{3} \\ +\frac{4}{3} \\ \hline 5 \end{array}$

43. $\begin{array}{r} \frac{7}{6} \\ +\frac{13}{16} \\ \hline \frac{95}{48} \end{array}$

44. $\begin{array}{r} \frac{9}{2} \\ +\frac{7}{3} \\ \hline \frac{41}{6} \end{array}$

45. $\begin{array}{r} \frac{1}{2} \\ +\frac{5}{2} \\ \hline 3 \end{array}$

46. $\begin{array}{r} \frac{19}{25} \\ +\frac{4}{5} \\ \hline \frac{39}{25} \end{array}$

47. $\begin{array}{r} \frac{13}{6} \\ +\frac{5}{8} \\ \hline \frac{67}{24} \end{array}$

48. $\begin{array}{r} \frac{3}{10} \\ +\frac{17}{100} \\ \hline \frac{47}{100} \end{array}$

49. $\begin{array}{r} \frac{1}{12} \\ +\frac{3}{16} \\ \hline \frac{13}{48} \end{array}$

50. $\begin{array}{r} \frac{3}{8} \\ +\frac{7}{8} \\ \hline \frac{5}{4} \end{array}$

51–100. Subtract. Write each result as a whole number or as a fraction in lowest terms.

51. $\frac{6}{7} - \frac{4}{7}$ $\frac{2}{7}$
52. $\frac{4}{3} - \frac{2}{3}$ $\frac{2}{3}$
53. $\frac{6}{8} - \frac{3}{8}$ $\frac{3}{8}$
54. $\frac{1}{3} - \frac{1}{4}$ $\frac{1}{12}$
55. $\frac{3}{4} - \frac{1}{6}$ $\frac{7}{12}$
56. $\frac{5}{9} - \frac{4}{9}$ $\frac{1}{9}$
57. $\frac{1}{2} - \frac{1}{3}$ $\frac{1}{6}$
58. $\frac{11}{11} - \frac{2}{11}$ $\frac{9}{11}$
59. $\frac{2}{3} - \frac{1}{2}$ $\frac{1}{6}$
60. $\frac{13}{6} - \frac{5}{6}$ $\frac{4}{3}$
61. $\frac{5}{6} - \frac{2}{3}$ $\frac{1}{6}$
62. $\frac{5}{2} - \frac{3}{2}$ 1
63. $\frac{8}{11} - \frac{5}{11}$ $\frac{3}{11}$
64. $\frac{3}{7} - \frac{1}{3}$ $\frac{2}{21}$
65. $\frac{2}{9} - \frac{0}{2}$ $\frac{2}{9}$
66. $\frac{3}{5} - \frac{1}{2}$ $\frac{1}{10}$
67. $\frac{13}{15} - \frac{4}{15}$ $\frac{3}{5}$
68. $\frac{4}{4} - \frac{4}{4}$ 0
69. $\frac{7}{8} - \frac{2}{3}$ $\frac{5}{24}$
70. $\frac{9}{10} - \frac{3}{5}$ $\frac{3}{10}$
71. $\frac{12}{5} - \frac{4}{10}$ 2
72. $\frac{8}{9} - \frac{2}{9}$ $\frac{2}{3}$
73. $\frac{11}{10} - \frac{3}{3}$ $\frac{1}{10}$
74. $\frac{7}{10} - \frac{2}{10}$ $\frac{1}{2}$
75. $1 - \frac{5}{16}$ $\frac{11}{16}$
76. $\frac{9}{10} - \frac{4}{10}$ $\frac{1}{2}$
77. $\frac{7}{11} - \frac{0}{11}$ $\frac{7}{11}$
78. $\frac{10}{13} - \frac{4}{13}$ $\frac{6}{13}$
79. $\frac{9}{2} - 2$ $\frac{5}{2}$
80. $\frac{7}{5} - \frac{3}{5}$ $\frac{4}{5}$

81. $\begin{array}{r} \frac{2}{5} \\ -\frac{1}{10} \\ \hline \frac{3}{10} \end{array}$
82. $\begin{array}{r} \frac{13}{9} \\ -\frac{7}{9} \\ \hline \frac{2}{3} \end{array}$
83. $\begin{array}{r} \frac{8}{20} \\ -\frac{2}{5} \\ \hline 0 \end{array}$
84. $\begin{array}{r} \frac{8}{5} \\ -1 \\ \hline \frac{3}{5} \end{array}$
85. $\begin{array}{r} \frac{6}{7} \\ -\frac{3}{4} \\ \hline \frac{3}{28} \end{array}$
86. $\begin{array}{r} \frac{7}{20} \\ -\frac{3}{20} \\ \hline \frac{1}{5} \end{array}$
87. $\begin{array}{r} \frac{5}{9} \\ -\frac{2}{5} \\ \hline \frac{7}{45} \end{array}$
88. $\begin{array}{r} \frac{13}{12} \\ -\frac{7}{12} \\ \hline \frac{1}{2} \end{array}$
89. $\begin{array}{r} \frac{11}{6} \\ -\frac{4}{3} \\ \hline \frac{1}{2} \end{array}$
90. $\begin{array}{r} 3 \\ -\frac{7}{5} \\ \hline \frac{8}{5} \end{array}$
91. $\begin{array}{r} \frac{7}{8} \\ -\frac{1}{8} \\ \hline \frac{3}{4} \end{array}$
92. $\begin{array}{r} \frac{5}{8} \\ -\frac{3}{8} \\ \hline \frac{1}{4} \end{array}$
93. $\begin{array}{r} \frac{7}{12} \\ -\frac{1}{4} \\ \hline \frac{1}{3} \end{array}$
94. $\begin{array}{r} \frac{17}{25} \\ -\frac{7}{25} \\ \hline \frac{2}{5} \end{array}$
95. $\begin{array}{r} \frac{19}{20} \\ -\frac{1}{5} \\ \hline \frac{3}{4} \end{array}$
96. $\begin{array}{r} \frac{9}{4} \\ -\frac{1}{4} \\ \hline 2 \end{array}$
97. $\begin{array}{r} \frac{11}{12} \\ -\frac{7}{12} \\ \hline \frac{1}{3} \end{array}$
98. $\begin{array}{r} \frac{5}{6} \\ -\frac{7}{18} \\ \hline \frac{4}{9} \end{array}$
99. $\begin{array}{r} \frac{12}{7} \\ -\frac{5}{7} \\ \hline 1 \end{array}$
100. $\begin{array}{r} \frac{3}{12} \\ -\frac{4}{16} \\ \hline 0 \end{array}$

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Multiplying Fractions

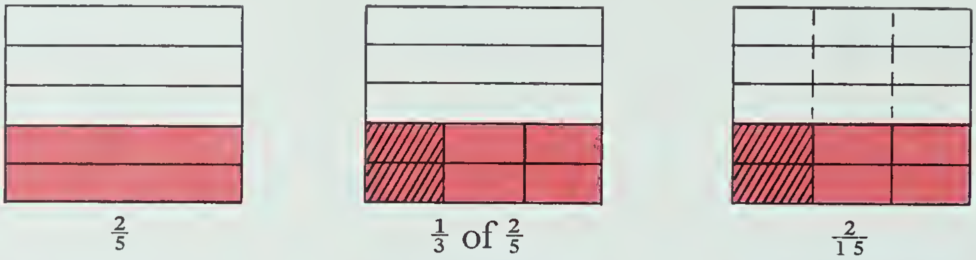
Performance Objective:
The student can

- multiply two or more fractions, or whole numbers and fractions.
- use the multiplication properties of zero, one, and reciprocals when appropriate.

$$\frac{1}{3} \times \frac{2}{5} \text{ means } \begin{cases} \frac{1}{3} \textit{ times } \frac{2}{5} \\ \frac{1}{3} \textit{ of } \frac{2}{5} \end{cases}$$

Examples:

1.



$$\text{That is, } \frac{1}{3} \times \frac{2}{5} = \frac{1 \times 2}{3 \times 5}$$

$$= \frac{2}{15}$$

These steps are often not written.

2. $\frac{1}{4} \times \frac{3}{5} \times \frac{3}{2} = \frac{1 \times 3 \times 3}{4 \times 5 \times 2}$

$$= \frac{9}{40}$$

3. $\frac{3}{8} \times 5 = \frac{3}{8} \times \frac{5}{1}$

$$= \frac{3 \times 5}{8 \times 1}$$

$$= \frac{15}{8}$$

Multiplying a number by 0 gives a product of 0.

4. a. $0 \times \frac{3}{4} = \frac{0}{4}$

$$= 0$$

b. $\frac{7}{8} \times \frac{0}{3} = \frac{0}{24}$

$$= 0$$

Multiplying a number by 1 does not change the value of the number.

5. a. $6 \times \frac{5}{5} = 6 \times 1$

$$= 6$$

b. $\frac{5}{8} \times \frac{3}{3} = \frac{5}{8} \times 1$

$$= \frac{5}{8}$$

The product of **reciprocals** is 1.

6. a. $\underbrace{\frac{2}{3} \times \frac{3}{2}}_{\text{reciprocals}} = \frac{6}{6}$

$$= 1$$

b. $\underbrace{5 \times \frac{1}{5}}_{\text{reciprocals}} = \frac{5}{1} \times \frac{1}{5}$

$$= \frac{5}{5}$$

$$= 1$$

Exercises

1–42. Multiply. Write the answer as a whole number if possible.

1. $\frac{1}{3} \times \frac{4}{5}$ $\frac{4}{15}$ **2.** $\frac{2}{3} \times \frac{2}{5}$ $\frac{4}{15}$ **3.** $\frac{1}{4} \times \frac{2}{5}$ $\frac{2}{20}$ or $\frac{1}{10}$ **4.** $\frac{3}{5} \times \frac{2}{4}$ $\frac{6}{20}$ or $\frac{3}{10}$

5. $\frac{2}{3} \times \frac{1}{3}$ $\frac{2}{9}$ **6.** $\frac{1}{2} \times \frac{3}{4}$ $\frac{3}{8}$ **7.** $\frac{3}{4} \times \frac{5}{2}$ $\frac{15}{8}$ **8.** $\frac{5}{3} \times \frac{1}{2}$ $\frac{5}{6}$

9. $\frac{3}{2} \times \frac{3}{5}$ $\frac{9}{10}$ **10.** $\frac{7}{3} \times \frac{3}{4}$ $\frac{21}{12}$ or $\frac{7}{4}$ **11.** $\frac{5}{7} \times \frac{5}{2}$ $\frac{25}{14}$ **12.** $\frac{4}{9} \times \frac{2}{3}$ $\frac{8}{27}$

13. $\frac{9}{2} \times \frac{5}{8} \times \frac{1}{2}$ $\frac{45}{32}$ **14.** $\frac{2}{3} \times \frac{4}{5} \times \frac{2}{5} \times 10$ $\frac{32}{15}$ **15.** $\frac{3}{5} \times \frac{1}{2} \times \frac{7}{4}$ $\frac{21}{40}$

16. $\frac{1}{4} \times 3 \times \frac{3}{5}$ $\frac{9}{20}$ **17.** $\frac{1}{2} \times 4 \times 6 \times 6$ 72 **18.** $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ $\frac{1}{24}$

19. $\frac{1}{3} \times 6$ 2 **20.** $9 \times \frac{2}{3}$ 6 **21.** $\frac{5}{6} \times 5$ $\frac{25}{6}$ **22.** $10 \times \frac{1}{2}$ 5

23. $8 \times \frac{3}{5}$ $\frac{24}{5}$ **24.** $7 \times \frac{2}{15}$ $\frac{14}{15}$ **25.** $\frac{3}{4} \times 4$ 3 **26.** $\frac{2}{2} \times 8$ 8

27. $\frac{3}{5} \times \frac{0}{8}$ 0 **28.** $\frac{7}{7} \times 4$ 4 **29.** $\frac{7}{10} \times \frac{3}{3}$ $\frac{7}{10}$ **30.** $\frac{0}{5} \times 10$ 0

31–42. Each product is 1.

31. $\frac{1}{2} \times 2$ **32.** $\frac{9}{7} \times \frac{7}{9}$ **33.** $9 \times \frac{1}{9}$ **34.** $\frac{2}{5} \times \frac{5}{2}$

35. $\frac{6}{5} \times \frac{5}{6}$ **36.** $\frac{2}{7} \times \frac{7}{2}$ **37.** $\frac{3}{8} \times \frac{8}{3}$ **38.** $\frac{5}{5} \times \frac{2}{2}$

39. $\frac{7}{10} \times \frac{10}{7}$ **40.** $\frac{1}{25} \times 25$ **41.** $\frac{6}{10} \times \frac{5}{3}$ **42.** $\frac{3}{6} \times 2$

43–54. Tell by what number the ? should be replaced.

43. $? \times \frac{3}{4} = 1$ $\frac{4}{3}$ **44.** $4 \times ? = 1$ $\frac{1}{4}$ **45.** $\frac{3}{5} \times ? = 0$ 0

46. $\frac{?}{2} \times 6 = 0$ 0 **47.** $\frac{2}{9} \times \frac{?}{3} = \frac{2}{9}$ 3 **48.** $\frac{?}{2} \times \frac{5}{5} = \frac{3}{2}$ 3

49. $\frac{1}{6} \times ? = 1$ 6 **50.** $\frac{7}{8} \times ? = 1$ $\frac{8}{7}$ **51.** $9 \times \frac{?}{10} = 9$ 10

52. $\frac{4}{?} \times \frac{4}{7} = \frac{4}{7}$ 4 **53.** $? \times \frac{3}{3} = 4$ 4 **54.** $\frac{?}{5} \times \frac{1}{3} = 0$ 0

55–62. Write the reciprocal of the number as a whole number or as a fraction in lowest terms.

55. $\frac{3}{8}$ $\frac{8}{3}$ **56.** $\frac{11}{7}$ $\frac{7}{11}$ **57.** $\frac{8}{10}$ $\frac{5}{4}$ **58.** $\frac{1}{8}$ 8

59. 7 $\frac{1}{7}$ **60.** 13 $\frac{1}{13}$ **61.** $\frac{3}{12}$ 4 **62.** 1 1

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Products in Lowest Terms

Performance Objective: The student can reduce a fractional product to lowest terms.

Examples:

METHOD I

1. $\frac{2}{3} \times \frac{1}{4}$ Think: $\begin{array}{l} 2 \times 1 = 2 \\ 3 \times 4 = 12 \end{array}$

$= \frac{2}{12}$ ◀ not in lowest terms

$= \frac{2 \div 2}{12 \div 2}$

$= \frac{1}{6}$

METHOD II

2. $\frac{2}{3} \times \frac{1}{4} = \frac{\overset{1}{\cancel{2}} \times 1}{3 \times \underset{2}{\cancel{4}}}$ ◀ Divide numerator and denominator by 2 *before* multiplying.

$= \frac{1}{6}$

or $\frac{\overset{1}{\cancel{2}}}{3} \times \frac{1}{\underset{2}{\cancel{4}}} = \frac{1}{6}$ ◀ to avoid writing the first step above

3. $\frac{3}{5} \times \frac{10}{3} = \frac{\overset{1}{\cancel{3}} \times \overset{2}{\cancel{10}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{3}}}$ ◀ Divide numerator and denominator by 3 and then by 5 before (or after) multiplying.

$= \frac{2}{1}$

$= 2$

4. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{5}{\cancel{25}}} = \frac{1}{15}$ ◀ Divide by 3, 5, and 2.

Exercises

1–30. Multiply. Write the product as a whole number or as a fraction in lowest terms.

1. $\frac{4}{5} \times \frac{1}{4}$ $\frac{1}{5}$
2. $\frac{1}{3} \times \frac{3}{4}$ $\frac{1}{4}$
3. $\frac{7}{5} \times \frac{1}{7}$ $\frac{1}{5}$
4. $\frac{3}{4} \times \frac{8}{5}$ $\frac{6}{5}$
5. $\frac{1}{2} \times \frac{4}{7}$ $\frac{2}{7}$
6. $\frac{3}{4} \times \frac{5}{6}$ $\frac{5}{8}$
7. $\frac{2}{5} \times \frac{3}{4}$ $\frac{3}{10}$
8. $\frac{3}{16} \times \frac{4}{7}$ $\frac{3}{28}$
9. $\frac{3}{4} \times \frac{1}{15}$ $\frac{1}{20}$
10. $\frac{3}{4} \times \frac{4}{9}$ $\frac{1}{3}$
11. $\frac{4}{25} \times \frac{5}{8}$ $\frac{1}{10}$
12. $\frac{5}{12} \times \frac{3}{10}$ $\frac{1}{8}$
13. $\frac{2}{3} \times \frac{6}{10}$ $\frac{2}{5}$
14. $\frac{3}{2} \times \frac{4}{9}$ $\frac{2}{3}$
15. $\frac{4}{15} \times \frac{5}{8}$ $\frac{1}{6}$
16. $\frac{5}{12} \times \frac{3}{5}$ $\frac{1}{4}$
17. $\frac{9}{10} \times \frac{5}{3}$ $\frac{3}{2}$
18. $\frac{18}{25} \times \frac{5}{9}$ $\frac{2}{5}$
19. $\frac{1}{2} \times \frac{4}{5} \times \frac{1}{3}$ $\frac{2}{15}$
20. $\frac{3}{4} \times \frac{1}{2} \times \frac{5}{6}$ $\frac{5}{16}$
21. $\frac{5}{9} \times \frac{7}{10} \times \frac{3}{2}$ $\frac{7}{12}$
22. $\frac{7}{8} \times \frac{4}{5} \times \frac{3}{7}$ $\frac{3}{10}$
23. $\frac{2}{5} \times \frac{1}{4} \times \frac{10}{3}$ $\frac{1}{3}$
24. $\frac{4}{3} \times \frac{5}{6} \times \frac{9}{16}$ $\frac{5}{8}$
25. $\frac{7}{8} \times 12 \times \frac{3}{14}$ $\frac{9}{4}$
26. $\frac{21}{100} \times \frac{4}{7} \times 50$ 6
27. $\frac{8}{3} \times \frac{27}{100} \times \frac{1}{2}$ $\frac{9}{25}$
28. $\frac{5}{12} \times \frac{3}{10} \times \frac{0}{3}$ 0
29. $\frac{7}{10} \times \frac{15}{14} \times \frac{4}{3} \times 6$ 6
30. $\frac{5}{4} \times \frac{5}{9} \times \frac{24}{25} \times 10$ $\frac{20}{3}$

31–39. Tell what whole numbers less than 10 can replace the ? so that multiplying will give a fraction in lowest terms.

31. $\frac{1}{2} \times \frac{?}{15}$ 1, 7
32. $\frac{3}{2} \times \frac{5}{?}$ 1, 2, 4, 7, 8
33. $6 \times \frac{7}{?}$ 1, 5
34. $\frac{9 \times ?}{10 \times 7}$ 1, 3, 9
35. $\frac{5 \times ?}{6 \times 3}$ 1, 5, 7
36. $\frac{3 \times ?}{7 \times 2}$ 1, 3, 5, 9
37. $\frac{4}{5} \times \frac{3}{?}$ 1, 5, 7
38. $\frac{5 \times ?}{3 \times 7}$ 1, 2, 4, 5, 8
39. $\frac{2}{3} \times \frac{5}{7} \times ?$ 1, 2, 4, 5, 8

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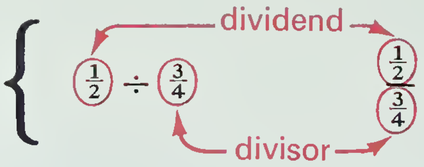
Dividing Fractions

Performance Objective:
The student can
- divide fractions by multiplying the dividend by the reciprocal of the divisor.
- use the division property of one.

Both forms mean *divide* $\frac{1}{2}$ *by* $\frac{3}{4}$.



dividend
$\frac{1}{2} \div \frac{3}{4}$
$\frac{\frac{1}{2}}{\frac{3}{4}}$
divisor

◀ **complex fraction**

$$\frac{1}{2} \div \frac{3}{4} \rightarrow \frac{\frac{1}{2} \times \frac{4}{3}}{\underset{\uparrow}{\frac{3}{4}} \times \underset{\uparrow}{\frac{4}{3}}} = \frac{\frac{1}{2} \times \frac{4}{3}}{1}$$

reciprocals

$$= \frac{1}{2} \times \frac{4}{3}$$

reciprocals

That is, $\frac{1}{2} \div \overset{\downarrow}{\frac{3}{4}} = \frac{1}{2} \times \overset{\downarrow}{\frac{4}{3}}$.

If *either* the dividend or the divisor is a fraction, multiply the dividend by the *reciprocal* of the divisor.

When the divisor is a fraction, the rule above is sometimes stated, Invert the divisor and multiply.

Examples:

1. $\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3}$

◀The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

$= \frac{5}{6}$

CHECK: $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} = \frac{1}{2}$

---

2. $5 \div \frac{2}{3} = \frac{5}{1} \times \frac{3}{2}$

◀Write 5 as a fraction;
the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$= \frac{15}{2}$

CHECK: $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{2}}} = \frac{5}{1} = 5$

3. $\frac{1}{5} \div 3 = \frac{1}{5} \times \frac{1}{3}$

◀The reciprocal of 3 is $\frac{1}{3}$.

$= \frac{1}{15}$

CHECK: $\frac{\overset{1}{\cancel{3}}}{1} \times \frac{1}{\underset{5}{\cancel{15}}} = \frac{1}{5}$

Dividing a number by 1 does not change the value of the number.

4. a. $\frac{3}{5} \div \frac{4}{4} = \frac{3}{5} \div 1$

$= \frac{3}{5}$

b. $7 \div \frac{3}{3} = 7 \div 1$

$= 7$

Remember that division by 0 is not possible.

Exercises

1–10. Name the reciprocal.

1. $\frac{3}{5}$ $\frac{5}{3}$
2. $\frac{1}{4}$ 4
3. $\frac{7}{8}$ $\frac{8}{7}$
4. $\frac{9}{2}$ $\frac{2}{9}$
5. 6 $\frac{1}{6}$
6. $\frac{3}{3}$ 1
7. 12 $\frac{1}{12}$
8. $\frac{1}{2}$ 2
9. $\frac{2}{4}$ 2
10. $\frac{6}{9}$ $\frac{3}{2}$

11–46. Divide. Write each quotient as a whole number or as a fraction in lowest terms. Check by multiplying.

11. $\frac{1}{4} \div \frac{3}{5}$ $\frac{5}{12}$
12. $\frac{3}{5} \div \frac{1}{4}$ $\frac{12}{5}$
13. $\frac{2}{3} \div \frac{7}{8}$ $\frac{16}{21}$
14. $2 \div \frac{9}{2}$ $\frac{4}{9}$
15. $4 \div \frac{3}{5}$ $\frac{20}{3}$
16. $5 \div \frac{1}{4}$ 20
17. $\frac{1}{2} \div 6$ $\frac{1}{12}$
18. $\frac{3}{5} \div 4$ $\frac{3}{20}$
19. $\frac{2}{3} \div 5$ $\frac{2}{15}$
20. $\frac{3}{4} \div \frac{5}{5}$ $\frac{3}{4}$
21. $\frac{3}{4} \div \frac{3}{5}$ $\frac{5}{4}$
22. $\frac{3}{8} \div \frac{5}{4}$ $\frac{3}{10}$
23. $\frac{8}{9} \div \frac{2}{3}$ $\frac{4}{3}$
24. $\frac{4}{9} \div \frac{4}{9}$ 1
25. $\frac{6}{7} \div \frac{3}{4}$ $\frac{8}{7}$
26. $\frac{1}{8} \div \frac{7}{8}$ $\frac{1}{7}$
27. $\frac{3}{3} \div 4$ $\frac{1}{4}$
28. $\frac{5}{9} \div \frac{2}{3}$ $\frac{5}{6}$
29. $\frac{0}{5} \div \frac{3}{10}$ 0
30. $6 \div \frac{3}{7}$ 14
31. $\frac{21}{100} \div 7$ $\frac{3}{100}$
32. $\frac{3}{5} \div \frac{4}{5}$ $\frac{3}{4}$
33. $\frac{2}{3} \div \frac{3}{2}$ $\frac{4}{9}$
34. $\frac{3}{5} \div \frac{3}{4}$ $\frac{4}{5}$
35. $\frac{4}{7} \div 2$ $\frac{2}{7}$
36. $\frac{2}{3} \div \frac{4}{6}$ 1
37. $5 \div \frac{10}{3}$ $\frac{3}{2}$
38. $\dfrac{\frac{3}{5}}{\frac{1}{8}}$ $\frac{24}{5}$
39. $\dfrac{\frac{1}{3}}{\frac{2}{3}}$ $\frac{1}{2}$
40. $\dfrac{\frac{5}{8}}{\frac{5}{6}}$ $\frac{3}{4}$
41. $\dfrac{\frac{4}{5}}{3}$ $\frac{4}{15}$
42. $\dfrac{\frac{5}{8}}{6}$ $\frac{5}{48}$
43. $\dfrac{\frac{0}{3}}{9}$ 0
44. $\dfrac{5}{\frac{3}{2}}$ $\frac{10}{3}$
45. $\dfrac{10}{\frac{2}{3}}$ 15
46. $\dfrac{2}{\frac{5}{2}}$ $\frac{4}{5}$

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Practice Multiplying and Dividing Fractions

Performance Objective: The student practices multiplication with fractions and whole numbers.

1–50. Multiply. Write the product as a whole number or as a fraction in lowest terms.

1. $\frac{2}{3} \times \frac{5}{9}$ $\frac{10}{27}$

2. $\frac{4}{5} \times \frac{3}{5}$ $\frac{12}{25}$

3. $\frac{1}{4} \times \frac{7}{8}$ $\frac{7}{32}$

4. $\frac{7}{8} \times \frac{4}{5}$ $\frac{7}{10}$

5. $\frac{7}{9} \times \frac{0}{5}$ 0

6. $\frac{3}{4} \times \frac{5}{9}$ $\frac{5}{12}$

7. $\frac{5}{6} \times \frac{1}{2}$ $\frac{5}{12}$

8. $\frac{5}{8} \times \frac{8}{5}$ 1

9. $\frac{5}{2} \times \frac{8}{3}$ $\frac{20}{3}$

10. $\frac{4}{9} \times \frac{3}{3}$ $\frac{4}{9}$

11. $\frac{3}{4} \times \frac{3}{5}$ $\frac{9}{20}$

12. $\frac{5}{12} \times 8$ $\frac{10}{3}$

13. $\frac{9}{10} \times \frac{1}{3}$ $\frac{3}{10}$

14. $15 \times \frac{3}{5}$ 9

15. $\frac{4}{5} \times \frac{2}{3}$ $\frac{8}{15}$

16. $\frac{3}{14} \times 7$ $\frac{3}{2}$

17. $\frac{4}{5} \times \frac{10}{11}$ $\frac{8}{11}$

18. $\frac{1}{7} \times 7$ 1

19. $\frac{3}{4} \times \frac{16}{21}$ $\frac{4}{7}$

20. $\frac{8}{3} \times \frac{9}{2}$ 12

21. $\frac{1}{6} \times \frac{12}{13}$ $\frac{2}{13}$

22. $\frac{2}{9} \times \frac{18}{4}$ 1

23. $\frac{4}{15} \times \frac{3}{8}$ $\frac{1}{10}$

24. $\frac{5}{9} \times \frac{3}{15}$ $\frac{1}{9}$

25. $\frac{5}{9} \times \frac{9}{5}$ 1

26. $\frac{8}{27} \times \frac{9}{2}$ $\frac{4}{3}$

27. $\frac{2}{3} \times 24$ 16

28. $\frac{12}{13} \times \frac{26}{3}$ 8

29. $\frac{3}{10} \times 0$ 0

30. $\frac{12}{5} \times \frac{15}{24}$ $\frac{3}{2}$

31. $\frac{4}{5} \times \frac{25}{32}$ $\frac{5}{8}$

32. $\frac{0}{2} \times \frac{4}{3}$ 0

33. $8 \times \frac{7}{2}$ 28

34. $\frac{7}{8} \times 1$ $\frac{7}{8}$

35. $\frac{11}{5} \times \frac{7}{22}$ $\frac{7}{10}$

36. $\frac{1}{20} \times 15$ $\frac{3}{4}$

37. $\frac{3}{4} \times \frac{8}{15} \times \frac{7}{3}$ $\frac{14}{15}$

38. $\frac{7}{12} \times \frac{4}{5} \times \frac{3}{14}$ $\frac{1}{10}$

39. $\frac{5}{9} \times \frac{7}{50} \times \frac{27}{14}$ $\frac{3}{20}$

40. $\frac{7}{8} \times \frac{8}{7} \times \frac{3}{2}$ $\frac{3}{2}$

41. $\frac{12}{5} \times \frac{15}{16} \times \frac{4}{9}$ 1

42. $\frac{121}{13} \times \frac{39}{11} \times \frac{3}{44}$ $\frac{9}{4}$

43. $\frac{8}{9} \times \frac{0}{4} \times \frac{3}{2}$ 0

44. $\frac{5}{8} \times 4 \times \frac{3}{25}$ $\frac{3}{10}$

45. $\frac{5}{6} \times 4 \times 4$ $\frac{40}{3}$

46. $6 \times \frac{7}{8} \times 2 \times 2$ 21

47. $\frac{3}{4} \times \frac{1}{8} \times 10 \times 10$ $\frac{75}{8}$

48. $\frac{7}{54} \times 3 \times 3 \times 6$ 7

49. $\frac{5}{12} \times \frac{1}{2} \times 6 \times 6 \times 3$ $\frac{45}{2}$

50. $\frac{3}{8} \times 3 \times \frac{4}{9} \times 7 \times 7$ $\frac{49}{2}$

51–100. Divide. Write each quotient as a whole number or as a fraction in lowest terms.

51. $\frac{2}{3} \div \frac{3}{4}$ $\frac{8}{9}$

52. $\frac{5}{8} \div \frac{2}{3}$ $\frac{15}{16}$

53. $\frac{3}{5} \div \frac{5}{6}$ $\frac{18}{25}$

54. $\frac{1}{2} \div \frac{2}{3}$ $\frac{3}{4}$

55. $\frac{1}{3} \div \frac{3}{2}$ $\frac{2}{9}$

56. $\frac{3}{4} \div \frac{2}{5}$ $\frac{15}{8}$

57. $\frac{4}{7} \div \frac{1}{4}$ $\frac{16}{7}$

58. $\frac{5}{6} \div \frac{5}{9}$ $\frac{3}{2}$

59. $\frac{9}{10} \div \frac{3}{5}$ $\frac{3}{2}$

60. $\frac{4}{9} \div \frac{2}{7}$ $\frac{14}{9}$

61. $\frac{5}{12} \div \frac{10}{21}$ $\frac{7}{8}$

62. $\frac{7}{8} \div \frac{1}{4}$ $\frac{7}{2}$

63. $\frac{0}{7} \div \frac{2}{3}$ 0

64. $\frac{8}{15} \div \frac{2}{3}$ $\frac{4}{5}$

65. $\frac{9}{20} \div \frac{3}{5}$ $\frac{3}{4}$

66. $\frac{8}{9} \div \frac{4}{6}$ $\frac{1}{3}$

67. $\frac{1}{6} \div \frac{5}{6}$ $\frac{1}{5}$

68. $\frac{2}{5} \div \frac{4}{15}$ $\frac{3}{2}$

69. $\frac{1}{4} \div \frac{3}{16}$ $\frac{4}{3}$

70. $1 \div \frac{9}{5}$ $\frac{5}{9}$

71. $\frac{3}{10} \div \frac{7}{20}$ $\frac{6}{7}$

72. $\frac{8}{9} \div \frac{8}{9}$ 1

73. $\frac{7}{12} \div \frac{21}{36}$ 1

74. $\frac{3}{8} \div \frac{1}{6}$ $\frac{9}{4}$

75. $\frac{8}{8} \div \frac{3}{2}$ $\frac{2}{3}$

76. $\frac{7}{10} \div \frac{14}{15}$ $\frac{3}{4}$

77. $\frac{3}{7} \div \frac{7}{3}$ $\frac{9}{49}$

78. $\frac{8}{11} \div \frac{4}{3}$ $\frac{6}{11}$

79. $0 \div \frac{3}{5}$ 0

80. $\frac{9}{14} \div \frac{3}{7}$ $\frac{3}{2}$

81. $\frac{15}{16} \div \frac{5}{12}$ $\frac{9}{4}$

82. $\frac{5}{18} \div \frac{25}{27}$ $\frac{3}{10}$

83. $\frac{8}{9} \div 4$ $\frac{2}{9}$

84. $\frac{4}{7} \div \frac{3}{3}$ $\frac{4}{7}$

85. $\frac{27}{100} \div \frac{3}{10}$ $\frac{9}{10}$

86. $\frac{14}{25} \div \frac{7}{10}$ $\frac{4}{5}$

87. $\frac{4}{15} \div \frac{4}{5}$ $\frac{1}{3}$

88. $9 \div \frac{3}{4}$ 12

89. $\frac{1}{10} \div 10$ $\frac{1}{100}$

90. $8 \div \frac{4}{5}$ 10

91. $\frac{8}{8} \div \frac{4}{5}$ $\frac{5}{4}$

92. $\frac{9}{10} \div 15$ $\frac{3}{50}$

93. $\dfrac{\frac{7}{8}}{\frac{3}{4}}$ $\frac{7}{6}$

94. $\dfrac{\frac{13}{15}}{\frac{3}{10}}$ $\frac{26}{9}$

95. $\dfrac{\frac{7}{3}}{\frac{10}{3}}$ $\frac{7}{10}$

96. $\dfrac{\frac{5}{11}}{\frac{5}{6}}$ $\frac{6}{11}$

97. $\dfrac{\frac{4}{15}}{\frac{2}{5}}$ $\frac{2}{3}$

98. $\dfrac{\frac{7}{12}}{\frac{1}{3}}$ $\frac{7}{4}$

99. $\dfrac{\frac{21}{12}}{\frac{3}{4}}$ $\frac{7}{3}$

100. $\dfrac{\frac{0}{6}}{\frac{7}{3}}$ 0

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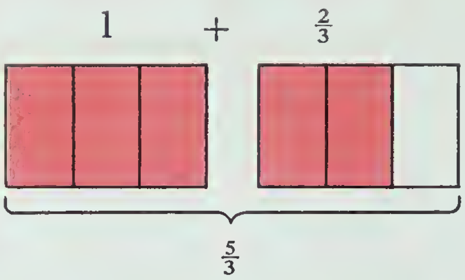
Meaning of Mixed Numbers

Performance Objective:
The student can read mixed numbers and interpret them either as sums or by using models.



$$1 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3}$$
$$= \frac{5}{3}$$

$1 + \frac{2}{3}$ can also be written as the **mixed number** $1\frac{2}{3}$.

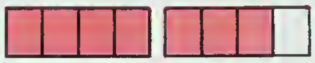
▼ Read *one and two-thirds.*

> A **mixed number** stands for the *sum* of a *whole number* and a *fraction.*

Exercises

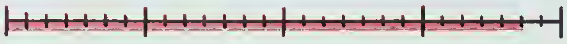
1–6. Use a mixed number to tell how much is in red.

1.



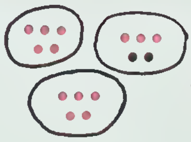
$1\frac{3}{4}$

2.



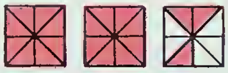
$3\frac{5}{7}$

3.



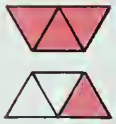
$2\frac{3}{5}$

4.



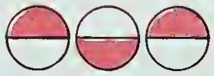
$2\frac{3}{8}$

5.



$1\frac{1}{3}$

6.



$1\frac{1}{2}$

7–14. Write each sum as a mixed number.

7. $5 + \frac{3}{8}$ $5\frac{3}{8}$

8. $4 + \frac{3}{4}$ $4\frac{3}{4}$

9. $\frac{2}{5} + 3$ $3\frac{2}{5}$

10. $\frac{7}{8} + 8$ $8\frac{7}{8}$

11. $\frac{7}{1} + \frac{3}{7}$ $7\frac{3}{7}$

12. $\frac{3}{10} + \frac{6}{1}$ $6\frac{3}{10}$

13. $\frac{4}{4} + \frac{1}{2}$ $1\frac{1}{2}$

14. $\frac{5}{6} + \frac{6}{6}$ $1\frac{5}{6}$

15–22. Write in words.

15. $12\frac{1}{2}$

16. $27\frac{3}{5}$

17. $9\frac{7}{10}$

18. $36\frac{21}{100}$

15. Twelve and one-half
16. Twenty-seven and three-fifths
17. Nine and seven-tenths
18. Thirty-six and twenty-one hundredths

19. $8\frac{6}{7}$ Eight and six-sevenths

20. $3\frac{2}{3}$ Three and two-thirds

21. $9\frac{5}{22}$ Nine and five twenty-seconds

22. $15\frac{3}{50}$ Fifteen and three-fiftieths

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Mixed Numbers and Decimals

Performance Objective:
The student can
- change a mixed number to a decimal.
- change a decimal greater than one to a mixed number.

To change a mixed number to a decimal:

① Change the fraction to a decimal.

② Add the whole number and the decimal.

Examples:

1. Change $4\frac{7}{8}$ to a decimal.

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{6\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

So $4\frac{7}{8} = 4.875$

2. Change $5\frac{1}{6}$ to a decimal.

$$\begin{array}{r} 0.16\,66\cdots \\ 6\overline{)1.00} \\ \underline{6} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

The remainder 4 repeats, . . .

. . . so the digit 6 in the quotient repeats.

So $\frac{1}{6} = 0.1\overline{6}$ and $5\frac{1}{6} = 5.1\overline{6}$

To change a decimal to a mixed number:

① Change the part to the right of the decimal point to a fraction in lowest terms.

② Add the whole-number part to this fraction.

3. Change 35.25 to a mixed number.

$$\left.\begin{aligned} 0.25 &= \tfrac{25}{100} \\ &= \tfrac{1}{4} \end{aligned}\right\rangle \quad \text{So } 35.\underbrace{25}_{} = 35 + \tfrac{1}{4} = 35\tfrac{1}{4}$$

Exercises

1–12. Change to a decimal.

1. $2\frac{1}{2}$ 2.5
2. $7\frac{1}{5}$ 7.2
3. $12\frac{3}{10}$ 12.3
4. $9\frac{1}{4}$ 9.25
5. $5\frac{1}{8}$ 5.125
6. $1\frac{3}{16}$ 1.1875
7. $4\frac{3}{5}$ 4.6
8. $16\frac{33}{100}$ 16.33
9. $13\frac{4}{25}$ 13.16
10. $6\frac{7}{20}$ 6.35
11. $14\frac{2}{3}$ $14.\overline{6}$
12. $8\frac{21}{50}$ 8.42

13–24. Change to a mixed number.

13. 5.7 $5\frac{7}{10}$
14. 3.4 $3\frac{2}{5}$
15. 9.75 $9\frac{3}{4}$
16. 13.8 $13\frac{4}{5}$
17. 1.1 $1\frac{1}{10}$
18. 2.125 $2\frac{1}{8}$
19. 6.9 $6\frac{9}{10}$
20. 19.25 $19\frac{1}{4}$
21. 4.625 $4\frac{5}{8}$
22. 7.375 $7\frac{3}{8}$
23. 12.3125 $12\frac{5}{16}$
24. 8.0625 $8\frac{1}{16}$

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Mixed Numbers and Fractions

Performance Objective:
The student can
- change a mixed number to an improper fraction.
- change a proper or an improper fraction or a whole number to a mixed number.

To change a mixed number to a fraction:

① Write the mixed number as a sum of like fractions.

② Add.

Examples:

1. $3\frac{5}{8} = \frac{3}{1} + \frac{5}{8}$

$$= \frac{3 \times 8}{1 \times 8} + \frac{5}{8}$$

$$= \frac{24}{8} + \frac{5}{8}$$

$$= \frac{29}{8}$$

To do this in your head, think

$$3\frac{5}{8}$$

$$(3 \times 8) + 5 = 24 + 5$$

$$= 29$$

So $3\frac{5}{8} = \frac{29}{8}$

To change an improper fraction to a mixed number:

① Divide numerator by denominator to get a whole-number quotient and a remainder.

② Write the whole number followed by a fraction made up of the remainder over the divisor.

2.

$$\frac{30}{7} \;\blacktriangleright\; \begin{array}{rl} 4 & \blacktriangleleft \text{ whole number} \\ 7\overline{)30} & \\ \underline{28} & \\ 2 & \blacktriangleleft \text{ remainder} \end{array}$$

So $\frac{30}{7} = 4\frac{2}{7}$

whole number
remainder
divisor

You can think of a proper fraction as a mixed number with zero as the whole number part.

3. $\frac{2}{3} = 0\frac{2}{3}$ Remember, this means $0 + \frac{2}{3}$ (*not* $0 \times \frac{2}{3}$).

Every whole number can be written as a mixed number with any given number as the denominator of the fraction part.

4. $7 = 6 + 1 = 6\frac{1}{1} = 6\frac{2}{2} = 6\frac{3}{3} = 6\frac{4}{4} = \cdots$

$$= 5 + 2 = 5\frac{2}{1} = 5\frac{4}{2} = 5\frac{6}{3} = 5\frac{8}{4} = \cdots$$

$$= 4 + 3 = 4\frac{3}{1} = 4\frac{6}{2} = 4\frac{9}{3} = 4\frac{12}{4} = \cdots$$

$$\vdots$$

Exercises

1–32. Change to a fraction.

1. $1\frac{7}{8}$	$\frac{15}{8}$	**2.** $4\frac{3}{5}$	$\frac{23}{5}$	**3.** $2\frac{1}{6}$	$\frac{13}{6}$	**4.** $3\frac{3}{4}$	$\frac{15}{4}$
5. $7\frac{1}{2}$	$\frac{15}{2}$	**6.** $8\frac{3}{5}$	$\frac{43}{5}$	**7.** $4\frac{3}{10}$	$\frac{43}{10}$	**8.** $2\frac{3}{100}$	$\frac{203}{100}$
9. $5\frac{4}{5}$	$\frac{29}{5}$	**10.** $8\frac{7}{10}$	$\frac{87}{10}$	**11.** $2\frac{4}{9}$	$\frac{22}{9}$	**12.** $6\frac{4}{7}$	$\frac{46}{7}$
13. $6\frac{2}{3}$	$\frac{20}{3}$	**14.** $9\frac{1}{5}$	$\frac{46}{5}$	**15.** $0\frac{3}{4}$	$\frac{3}{4}$	**16.** $1\frac{43}{50}$	$\frac{93}{50}$
17. $2\frac{1}{7}$	$\frac{15}{7}$	**18.** $4\frac{2}{9}$	$\frac{38}{9}$	**19.** $7\frac{1}{4}$	$\frac{29}{4}$	**20.** $3\frac{6}{7}$	$\frac{27}{7}$
21. $5\frac{7}{9}$	$\frac{52}{9}$	**22.** $6\frac{9}{10}$	$\frac{69}{10}$	**23.** $9\frac{1}{3}$	$\frac{28}{3}$	**24.** $7\frac{2}{5}$	$\frac{37}{5}$
25. $5\frac{5}{6}$	$\frac{35}{6}$	**26.** $5\frac{5}{7}$	$\frac{40}{7}$	**27.** $4\frac{3}{8}$	$\frac{35}{8}$	**28.** $6\frac{8}{9}$	$\frac{62}{9}$
29. $1\frac{3}{7}$	$\frac{10}{7}$	**30.** $3\frac{5}{9}$	$\frac{32}{9}$	**31.** $10\frac{2}{7}$	$\frac{72}{7}$	**32.** $11\frac{1}{8}$	$\frac{89}{8}$

33–64. Change to a mixed number or, if possible, to a whole number.

33. $\frac{17}{2}$	$8\frac{1}{2}$	**34.** $\frac{19}{4}$	$4\frac{3}{4}$	**35.** $\frac{37}{6}$	$6\frac{1}{6}$	**36.** $\frac{67}{10}$	$6\frac{7}{10}$
37. $\frac{25}{4}$	$6\frac{1}{4}$	**38.** $\frac{16}{7}$	$2\frac{2}{7}$	**39.** $\frac{43}{2}$	$21\frac{1}{2}$	**40.** $\frac{59}{6}$	$9\frac{5}{6}$
41. $\frac{38}{4}$	$9\frac{1}{2}$	**42.** $\frac{20}{6}$	$3\frac{1}{3}$	**43.** $\frac{44}{10}$	$4\frac{2}{5}$	**44.** $\frac{28}{6}$	$4\frac{2}{3}$
45. $\frac{42}{9}$	$4\frac{2}{3}$	**46.** $\frac{63}{8}$	$7\frac{7}{8}$	**47.** $\frac{91}{9}$	$10\frac{1}{9}$	**48.** $\frac{23}{7}$	$3\frac{2}{7}$
49. $\frac{65}{8}$	$8\frac{1}{8}$	**50.** $\frac{93}{5}$	$18\frac{3}{5}$	**51.** $\frac{68}{5}$	$13\frac{3}{5}$	**52.** $\frac{85}{3}$	$28\frac{1}{3}$
53. $\frac{108}{5}$	$21\frac{3}{5}$	**54.** $\frac{17}{5}$	$3\frac{2}{5}$	**55.** $\frac{79}{3}$	$26\frac{1}{3}$	**56.** $\frac{94}{8}$	$11\frac{3}{4}$
57. $\frac{25}{3}$	$8\frac{1}{3}$	**58.** $\frac{87}{3}$	29	**59.** $\frac{95}{3}$	$31\frac{2}{3}$	**60.** $\frac{62}{8}$	$7\frac{3}{4}$
61. $\frac{17}{8}$	$2\frac{1}{8}$	**62.** $\frac{23}{8}$	$2\frac{7}{8}$	**63.** $\frac{25}{9}$	$2\frac{7}{9}$	**64.** $\frac{37}{8}$	$4\frac{5}{8}$

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Adding Mixed Numbers

Performance Objective:
The student can
- add two or more mixed numbers.
- add mixed numbers and fractions or whole numbers.

To add mixed numbers (METHOD I):

1 Add the whole numbers.

2 Add the fractions; if possible, write the sum as a whole number or as a mixed number.

③ Add the results of Steps 1 and 2.

Examples:

1.

$$\begin{array}{lrl} & 3\frac{4}{5} & \\ & +4\frac{3}{5} & \\ \hline \text{Step 1} & 7\cancel{\frac{7}{5}} & \text{Step 2} \\ & 1\frac{2}{5} & \\ \hline & 8\frac{2}{5} & \blacktriangleleft \text{Step 3} \end{array}$$

2. $2\frac{1}{2} + 3\frac{1}{3}$

$2 + 3 = 5$ Step 1

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$ Step 2

$= \frac{5}{6}$

$5 + \frac{5}{6} = 5\frac{5}{6}$ ◀ Step 3

To add mixed numbers (METHOD II):

1 Change the mixed numbers to like fractions.

2 Add.

3.

$$\begin{array}{rcll} 3\frac{4}{5} & = & \frac{19}{5} & \\ +4\frac{3}{5} & = & \frac{23}{5} & \text{Step 1} \\ \hline & & \frac{42}{5} & \blacktriangleleft \text{Step 2} \end{array}$$

4.

$$\begin{array}{rcccll} 2\frac{1}{2} & = & \frac{5}{2} & = & \frac{15}{6} & \\ +3\frac{1}{3} & = & \frac{10}{3} & = & \frac{20}{6} & \text{Step 1} \\ \hline & & & & \frac{35}{6} & \blacktriangleleft \text{Step 2} \end{array}$$

With like fractions, METHOD I may be easier. With unlike fractions, METHOD II may be easier. But *the results are equivalent.* Fractions in the final results should be in lowest terms.

5. To add a proper fraction ▶ $\frac{5}{12} + 3\frac{1}{5}$ ◀ to a mixed number,

think $0\frac{5}{12} + 3\frac{1}{5}$ and use METHOD I

or change to $\frac{5}{12} + \frac{16}{5}$ and use METHOD II.

6. To add a whole number ▶ $5 + 3\frac{2}{3}$ ◀ to a mixed number,

add the whole numbers $8\frac{2}{3}$ and write the fraction.

Exercises

1–45. Add. Use any method, but unless the sum is a whole number, write it both as a mixed number and as an improper fraction in lowest terms.

1. $\begin{array}{r} 3\frac{1}{3} \\ +7\frac{1}{3} \\ \hline 10\frac{2}{3} = \frac{32}{3} \end{array}$ **2.** $\begin{array}{r} 9\frac{2}{7} \\ +3\frac{3}{7} \\ \hline 12\frac{5}{7} = \frac{89}{7} \end{array}$ **3.** $\begin{array}{r} 4\frac{2}{9} \\ +7\frac{5}{9} \\ \hline 11\frac{7}{9} = \frac{106}{9} \end{array}$ **4.** $\begin{array}{r} 2\frac{1}{8} \\ +3\frac{5}{8} \\ \hline 5\frac{3}{4} = \frac{23}{4} \end{array}$

5. $\begin{array}{r} 7\frac{3}{10} \\ +2\frac{1}{10} \\ \hline 9\frac{2}{5} = \frac{47}{5} \end{array}$ **6.** $\begin{array}{r} 5\frac{4}{9} \\ +2\frac{2}{9} \\ \hline 7\frac{2}{3} = \frac{23}{3} \end{array}$ **7.** $\begin{array}{r} 2\frac{1}{2} \\ +5\frac{1}{2} \\ \hline 8 \end{array}$ **8.** $\begin{array}{r} 8\frac{2}{3} \\ +3\frac{1}{3} \\ \hline 12 \end{array}$

9. $\begin{array}{r} 2\frac{4}{7} \\ +9\frac{3}{7} \\ \hline 12 \end{array}$ **10.** $\begin{array}{r} 2\frac{7}{9} \\ +3\frac{7}{9} \\ \hline 6\frac{5}{9} = \frac{59}{9} \end{array}$ **11.** $\begin{array}{r} 6\frac{3}{4} \\ +2\frac{3}{4} \\ \hline 9\frac{1}{2} = \frac{19}{2} \end{array}$ **12.** $\begin{array}{r} 4\frac{7}{8} \\ +3\frac{5}{8} \\ \hline 8\frac{1}{2} = \frac{17}{2} \end{array}$

13. $4\frac{1}{8} + 3\frac{1}{4}$ $7\frac{3}{8} = \frac{59}{8}$ **14.** $5\frac{1}{6} + 7\frac{2}{3}$ $12\frac{5}{6} = \frac{77}{6}$ **15.** $2\frac{1}{4} + 3\frac{1}{3}$ $5\frac{7}{12} = \frac{67}{12}$

16. $2\frac{3}{4} + 3\frac{1}{2}$ $6\frac{1}{4} = \frac{25}{4}$ **17.** $3\frac{5}{6} + 5\frac{2}{3}$ $9\frac{1}{2} = \frac{19}{2}$ **18.** $2\frac{7}{8} + 5\frac{1}{4}$ $8\frac{1}{8} = \frac{65}{8}$

19. $2\frac{2}{5} + 7\frac{3}{4}$ $10\frac{3}{20} = \frac{203}{20}$ **20.** $10\frac{2}{3} + 3\frac{3}{4}$ $14\frac{5}{12} = \frac{173}{12}$ **21.** $4\frac{2}{3} + 2\frac{4}{5}$ $7\frac{7}{15} = \frac{112}{15}$

22. $\frac{1}{3} + 2\frac{1}{6}$ $2\frac{1}{2} = \frac{5}{2}$ **23.** $5\frac{1}{4} + \frac{7}{8}$ $6\frac{1}{8} = \frac{49}{8}$ **24.** $\frac{7}{9} + 3\frac{5}{6}$ $4\frac{11}{18} = \frac{83}{18}$

25. $9 + 3\frac{3}{8}$ $12\frac{3}{8} = \frac{99}{8}$ **26.** $4\frac{1}{6} + 5$ $9\frac{1}{6} = \frac{55}{6}$ **27.** $13 + 21\frac{3}{4}$ $34\frac{3}{4} = \frac{139}{4}$

28. $6\frac{3}{10} + 4\frac{1}{10} + 3\frac{3}{10}$
$13\frac{7}{10} = \frac{137}{10}$

29. $5\frac{1}{8} + 2\frac{3}{8} + 9\frac{3}{8}$
$16\frac{7}{8} = \frac{135}{8}$

30. $10\frac{1}{9} + 1\frac{5}{9} + 5\frac{2}{9}$
$16\frac{8}{9} = \frac{152}{9}$

31. $5\frac{1}{6} + 4\frac{1}{3} + 2\frac{1}{12}$
$11\frac{7}{12} = \frac{139}{12}$

32. $1\frac{1}{16} + 3\frac{1}{8} + 2\frac{1}{4}$
$6\frac{7}{16} = \frac{103}{16}$

33. $5\frac{1}{2} + 3\frac{1}{4} + 6\frac{1}{8}$
$14\frac{7}{8} = \frac{119}{8}$

34. $12\frac{1}{4} + 2\frac{3}{8} + 1\frac{13}{16}$
$16\frac{7}{16} = \frac{263}{16}$

35. $7\frac{3}{10} + 8\frac{9}{10} + 11\frac{1}{10}$
$27\frac{3}{10} = \frac{273}{10}$

36. $2\frac{3}{8} + 4\frac{1}{2} + 9\frac{3}{4}$
$16\frac{5}{8} = \frac{133}{8}$

37. $4\frac{1}{3} + 2\frac{5}{6} + 1\frac{1}{2}$
$8\frac{2}{3} = \frac{26}{3}$

38. $6\frac{1}{3} + 7\frac{1}{2} + 3\frac{2}{3}$
$17\frac{1}{2} = \frac{35}{2}$

39. $11\frac{3}{5} + 5\frac{7}{10} + 1\frac{1}{5}$
$18\frac{1}{2} = \frac{37}{2}$

40. $6\frac{2}{5} + 1\frac{1}{4} + 9\frac{2}{3}$
$17\frac{19}{60} = \frac{1039}{60}$

41. $8\frac{1}{2} + 3\frac{7}{10} + 2\frac{1}{5}$
$14\frac{2}{5} = \frac{72}{5}$

42. $12\frac{1}{3} + 7\frac{5}{12} + 4\frac{3}{5}$
$24\frac{7}{20} = \frac{487}{20}$

43. $8\frac{1}{2} + \frac{2}{3} + 3\frac{1}{6}$
$12\frac{1}{3} = \frac{37}{3}$

44. $4\frac{1}{4} + 7 + 3\frac{1}{3}$
$14\frac{7}{12} = \frac{175}{12}$

45. $5\frac{2}{3} + \frac{7}{8} + 9\frac{1}{2}$
$16\frac{1}{24} = \frac{385}{24}$

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Subtracting Mixed Numbers

Performance Objective:
The student can
- subtract a mixed number, a fraction, or a whole number from a mixed number.
- subtract a mixed number or a proper fraction from a whole number.

To subtract mixed numbers (METHOD I):

1. Subtract the fractions; write the result in lowest terms.
2. Subtract the whole numbers.
3. Add the results of Steps 1 and 2.

Examples:

1.

$$\begin{array}{rr} 7 & \frac{5}{6} \\ -4 & \frac{1}{6} \\ \hline & \cancel{\frac{4}{6}} \\ 3 & \frac{2}{3} \end{array}$$

Step 1

Step 2

Step 3

2. $5\frac{3}{4} - 2\frac{1}{2}$

$$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4}$$

$$= \frac{1}{4}$$

Step 1

$$5 - 2 = 3$$

Step 2

$$3\frac{1}{4}$$

Step 3

3. $6\frac{1}{2} - 4 = 6\frac{1}{2} - 4\frac{0}{2}$

$$\frac{1}{2} - \frac{0}{2} = \frac{1}{2}$$

Step 1

$$6 - 4 = 2$$

Step 2

$$2\frac{1}{2}$$

Step 3

4.

$$\begin{array}{r} 3\frac{1}{3} \\ -1\frac{2}{3} \\ \hline \end{array}$$

$\frac{1}{3}$ is smaller than $\frac{2}{3}$.

Take 1 from the whole number part.

Add $\frac{3}{3}$ to the fraction part.

$$\begin{array}{rcrr} 3\frac{1}{3} & = & 2 & \frac{4}{3} \\ -1\frac{2}{3} & = & 1 & \frac{2}{3} \\ \hline & & 1 & \frac{2}{3} \end{array}$$

Step 2

Step 1

Step 3

To subtract mixed numbers (METHOD II):

1. Change the mixed numbers to like fractions.
2. Subtract.

5.

$$\begin{array}{rcl} 7\frac{5}{6} & = & \frac{47}{6} \\ -4\frac{1}{6} & = & \frac{25}{6} \\ \hline & & \frac{22}{6} \end{array}$$

Step 1

Step 2

$$\frac{11}{3}$$

in lowest terms

6.

$$\begin{array}{rcccl} 5\frac{3}{4} & = & \frac{23}{4} & = & \frac{23}{4} \\ -2\frac{1}{2} & = & \frac{5}{2} & = & \frac{10}{4} \\ \hline & & & & \frac{13}{4} \end{array}$$

Step 1

Step 2

To subtract a mixed number (or a proper fraction) from a whole number:

① Change the whole number to a mixed number so the fractions are like fractions.

② Subtract by METHOD I.

or

① Change the whole number and the mixed number to like fractions.

② Subtract by METHOD II.

7.
$$\begin{array}{rcl} 5 & = & 4\frac{2}{2} \quad \text{Step 1} \\ -3\frac{1}{2} & = & 3\frac{1}{2} \\ \hline & & 1\frac{1}{2} \quad \text{Step 2} \end{array}$$

8.
$$\begin{array}{rcl} 9 & = & 8\frac{4}{4} \quad \text{Step 1} \\ -\frac{3}{4} & = & 0\frac{3}{4} \\ \hline & & 8\frac{1}{4} \quad \text{Step 2} \end{array}$$

9.
$$\begin{array}{rcl} 5 & = & \frac{10}{2} \quad \text{Step 1} \\ -3\frac{1}{2} & = & \frac{7}{2} \\ \hline & & \frac{3}{2} \quad \text{Step 2} \end{array}$$

Exercises

1–41. Subtract. Use any method. Check by adding.
Answers are given in two forms.

1. $\begin{array}{r} 4\frac{7}{9} \\ -2\frac{3}{9} \\ \hline 2\frac{4}{9} = \frac{22}{9} \end{array}$ **2.** $\begin{array}{r} 6\frac{4}{5} \\ -5\frac{2}{5} \\ \hline 1\frac{2}{5} = \frac{7}{5} \end{array}$ **3.** $\begin{array}{r} 7\frac{6}{7} \\ -5\frac{3}{7} \\ \hline 2\frac{3}{7} = \frac{17}{7} \end{array}$ **4.** $\begin{array}{r} 9\frac{8}{9} \\ -2\frac{7}{9} \\ \hline 7\frac{1}{9} = \frac{64}{9} \end{array}$

5. $\begin{array}{r} 5\frac{3}{4} \\ -4\frac{5}{8} \\ \hline 1\frac{1}{8} = \frac{9}{8} \end{array}$ **6.** $\begin{array}{r} 8\frac{3}{4} \\ -6\frac{1}{2} \\ \hline 2\frac{1}{4} = \frac{9}{4} \end{array}$ **7.** $\begin{array}{r} 7\frac{7}{8} \\ -3\frac{1}{4} \\ \hline 4\frac{5}{8} = \frac{37}{8} \end{array}$ **8.** $\begin{array}{r} 6\frac{4}{5} \\ -5\frac{3}{10} \\ \hline 1\frac{1}{2} = \frac{3}{2} \end{array}$

9. $7\frac{1}{3} - 4\frac{1}{6}$ $3\frac{1}{6} = \frac{19}{6}$ **10.** $8\frac{5}{9} - 2\frac{1}{3}$ $6\frac{2}{9} = \frac{56}{9}$ **11.** $12\frac{5}{6} - 3\frac{5}{9}$ $9\frac{5}{18} = \frac{167}{18}$

12. $9\frac{1}{6} - 2\frac{5}{6}$ $6\frac{1}{3} = \frac{19}{3}$ **13.** $11\frac{1}{5} - 7\frac{3}{5}$ $3\frac{3}{5} = \frac{18}{5}$ **14.** $9\frac{3}{10} - 7\frac{7}{10}$ $1\frac{3}{5} = \frac{8}{5}$

15. $3\frac{1}{4} - 1\frac{3}{4}$ $1\frac{1}{2} = \frac{3}{2}$ **16.** $7\frac{1}{8} - 3\frac{5}{8}$ $3\frac{1}{2} = \frac{7}{2}$ **17.** $4\frac{1}{3} - 3\frac{2}{3}$ $\frac{2}{3}$

18. $12\frac{2}{3} - 7$ $5\frac{2}{3} = \frac{17}{3}$ **19.** $4\frac{5}{8} - 3$ $1\frac{5}{8} = \frac{13}{8}$ **20.** $7\frac{3}{5} - 5$ $2\frac{3}{5} = \frac{13}{5}$

21. $5\frac{1}{2} - 3\frac{5}{8}$ $1\frac{7}{8} = \frac{15}{8}$ **22.** $4\frac{1}{10} - 2\frac{3}{5}$ $1\frac{1}{2} = \frac{3}{2}$ **23.** $12\frac{1}{6} - 7\frac{2}{3}$ $4\frac{1}{2} = \frac{9}{2}$

24. $8\frac{3}{8} - 4\frac{3}{4}$ $3\frac{5}{8} = \frac{29}{8}$ **25.** $5\frac{1}{4} - 3\frac{1}{2}$ $1\frac{3}{4} = \frac{7}{4}$ **26.** $7\frac{1}{6} - 3\frac{2}{9}$ $3\frac{17}{18} = \frac{71}{18}$

27. $6 - 3\frac{1}{2}$ $2\frac{1}{2} = \frac{5}{2}$ **28.** $7 - 5\frac{3}{8}$ $1\frac{5}{8} = \frac{13}{8}$ **29.** $9 - 5\frac{8}{10}$ $3\frac{1}{5} = \frac{16}{5}$

30. $5 - \frac{3}{16}$ $4\frac{13}{16} = \frac{77}{16}$ **31.** $8 - \frac{5}{12}$ $7\frac{7}{12} = \frac{91}{12}$ **32.** $13 - \frac{3}{7}$ $12\frac{4}{7} = \frac{88}{7}$

33. $9\frac{1}{2} - 4\frac{1}{2}$ 5 **34.** $12\frac{3}{5} - 7\frac{6}{10}$ 5 **35.** $8\frac{90}{100} - 5\frac{9}{10}$ 3

36. $2\frac{1}{2} - 2\frac{3}{6}$ 0 **37.** $13\frac{1}{2} - 0$ $13\frac{1}{2} = \frac{27}{2}$ **38.** $12 - 11\frac{1}{5}$ $\frac{4}{5}$

39. $4\frac{3}{8} - 4\frac{1}{4}$ $\frac{1}{8}$ **40.** $10\frac{3}{5} - \frac{0}{5}$ $10\frac{3}{5} = \frac{53}{5}$ **41.** $8\frac{5}{12} - 8$ $\frac{5}{12}$

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Multiplying and Dividing Mixed Numbers

Performance Objective: The student can multiply and divide with mixed numbers.

To multiply (or to divide) with mixed numbers:

① Change the mixed numbers to fractions.

② Multiply (or divide); write the product (or the quotient) as a fraction or as a mixed number.

Examples:

1. $$\begin{aligned} 2\tfrac{1}{4} \times 4\tfrac{3}{5} &= \tfrac{9}{4} \times \tfrac{23}{5} \\ &= \frac{9 \times 23}{4 \times 5} \\ &= \tfrac{207}{20} \\ &= 10\tfrac{7}{20} \end{aligned}$$

2. $$\begin{aligned} 3\tfrac{1}{2} \div 2\tfrac{2}{5} &= \tfrac{7}{2} \div \tfrac{12}{5} \\ &= \tfrac{7}{2} \times \tfrac{5}{12} \\ &= \tfrac{35}{24} \\ &= 1\tfrac{11}{24} \end{aligned}$$

3. $$\begin{aligned} 4\tfrac{3}{4} \times \tfrac{4}{5} &= \frac{19}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{5} \\ &= \tfrac{19}{5} \\ &= 3\tfrac{4}{5} \end{aligned}$$

4. $$\begin{aligned} \tfrac{3}{4} \div 3\tfrac{1}{2} &= \tfrac{3}{4} \div \tfrac{7}{2} \\ &= \frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{7} \\ &= \tfrac{3}{14} \end{aligned}$$

5. $$\begin{aligned} 1\tfrac{2}{5} \times 4 &= \tfrac{7}{5} \times \tfrac{4}{1} \\ &= \tfrac{28}{5} \\ &= 5\tfrac{3}{5} \end{aligned}$$

6. $$\begin{aligned} 2 \div 1\tfrac{3}{4} &= \tfrac{2}{1} \div \tfrac{7}{4} \\ &= \tfrac{2}{1} \times \tfrac{4}{7} \\ &= \tfrac{8}{7} \\ &= 1\tfrac{1}{7} \end{aligned}$$

The form in which you write the result usually depends on what the result is to be used for. Fractions in the final results should be in lowest terms.

Exercises

1–24. Multiply. Write each product as a whole number, a mixed number, or a proper fraction in lowest terms.

1. $3\frac{1}{2} \times 2\frac{1}{4}$ $7\frac{7}{8}$ **2.** $2\frac{3}{4} \times 1\frac{1}{2}$ $4\frac{1}{8}$ **3.** $2\frac{1}{8} \times 3\frac{1}{2}$ $7\frac{7}{16}$

4. $3\frac{2}{3} \times 1\frac{2}{5}$ $5\frac{2}{15}$ **5.** $3\frac{3}{5} \times 2\frac{1}{3}$ $8\frac{2}{5}$ **6.** $3\frac{2}{3} \times 5\frac{4}{5}$ $21\frac{4}{15}$

7. $1\frac{1}{2} \times 2\frac{3}{8}$ $3\frac{9}{16}$ **8.** $6\frac{1}{8} \times 5\frac{1}{2}$ $33\frac{11}{16}$ **9.** $2\frac{3}{4} \times 2\frac{2}{5}$ $6\frac{3}{5}$

10. $3\frac{1}{3} \times 2\frac{3}{4}$ $9\frac{1}{6}$ **11.** $6\frac{5}{8} \times 1\frac{2}{3}$ $11\frac{1}{24}$ **12.** $6\frac{1}{3} \times 2\frac{4}{5}$ $17\frac{11}{15}$

13. $7\frac{2}{3} \times \frac{4}{5}$ $6\frac{2}{15}$ **14.** $8\frac{1}{2} \times \frac{2}{3}$ $5\frac{2}{3}$ **15.** $\frac{3}{10} \times 5\frac{2}{3}$ $1\frac{7}{10}$

16. $3\frac{1}{2} \times \frac{2}{7}$ 1 **17.** $\frac{5}{12} \times 4\frac{2}{3}$ $1\frac{17}{18}$ **18.** $\frac{5}{8} \times 1\frac{3}{5}$ 1

19. $6\frac{7}{8} \times \frac{3}{3}$ $6\frac{7}{8}$ **20.** $6\frac{2}{3} \times \frac{3}{2}$ 10 **21.** $\frac{0}{7} \times 5\frac{1}{2}$ 0

22. $2\frac{1}{3} \times 5 \times 1\frac{2}{7}$ 15 **23.** $7 \times 5\frac{1}{2} \times \frac{3}{7}$ $16\frac{1}{2}$ **24.** $1\frac{2}{9} \times 3 \times 2\frac{2}{3}$ $9\frac{7}{9}$

25–48. Divide. Write each quotient as a whole number, a mixed number, or a proper fraction in lowest terms. Check by multiplying.

25. $4\frac{1}{2} \div 2\frac{1}{2}$ $1\frac{4}{5}$ **26.** $7\frac{1}{3} \div 5\frac{2}{3}$ $1\frac{5}{17}$ **27.** $6\frac{2}{5} \div 3\frac{4}{5}$ $1\frac{13}{19}$

28. $3\frac{1}{2} \div 4\frac{3}{4}$ $\frac{14}{19}$ **29.** $2\frac{1}{2} \div 3\frac{3}{4}$ $\frac{2}{3}$ **30.** $6\frac{3}{8} \div 2\frac{3}{4}$ $2\frac{7}{22}$

31. $2\frac{1}{2} \div 4\frac{3}{4}$ $\frac{10}{19}$ **32.** $2\frac{3}{8} \div 6\frac{1}{2}$ $\frac{19}{52}$ **33.** $2\frac{2}{3} \div 3\frac{3}{5}$ $\frac{20}{27}$

34. $4\frac{1}{5} \div 2\frac{1}{3}$ $1\frac{4}{5}$ **35.** $2\frac{3}{6} \div 2\frac{1}{2}$ 1 **36.** $3\frac{2}{3} \div 4\frac{2}{3}$ $\frac{11}{14}$

37. $5\frac{1}{5} \div \frac{4}{5}$ $6\frac{1}{2}$ **38.** $\frac{4}{5} \div 2\frac{2}{3}$ $\frac{3}{10}$ **39.** $6\frac{7}{8} \div \frac{5}{5}$ $6\frac{7}{8}$

40. $\frac{16}{5} \div 3\frac{1}{5}$ 1 **41.** $4\frac{5}{6} \div \frac{2}{3}$ $7\frac{1}{4}$ **42.** $\frac{7}{8} \div 3\frac{1}{4}$ $\frac{7}{26}$

43. $7\frac{2}{3} \div 3$ $2\frac{5}{9}$ **44.** $4\frac{4}{5} \div 6$ $\frac{4}{5}$ **45.** $8 \div 2\frac{3}{8}$ $3\frac{7}{19}$

46. $1 \div 5\frac{1}{2}$ $\frac{2}{11}$ **47.** $3\frac{5}{9} \div 32$ $\frac{1}{9}$ **48.** $0 \div 12\frac{1}{3}$ 0

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Practice with Mixed Numbers

Performance Objective: The student practices the four basic operations with mixed numbers.

1–36. Add. Write the sum as a whole number or as a mixed number.

1. $2\frac{2}{3} + 5\frac{2}{3}$ $8\frac{1}{3}$
2. $9\frac{5}{8} + 3\frac{1}{8}$ $12\frac{3}{4}$
3. $4\frac{5}{12} + 3\frac{1}{12}$ $7\frac{1}{2}$
4. $3\frac{2}{3} + 2\frac{1}{3}$ 6
5. $7\frac{7}{8} + 3\frac{1}{8}$ 11
6. $8\frac{1}{4} + 2\frac{7}{12}$ $10\frac{5}{6}$
7. $4\frac{5}{6} + 2\frac{2}{9}$ $7\frac{1}{18}$
8. $4\frac{3}{4} + 5\frac{1}{6}$ $9\frac{11}{12}$
9. $2\frac{5}{6} + 7\frac{3}{8}$ $10\frac{5}{24}$
10. $4\frac{7}{8} + 7\frac{5}{12}$ $12\frac{7}{24}$
11. $9\frac{3}{5} + 5\frac{3}{4}$ $15\frac{7}{20}$
12. $5\frac{1}{2} + 6\frac{3}{5}$ $12\frac{1}{10}$
13. $9\frac{3}{7} + 4\frac{2}{5}$ $13\frac{29}{35}$
14. $7\frac{1}{6} + 5\frac{2}{7}$ $12\frac{19}{42}$
15. $3\frac{1}{2} + 6\frac{2}{3}$ $10\frac{1}{6}$
16. $5\frac{3}{4} + 3\frac{2}{3}$ $9\frac{5}{12}$
17. $6\frac{3}{5} + 7\frac{1}{6}$ $13\frac{23}{30}$
18. $5\frac{5}{6} + 8\frac{3}{10}$ $14\frac{2}{15}$
19. $6\frac{1}{2} + 4\frac{3}{7}$ $10\frac{13}{14}$
20. $8\frac{7}{10} + 6\frac{3}{8}$ $15\frac{3}{40}$
21. $3\frac{4}{5} + \frac{3}{8}$ $4\frac{7}{40}$
22. $7\frac{3}{5} + 27$ $34\frac{3}{5}$
23. $3\frac{2}{3} + 8\frac{4}{5}$ $12\frac{7}{15}$
24. $\frac{5}{12} + 8\frac{4}{9}$ $8\frac{31}{36}$
25. $6\frac{2}{3} + 2\frac{5}{8}$ $9\frac{7}{24}$
26. $5 + 8\frac{3}{4}$ $13\frac{3}{4}$
27. $4\frac{7}{10} + 11\frac{4}{15}$ $15\frac{29}{30}$
28. $4\frac{1}{7} + 1\frac{2}{7} + 9\frac{3}{7}$ $14\frac{6}{7}$
29. $6\frac{2}{3} + 1\frac{2}{3} + 8\frac{1}{3}$ $16\frac{2}{3}$
30. $4\frac{1}{10} + 9\frac{7}{10} + 13\frac{3}{10}$ $27\frac{1}{10}$
31. $3\frac{1}{8} + 9\frac{3}{4} + 2\frac{1}{12}$ $14\frac{23}{24}$
32. $7 + 5\frac{7}{11} + 12$ $24\frac{7}{11}$
33. $4\frac{2}{3} + 11\frac{1}{6} + 2\frac{5}{8}$ $18\frac{11}{24}$
34. $7\frac{2}{3} + 1\frac{1}{4} + 3\frac{1}{2}$ $12\frac{5}{12}$
35. $4\frac{1}{6} + 8 + \frac{7}{9}$ $12\frac{17}{18}$
36. $9\frac{1}{2} + 2\frac{3}{5} + \frac{3}{4}$ $12\frac{17}{20}$

37–72. Subtract. Write the difference as a whole number, a mixed number, or a proper fraction in lowest terms. Check by adding.

37. $9\frac{5}{7} - 3\frac{1}{7}$ $6\frac{4}{7}$
38. $12\frac{4}{5} - 7\frac{2}{5}$ $5\frac{2}{5}$
39. $4\frac{7}{9} - 3\frac{5}{9}$ $1\frac{2}{9}$
40. $8\frac{7}{8} - 2\frac{5}{8}$ $6\frac{1}{4}$
41. $6\frac{3}{10} - 4\frac{9}{10}$ $1\frac{2}{5}$
42. $11\frac{5}{12} - 7\frac{7}{12}$ $3\frac{5}{6}$
43. $5\frac{3}{4} - 4\frac{3}{4}$ 1
44. $8\frac{1}{2} - 3\frac{1}{2}$ 5
45. $7\frac{7}{8} - 7\frac{2}{3}$ $\frac{5}{24}$
46. $7\frac{6}{10} - 6\frac{4}{5}$ $\frac{4}{5}$
47. $7\frac{1}{3} - 4\frac{3}{4}$ $2\frac{7}{12}$
48. $4\frac{3}{8} - \frac{5}{16}$ $4\frac{1}{16}$
49. $10 - 6\frac{2}{3}$ $3\frac{1}{3}$
50. $8\frac{5}{9} - 3\frac{5}{6}$ $4\frac{13}{18}$
51. $12\frac{8}{15} - \frac{4}{5}$ $11\frac{11}{15}$
52. $11\frac{1}{5} - 7\frac{7}{10}$ $3\frac{1}{2}$
53. $5\frac{8}{11} - 3$ $2\frac{8}{11}$
54. $7\frac{1}{8} - 1\frac{5}{12}$ $5\frac{17}{24}$
55. $8 - 4\frac{3}{16}$ $3\frac{13}{16}$
56. $5\frac{1}{2} - 2\frac{3}{5}$ $2\frac{9}{10}$
57. $7 - \frac{3}{4}$ $6\frac{1}{4}$
58. $12\frac{5}{6} - 6\frac{7}{8}$ $5\frac{23}{24}$
59. $9\frac{2}{3} - \frac{5}{6}$ $8\frac{5}{6}$
60. $4\frac{3}{7} - 1\frac{1}{10}$ $3\frac{23}{70}$

61. $6 - \frac{4}{9}$ $5\frac{5}{9}$ **62.** $12\frac{5}{9} - 8\frac{7}{12}$ $3\frac{35}{36}$ **63.** $13\frac{9}{10} - 4\frac{1}{6}$ $9\frac{11}{15}$

64. $3\frac{1}{12} - 3\frac{1}{18}$ $\frac{1}{36}$ **65.** $9\frac{5}{8} - \frac{0}{6}$ $9\frac{5}{8}$ **66.** $21 - 5\frac{5}{9}$ $15\frac{4}{9}$

67. $4\frac{3}{8} - 1\frac{5}{6}$ $2\frac{13}{24}$ **68.** $4\frac{1}{6} - 3\frac{5}{9}$ $\frac{11}{18}$ **69.** $13 - \frac{3}{14}$ $12\frac{11}{14}$

70. $8\frac{3}{4} - 5\frac{7}{12}$ $3\frac{1}{6}$ **71.** $23\frac{1}{10} - 4\frac{1}{8}$ $18\frac{39}{40}$ **72.** $8\frac{3}{16} - 5\frac{7}{12}$ $2\frac{29}{48}$

73–96. Multiply. Write the product as a whole number or a mixed number.

73. $1\frac{3}{4} \times 2\frac{1}{2}$ $4\frac{3}{8}$ **74.** $2\frac{2}{3} \times 3\frac{1}{4}$ $8\frac{2}{3}$ **75.** $3\frac{2}{5} \times 1\frac{1}{3}$ $4\frac{8}{15}$

76. $2\frac{5}{8} \times 1\frac{1}{4}$ $3\frac{9}{32}$ **77.** $5\frac{1}{2} \times 3$ $16\frac{1}{2}$ **78.** $1\frac{7}{8} \times 2\frac{1}{2}$ $4\frac{11}{16}$

79. $4\frac{1}{3} \times 1\frac{2}{7}$ $5\frac{4}{7}$ **80.** $2\frac{1}{8} \times 1\frac{1}{5}$ $2\frac{11}{20}$ **81.** $2\frac{3}{7} \times 3\frac{1}{2}$ $8\frac{1}{2}$

82. $1\frac{4}{5} \times 4\frac{1}{2}$ $8\frac{1}{10}$ **83.** $4 \times 2\frac{1}{6}$ $8\frac{2}{3}$ **84.** $2\frac{2}{5} \times 2\frac{1}{6}$ $5\frac{1}{5}$

85. $\frac{4}{9} \times 3\frac{1}{4}$ $1\frac{4}{9}$ **86.** $2\frac{3}{4} \times 1\frac{1}{7}$ $3\frac{1}{7}$ **87.** $\frac{3}{4} \times 2\frac{1}{6}$ $1\frac{5}{8}$

88. $3\frac{1}{6} \times 2\frac{1}{2}$ $7\frac{11}{12}$ **89.** $7\frac{2}{3} \times \frac{0}{4}$ 0 **90.** $2\frac{1}{3} \times 3\frac{1}{6}$ $7\frac{7}{18}$

91. $3\frac{1}{4} \times 3$ $9\frac{3}{4}$ **92.** $1\frac{3}{4} \times 1\frac{1}{7}$ 2 **93.** $1\frac{3}{5} \times 1\frac{2}{7}$ $2\frac{2}{35}$

94. $5\frac{1}{2} \times 1\frac{1}{3} \times 2$ $14\frac{2}{3}$ **95.** $2\frac{1}{4} \times \frac{2}{5} \times 3$ $2\frac{7}{10}$ **96.** $3\frac{2}{3} \times 1\frac{3}{4} \times \frac{1}{2}$ $3\frac{5}{24}$

97–120. Divide. Write the quotient as a whole number, a mixed number, or a proper fraction in lowest terms. Check by multiplying.

97. $2\frac{1}{2} \div 3\frac{1}{4}$ $\frac{10}{13}$ **98.** $3\frac{2}{5} \div 2\frac{1}{6}$ $1\frac{37}{65}$ **99.** $3\frac{1}{2} \div 2\frac{2}{3}$ $1\frac{5}{16}$

100. $1\frac{2}{7} \div 1\frac{1}{5}$ $1\frac{1}{14}$ **101.** $4\frac{1}{3} \div 2\frac{1}{2}$ $1\frac{11}{15}$ **102.** $1\frac{3}{4} \div 2\frac{1}{6}$ $\frac{21}{26}$

103. $4\frac{1}{2} \div 1\frac{3}{5}$ $2\frac{13}{16}$ **104.** $1\frac{1}{7} \div \frac{3}{4}$ $1\frac{11}{21}$ **105.** $3\frac{1}{6} \div 2\frac{5}{8}$ $1\frac{13}{63}$

106. $1\frac{1}{4} \div 3$ $\frac{5}{12}$ **107.** $\frac{4}{9} \div 2\frac{1}{3}$ $\frac{4}{21}$ **108.** $\frac{0}{5} \div 1\frac{2}{7}$ 0

109. $2 \div 2\frac{1}{2}$ $\frac{4}{5}$ **110.** $2\frac{2}{3} \div 3\frac{1}{6}$ $\frac{16}{19}$ **111.** $7\frac{2}{3} \div 2\frac{2}{5}$ $3\frac{7}{36}$

112. $3\frac{1}{4} \div 1\frac{3}{8}$ $2\frac{4}{11}$ **113.** $2\frac{4}{5} \div 3\frac{1}{4}$ $\frac{56}{65}$ **114.** $2\frac{1}{6} \div 5\frac{1}{2}$ $\frac{13}{33}$

115. $4\frac{2}{3} \div 2\frac{1}{4}$ $2\frac{2}{27}$ **116.** $1\frac{3}{4} \div 2\frac{1}{3}$ $\frac{3}{4}$ **117.** $7\frac{1}{2} \div 3\frac{1}{2}$ $2\frac{1}{7}$

118. $8 \div 1\frac{3}{10}$ $6\frac{2}{13}$ **119.** $3\frac{1}{2} \div 4\frac{1}{6}$ $\frac{21}{25}$ **120.** $2\frac{3}{4} \div 1\frac{2}{5}$ $1\frac{27}{28}$

48

Meaning of Exponents

This product:	can be written this way:	and read this way:
3×3	3^2	*three squared*
$3 \times 3 \times 3$	3^3	*three cubed*
$3 \times 3 \times 3 \times 3$	3^4	*fourth power of three* or *three to the fourth power*
and so on		

Performance Objective:
The student can
- read powers of numbers and find the products they indicate.
- find products of factors some of which are indicated by powers.

$$\underset{\text{base}}{3}^{\overset{\text{exponent}}{2}}$$

3^2 is a **power** of 3.

Examples:

1. $3^2 = 3 \times 3$
   $= 9$

2. $2^4 = \underbrace{2 \times 2}_{} \times \underbrace{2 \times 2}_{}$
   $= 4 \times 4$
   $= 16$

3. $(\frac{3}{4})^2 = \frac{3}{4} \times \frac{3}{4}$
   $= \frac{9}{16}$

4. $$\overset{\text{exponent 2}}{10^2} = 10 \times 10 = 1\overbrace{00}^{\text{2 zeros}}$$
   $$\overset{\text{exponent 3}}{10^3} = 10 \times 10 \times 10 = 1\overbrace{000}^{\text{3 zeros}}$$
   $$\overset{\text{exponent 4}}{10^4} = 10 \times 10 \times 10 \times 10 = 1\overbrace{0\,000}^{\text{4 zeros}}$$

5. $$3^3 \times (0.2)^2 \times 5 = \underbrace{3 \times 3 \times 3}_{} \times \underbrace{0.2 \times 0.2}_{} \times 5$$
   $$= 27 \times 0.04 \times 5$$
   $$= 5.4$$

Exercises

1–6. Express in words.

1. 4^2 2. 5^3 3. 1^4 4. 6^2 5. 3^6 6. 12^3

1. Four squared
2. Five cubed
3. Fourth power of one, or one to the fourth power
4. Six squared
5. Sixth power of three, or three to the sixth power
6. Twelve cubed

7–24. Find the result.

7. 5^2 25
8. 4^3 64
9. 1^4 1
10. 3^3 27
11. 10^6 1 000 000
12. 3^4 81
13. 4^2
14. 10^5 100 000
15. 2^6 64
16. 0^3 0
17. $(1.2)^2$ 1.44
18. $(\frac{2}{3})^3$ $\frac{8}{27}$
19. $5^2 \times 10^3$ 25 000
20. $2^3 \times (\frac{7}{2})^2$ 98
21. 12×3^3 324
22. $9^2 \times 3^4 \times 0^3$ 0
23. $4 \times 6^2 \times 2^3$ 1152
24. $3 \times 5^2 \times 0.2$ 15

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Manual and Tests
MATHEMATICS
for Daily Use

Overview of Student's Text T2
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Overview of Student's Text

Mathematics for Daily Use provides a fresh approach to a comprehensive course in general mathematics. The content—a blend of basic computational skills and other practical skills—has been chosen to be relevant to today's students. The varied, highly visual presentation is designed to minimize such student shortcomings as lack of motivation, low reading ability, short attention span, and poor capacity for retention. The very flexible organization gives the teacher such options as concentrating the review of computational skills before the development of other skills or interspersing the review of computation among other topics. Flexibility is also built into the abundant practice exercises and special features of the text.

OVERALL BOOK ORGANIZATION. The book is organized into ten chapters and the Computing Skills Refresher. The latter presents review sections covering the four fundamental operations (addition, subtraction, multiplication, and division) of whole numbers, decimals, fractions, and mixed numbers. The ten chapters develop practical, usable skills pertaining to tables, graphs, formulas, ratio, percent, measurement, plane and solid figures, scale drawings, consumer topics, making estimates, and very basic probability and statistics. Each of the ten chapters also serves as a vehicle in which a large amount of basic computation has been imbedded. For example, in Chapter 6, Plane Figures, the exercises are predominantly on addition and multiplication of decimals or mixed numbers (in finding perimeters and areas).

CHAPTER ORGANIZATION. Each of Chapters 1–10 has the following consistent organization.

Preview Test. The first page of each chapter is a pretest on the chapter content. The test items are keyed to the sections in the chapter. These tests can be used for informal diagnostic purposes in different ways. For example, the teacher can pretest the class to determine what pacing will be required in the ensuing chapter. Or, an individual student can use a pretest for diagnostic self-help. Also, the tests may simply serve as a source of additional exercises.

Picture Preview. The next two pages of each chapter present a pictorial glimpse of applications for the mathematics that will be developed in the chapter. An accompanying paragraph gives a brief overview of the broad content goals of the chapter.

Basic Numbered Sections. Each basic section of the chapter is numbered (1.2, 2.5, 6.7, and so on) to show first the chapter number and next the sequence of the section in that chapter. These numbered sections are the basic daily lessons. Each begins with a lesson development in which the reading level has been rigidly controlled. Also, the amount of reading in each section is minimal. Whenever possible, visual aids supplement the reading.

The exercises are closely correlated to the examples in the lesson development. And exercise sets have the following structure.

- When appropriate, the first few exercises are of the type that can be done orally to summarize the preceding lesson development.

- The bulk of the exercises are for written practice, and the "odds" and "evens" form two subsets that are of equivalent content and difficulty. In almost all sections the odd-numbered exercises provide sufficient practice, and the even-numbered exercises are a backup set to be used when needed.

- The last few exercises in most sections are optional. These are somewhat more challenging and often involve problem solving.

The Daily Lesson-Plan Guide on pages T4–T16 suggests exercise assignments that are consistent with the foregoing structure of the exercise sets.

Chapter Test and Review. At the end of each chapter is a posttest which is parallel in content to the pretest. This posttest is followed by a chapter review. The exercises in both the posttest and the review are keyed back to the sections of the chapter. Thus, a student can complete the posttest to determine in which sections more work is needed, review those troublesome sections back in the chapter, and then proceed to the appropriate parts of the chapter review.

Special Topics. Interspersed within each chapter are several special topics, all of which are optional.

- *Quick Quiz.* These brief quizzes anticipate basic computing skills that the student will need in the next section or two. A student who does not do well on a particular Quick Quiz can use the appropriate section of the Computing Skills Refresher for remedial work before proceeding in the chapter. In this way, refresher work on computing skills can be individualized if the teacher so chooses.

- *Career and Consumer Corners.* These features highlight interesting topics from the everyday world of the worker or the consumer.

- *Other Special Topics.* A variety of other optional features chosen for their "hands-on," relevant nature is included. See contents page 8 of the text for a complete list.

Cumulative Review. Following every even-numbered chapter is a review of the two preceding chapters. The exercises are grouped by sections so that these reviews are usable even if parts of a preceding chapter have been omitted.

COMPUTING SKILLS REFRESHER ORGANIZATION. This comprehensive review of computing skills is organized into two parts. The first part is on whole numbers and decimals; the second is on fractions and mixed numbers. See page 465 of the text for a detailed table of contents, and see the suggestions below for various ways this material can be integrated into the course.

Daily Lesson-Plan Guide

This guide presents suggested daily lesson plans for three possible courses of study.

Course A	concentrates extensive refresher work with computing skills at the beginning of the year.
Course B	distributes extensive refresher work with computing skills throughout the year.
Course C	is set up so that refresher work with computing skills can be individualized by using the Quick Quizzes as suggested on page T3.

Before deciding which course is best suited for a certain class, the teacher may wish to administer the Computing Skills Pretests on pages T52–T53 and T56–T57.

Other variations in course structure, besides the three courses suggested here, are possible. For example,

1. Course A can be varied by interchanging Lessons 23–57 with Lessons 58–85. Thus, the refresher work with computing skills will be covered in two blocks, separated by the work with graphs and tables in Chapters 1 and 2.

2. Course C can be varied by omitting the Special Topics (see gray shaded cells in the following chart) and using the time for extending previous lessons or for work with computing skills.

After becoming familiar with the textbook, the teacher will find that courses A, B, and C can readily be tailored to meet particular needs.

Lesson	Course A	Course B	Course C
1	Sec. **1**, p. 466 *Orally:* 1–20 Sec. **2**, p. 467 *Written:* 1–20	Sec. **1**, p. 466 *Orally:* 1–20 Sec. **2**, p. 467 *Written:* 1–20	Preview Test, p. 9 (See p. T2.)
2	Sec. **3**, p. 468 *Orally:* 1–10 *Written:* 11–20 Sec. **4**, p. 469 *Written:* 1–39 odds	Sec. **3**, p. 468 *Orally:* 1–10 *Written:* 11–20 Sec. **4**, p. 469 *Written:* 1–39 odds	Sec. **1.1**, pp. 12–15 *Orally:* 1–8 *Written:* 9–26 Quick Quiz, p. 15
3	Sec. **5**, pp. 470–471 *Written:* 1–35 odds	Sec. **1.1**, pp. 12–15 *Orally:* 1–8 *Written:* 9–26	Sec. **1.2**, pp. 16–19 *Orally:* 1–10 *Written:* 11–40 Quick Quiz, p. 19
4	Sec. **6**, pp. 472–473 *Written:* 1–37 odds	Sec. **1.2**, pp. 16–19 *Orally:* 1–10 *Written:* 11–40	Sec. **1.3**, pp. 20–23 *Orally:* 1–12 *Written:* 13–26, 31–38
5	Sec. **7**, pp. 474–475 *Written:* 1–45 odds	Section. **5**, pp. 470–471 *Written:* 1–35 odds	Cross-Number Puzzle, p. 24
6	Sec. **8**, pp. 476–477 *Written:* 1–59 odds	Sec. **6**, pp. 472–473 *Written:* 1–37 odds	Sec. **1.4**, pp. 25–28 *Orally:* 1–18 *Written:* 19–36
7	Sec. **9**, pp. 478–479 *Written:* 1–43 odds	Sec. **1.3**, pp. 20–33 *Orally:* 1–12 *Written:* 13–26, 31–38	Sec. **1.5**, pp. 30–31 *Orally:* 1–20 Sec. **1.6**, pp. 32–33 *Orally:* 1–4 *Written:* 5–14
8	Sec. **9**, pp. 478–479 *Written:* 45–87 odds	Sec. **1.4**, pp. 25–28 *Orally:* 1–18 *Written:* 19–36	Sec. **1.7**, pp. 34–36 *Orally:* 1–6 *Written:* 7–27 odds Quick Quiz, p. 36
9	Sec. **10**, p. 480 *Orally:* 1–64 Sec. **11**, p. 481 *Written:* 1–47 odds	Sec. **1.5**, pp. 30–31 *Orally:* 1–20 Sec. **7**, pp. 474–475 *Written:* 1–45 odds	Sec. **1.8**, pp. 37–40 *Orally:* 1–2 *Written:* 3–19 odds, 27–37 odds
10	Sec. **12**, pp. 482–483 *Written:* 1–51 odds	Sec. **1.6**, pp. 32–33 *Orally:* 1–4 *Written:* 5–14 Sec. **8**, pp. 476–477 *Written:* 1–59 odds	Sec. **1.8**, pp. 37–40 *Written:* 4–20 evens, 28–38 evens
11	Sec. **13**, p. 484 *Written:* 1–24	Sec. **9**, pp. 478–479 *Written:* 1–31 odds 45–71 odds	Quick Quiz, p. 40 Sec. **1.9**, p. 41 *Orally:* 1–6 *Written:* 7–20
12	Sec. **14**, p. 485 *Written:* 1–26	Sec. **10**, p. 480 *Orally:* 1–64 Sec. **11**, p. 481 *Written:* 1–47 odds	Sec. **1.10**, pp. 42–44 *Orally:* 1–6 *Written:* 7–17 odds Quick Quiz, p. 44

Lesson	Course A	Course B	Course C
13	Sec. **15**, p. 486 *Orally:* 1–60 Sec. **16**, p. 487 *Written:* 1–51 odds	Sec. **12**, pp. 482–483 *Written:* 1–51 odds	Sec. **1.11**, pp. 46–48 *Orally:* 1–6 *Written:* 7–17 odds
14	Sec. **17**, pp. 488–489 *Written:* 1–59 odds	Sec. **1.7**, pp. 34–36 *Orally:* 1–6 *Written:* 7–27 odds	Test, p. 50 (See p. T3.) Start Review, pp. 51–52
15	Sec. **18**, pp. 490–491 *Written:* 1–21 odds, 37–43 odds	Sec. **13**, p. 484 *Written:* 1–23 odds Sec. **14**, p. 485 *Written:* 1–25 odds	Finish Review, pp. 51–52
16	Sec. **19**, pp. 492–493 *Written:* 1–23 odds, 41–55 odds	Sec. **15**, p. 486 *Orally:* 1–60 Sec. **16**, p. 487 *Written:* 1–51 odds	Test, pp. T40–T41
17	Sec. **20**, p. 494 *Orally:* 1–6 *Written:* 7–24	Sec. **17**, pp. 488–489 *Written:* 1–59 odds	Preview Test, p. 53 (See p. T2.)
18	Sec. **21**, p. 495 *Orally:* 1–8 *Written:* 9–31 odds	Sec. **1.8**, pp. 37–40 *Orally:* 1–2 *Written:* 3–19 odds, 27–37 odds	Sec. **2.1**, pp. 56–59 *Orally:* 1–6 *Written:* 7–42
19	Sec. **22**, pp. 496–497 *Written:* 1–39 odds	Sec. **1.8**, pp. 37–40 *Written:* 4–20 evens, 28–38 evens	**Numbers on the Map, p. 60**
20	Sec. **23**, pp. 498–499 *Written:* 1–51 odds	Sec. **18**, pp. 490–491 *Written:* 1–21 odds, 37–43 odds	Sec. **2.2**, pp. 61–62 *Orally:* 1–4 *Written:* 5–28 Quick Quiz, p. 62
21	Sec. **23**, pp. 498–499 *Written:* 53–109 odds	Sec. **19**, pp. 492–493 *Written:* 1–23 odds, 41–55 odds	**Fishy Graphs, p. 63**
22	Computing Skills Posttest, pp. T54–T55 (may require 2 days)	Sec. **20**, p. 494 *Orally:* 1–6 *Written:* 7–23 odds Sec. **1.9**, p. 41 *Orally:* 1–6 *Written:* 7–19 odds	Sec. **2.3**, pp. 64–65 *Orally:* 1–7 *Written:* 8, 9–19 odds
23	Sec. **24**, pp. 500–501 *Orally:* 1–38 *Written:* 39–56	Sec. **1.10**, pp. 42–44 *Orally:* 1–6 *Written:* 7–17 odds	Sec. **2.4**, pp. 66–69 *Orally:* 1–12 *Written:* 13–25 odds
24	Sec. **25**, pp. 502–503 *Orally:* 1–6 *Written:* 7–41 odds	Sec. **21**, p. 495 *Orally:* 1–8 *Written:* 9–31 odds	Sec. **2.5**, pp. 70–72 *Orally:* 1–6 *Written:* 7–10 Quick Quiz, p. 72
25	Sec. **26**, pp. 504–505 *Written:* 1–53 odds	Sec. **22**, pp. 496–497 *Written:* 1–39 odds	Sec. **2.6**, pp. 74–76 *Orally:* 1–12 *Written:* 13–19 odds

Lesson	Course A	Course B	Course C
26	Sec. **27**, p. 506 *Orally:* 1–4 *Written:* 5–24	Test, p. 50 (See p. T3.) Start Review, pp. 51–52	Sec. **2.7**, pp. 78–81 *Orally:* 1–8 *Written:* 9–34
27	Sec. **28**, p. 507 *Written:* 1–24	Finish Review, pp. 51–52	Sec. **2.8**, pp. 82–84 *Orally:* 1–6 *Written:* 7–17, 19–32
28	Sec. **29**, p. 508 *Written:* 1–30	Test, pp. T40–T41 (Omit item 32.)	Sec. **2.9**, pp. 86–89 *Orally:* 1–8 *Written:* 9–41 odds
29	Sec. **30**, p. 509 *Written:* 1–55 odds	Sec. **23**, pp. 498–499 *Written:* 1–51 odds	Test, p. 90 (See p. T3.) Start Review, pp. 91–92
30	Sec. **31**, pp. 510–511 *Orally:* 1–7 *Written:* 8–40	Sec. **23**, pp. 498–499 *Written:* 53–109 odds	Finish Review, pp. 91–92
31	Sec. **32**, p. 512 *Orally:* 1–6 *Written:* 7–30	Computing Skills Post-test, pp. T54–T55 (may require 2 days)	Test, pp. T42–T43
32	Sec. **33**, p. 513 *Orally:* 1–4 *Written:* 5–25	Sec. **2.1**, pp. 56–59 *Orally:* 1–6 *Written:* 7–42	Another Look at Chapters 1 and 2, pp. 93–96 *Written:* 1–26
33	Sec. **34**, pp. 514–515 *Written:* 1–29 odds	Sec. **2.2**, pp. 61–62 *Orally:* 1–4 *Written:* 5–28	Another Look at Chapters 1 and 2, pp. 93–96 *Written:* 27–42
34	Sec. **35**, pp. 516–517 *Written:* 1–49 odds	Sec. **24**, pp. 500–501 *Orally:* 1–38 *Written:* 39–56	Preview Test, p. 97 (See p. T2.)
35	Sec. **36**, pp. 518–519 *Written:* 1–49 odds	Sec. **25**, pp. 502–503 *Orally:* 1–6 *Written:* 7–41 odds	Sec. **3.1**, pp. 100–101 *Orally:* 1–14 *Written:* 15–30
36	Sec. **36**, pp. 518–519 *Written:* 51–99 odds	Sec. **2.3**, pp. 64–65 *Orally:* 1–7 *Written:* 8, 9–19 odds	Sec. **3.2**, pp. 102–103 *Orally:* 1–2 *Written:* 3–41 odds
37	Sec. **37**, pp. 520–521 *Written:* 1–61 odds	Sec. **26**, pp. 504–505 *Written:* 1–53 odds	Sec. **3.3**, pp. 104–105 *Orally:* 1–6 *Written:* 7–43 odds
38	Sec. **38**, pp. 522–523 *Written:* 1–29 odds	Sec. **27**, p. 506 *Written:* 1–23 odds Sec. **28**, p. 507 *Written:* 1–23 odds	Sec. **3.4**, pp. 107–109 *Orally:* 1–2 *Written:* 3–19 odds Quick Quiz, p. 109
39	Sec. **39**, pp. 524–525 *Orally:* 1–10 *Written:* 11–45 odds	Sec. **29**, p. 508 *Written:* 1–30	Sec. **3.5**, pp. 110–113 *Orally:* 1–6 *Written:* 7–43 odds Quick Quiz, p. 113
40	Sec. **40**, pp. 526–527 *Written:* 1–49 odds	Sec. **2.4**, pp. 66–69 *Orally:* 1–12 *Written:* 13–25 odds	Sec. **3.6**, pp. 114–117 *Orally:* 1–6 *Written:* 7–35 odds

Lesson	Course A	Course B	Course C
41	Sec. **40**, pp. 526–527 *Written:* 51–99 odds	Sec. **2.5**, pp. 70–72 *Orally:* 1–6 *Written:* 7–10	Sec. **3.7**, pp. 118–121 *Orally:* 1–10 *Written:* 11–39 odds
42	Sec. **41**, p. 528 *Written:* 1–22	Sec. **30**, p. 509 *Written:* 1–55 odds	Equation Cross-Number, p. 122
43	Sec. **42**, p. 529 *Written:* 1–24	Sec. **2.6**, pp. 74–76 *Orally:* 1–12 *Written:* 13–19 odds	Quick Quiz, p. 122 Sec. **3.8**, pp. 123–125 *Written:* 1–39 odds
44	Sec. **43**, pp. 530–531 *Written:* 1–31 odds	Sec. **2.7**, pp. 78–81 *Orally:* 1–8 *Written:* 9–34	Sec. **3.9**, pp. 126–129 *Orally:* 1–6 *Written:* 7–35 odds
45	Sec. **43**, pp. 530–531 *Written:* 33–63 odds	Test, p. 90 (See p. T3.) Start Review, pp. 91–92	Sec. **3.10**, pp. 130–134 *Orally:* 1–4 *Written:* 5–23 odds
46	Sec. **44**, pp. 532–533 *Written:* 1–23 odds	Finish Review, pp. 91–92	Test, p. 136 (See p. T3.) Start Review, pp. 139–140
47	Sec. **44**, pp. 532–533 *Written:* 25–45 odds	Test, pp. T42–T43 (Omit items 10–18.)	Finish Review, pp. 139–140
48	Sec. **45**, pp. 534–535 *Written:* 1–14	Another Look at Chapters 1 and 2, pp. 93–96 *Written:* 1–26	Test, p. T44
49	Sec. **45**, pp. 534–535 *Written:* 15–41 odds	Another Look at Chapters 1 and 2, pp. 93–96 *Written:* 28–37	Preview Test, p. 141 (See p. T2.)
50	Sec. **46**, pp. 536–537 *Written:* 1–23 odds	Sec. **3.1**, pp. 100–101 *Orally:* 1–14 *Written:* 15–30	Sec. **4.1**, pp. 144–146 *Orally:* 1–8 *Written:* 9–27 odds
51	Sec. **46**, pp. 536–537 *Written:* 25–47 odds	Sec. **3.2**, pp. 102–103 *Orally:* 1–2 *Written:* 3–41 odds	Sec. **4.2**, pp. 147–149 *Written:* 1–43 odds
52	Sec. **47**, pp. 538–539 *Written:* 1–35 odds	Sec. **3.3**, pp. 104–105 *Orally:* 1–6 *Written:* 7–43 odds	Graphing Equivalent Ratios, p. 150
53	Sec. **47**, pp. 538–539 *Written:* 37–71 odds	Sec. **3.4**, pp. 107–109 *Orally:* 1–2 *Written:* 3–19 odds	Sec. **4.3**, pp. 151–153 *Orally:* 1–4 *Written:* 5–25 odds
54	Sec. **47**, pp. 538–539 *Written:* 73–95 odds	Sec. **3.5**, pp. 110–113 *Orally:* 1–6 *Written:* 7–43 odds	Sec. **4.4**, pp. 154–157 *Written:* 1–23 odds
55	Sec. **47**, pp. 538–539 *Written:* 97–119 odds	Sec. **3.6**, pp. 114–117 *Orally:* 1–6 *Written:* 7–35 odds	Sec. **4.5**, pp. 158–161 *Written:* 1–13 odds
56	Sec. **48**, p. 540 *Orally:* 1–6 *Written:* 7–24	Sec. **31**, pp.510–511 *Orally:* 1–7 *Written:* 8–40	Sec. **4.6**, pp. 162–164 *Orally:* 1–2 *Written:* 3–25 odds Quick Quiz, p. 164

Lesson	Course A	Course B	Course C
57	Computing Skills Posttest, pp. T58–T59 (may require 2 days)	Sec. **32**, p. 512 *Orally:* 1–6 *Written:* 7–30	Sec. **4.7**, pp. 165–166 *Orally:* 1–2 *Written:* 3–23 odds
58	Sec. **1.1**, pp. 12–15 *Orally:* 1–8 *Written:* 9–26	Sec. **3.7**, pp. 118–121 *Orally:* 1–10 *Written:* 11–39 odds	Proportion Rummy, p. 168 Shopping in the Newspaper, pp. 168–169
59	Sec. **1.2**, pp. 16–19 *Orally:* 1–10 *Written:* 11–40	Test, p. 136 (See p. T3.) Start Review, pp. 139–140	Sec. **4.8**, pp. 170–172 *Orally:* 1–2 *Written:* 3–29 odds
60	Sec. **1.3**, pp. 20–23 *Orally:* 1–12 *Written:* 13–26, 31–38	Finish Review, pp. 139–140	Sec. **4.9**, pp. 173–174 *Orally:* 1–2 *Written:* 3–21 odds
61	Sec. **1.4**, pp. 25–28 *Orally:* 1–18 *Written:* 19–36	Test, p. T44 (Omit items 19–24.)	Sec. **4.10**, pp. 175–178 *Written:* 1–23 odds Quick Quiz, p. 178
62	Sec. **1.5**, pp. 30–31 *Orally:* 1–20 Sec. **1.6**, pp. 32–33 *Orally:* 1–4 *Written:* 5–14	Sec. **33**, p. 513 *Orally:* 1–4 *Written:* 5–25	Sec. **4.11**, pp. 179–181 *Written:* 1–39 odds
63	Sec. **1.7**, pp. 34–36 *Orally:* 1–6 *Written:* 7–27 odds	Sec. **34**, pp. 514–515 *Written:* 1–29 odds	Sec. **4.12**, pp. 183–185 *Written:* 1–33 odds
64	Sec. **1.8**, pp. 37–40 *Orally:* 1–2 *Written:* 3–19 odds, 27–37 odds	Sec. **4.1**, pp. 144–146 *Orally:* 1–8 *Written:* 9–27 odds	Test, p. 186 (See p. T3.) Start Review, pp. 187–188
65	Sec. **1.8**, pp. 37–40 *Written:* 4–20 evens, 28–38 evens	Sec. **4.2**, pp. 147–149 *Written:* 1–43 odds	Finish Review, pp. 187–188
66	Sec. **1.9**, p. 41 *Orally:* 1–6 *Written:* 7–20	Sec. **4.3**, pp. 151–153 *Orally:* 1–4 *Written:* 5–25 odds	Test, p. T45
67	Sec. **1.10**, pp. 42–44 *Orally:* 1–6 *Written:* 7–17 odds	Sec. **35**, pp. 516–517 *Written:* 1–49 odds	Another Look at Chapters 3 and 4, pp. 189–192 *Written:* 1–53 odds
68	Sec. **1.11**, pp. 46–48 *Orally:* 1–6 *Written:* 7–17 odds	Sec. **36**, pp. 518–519 *Written:* 1–49 odds	Another Look at Chapters 3 and 4, pp. 189–192 *Written:* 55–109 odds
69	Test, p. 50 (See p. T3.) Start Review, pp. 51–52	Sec. **4.4**, pp. 154–157 *Written:* 1–23 odds	Preview Test, p. 193 (See p. T2.)
70	Finish Review, pp. 51–52	Sec. **4.5**, pp. 158–161 *Written:* 1–13 odds	Sec. **5.1**, pp. 196–198 *Orally:* 1–30 *Written:* 31–49 odds

Lesson	Course A	Course B	Course C
71	Test, pp. T40–T41	Sec. **36**, pp. 518–519 *Written:* 51–99 odds	Sec. **5.2**, pp. 200–202 *Orally:* 1–14 *Written:* 15–25
72	Sec. **2.1**, pp. 56–59 *Orally:* 1–6 *Written:* 7–42	Sec. **37**, pp. 520–521 *Written:* 1–61 odds	Sec. **5.3**, pp. 203–205 *Orally:* 1–6 *Written:* 7–24
73	Sec. **2.2**, pp. 61–62 *Orally:* 1–4 *Written:* 5–28	Sec. **38**, pp. 522–523 *Written:* 1–29 odds	Measuring Activities, p. 206
74	Sec. **2.3**, pp. 64–65 *Orally:* 1–7 *Written:* 8, 9–19 odds	Sec. **4.6**, pp. 162–164 *Orally:* 1–2 *Written:* 3–25 odds	Quick Quiz, p. 206 Sec. **5.4**, pp. 207–208 *Orally:* 1–6 *Written:* 7–23 odds
75	Sec. **2.4**, pp. 66–69 *Orally:* 1–12 *Written:* 13–25 odds	Sec. **4.7**, pp. 165–166 *Orally:* 1–2 *Written:* 3–23 odds	Sec. **5.5**, pp. 210–213 *Orally:* 1–16 *Written:* 17–39 odds Quick Quiz, p. 213
76	Sec. **2.5**, pp. 70–72 *Orally:* 1–6 *Written:* 7–10	Sec. **4.8**, pp. 170–172 *Orally:* 1–2 *Written:* 3–29 odds	Sec. **5.6**, pp. 214–215 *Written:* 1–23 odds
77	Sec. **2.6**, pp. 74–76 *Orally:* 1–12 *Written:* 13–19 odds	Sec. **4.9**, pp. 173–174 *Orally:* 1–2 *Written:* 3–21 odds	Sec. **5.7**, pp. 216–219 *Orally:* 1–10 *Written:* 11–37 odds
78	Sec. **2.7**, pp. 78–81 *Orally:* 1–8 *Written:* 9–34	Test, p. 186 (See p. T3.) Start Review, pp. 187–188	Sec. **5.8**, pp. 220–222 *Orally:* 1–14 *Written:* 15–39 odds
79	Sec. **2.8**, pp. 82–84 *Orally:* 1–6 *Written:* 7–17, 19–32	Finish Review, pp. 187–188	Sec. **5.9**, pp. 223–225 *Orally:* 1–8 *Written:* 9–35 odds
80	Sec. **2.9**, pp. 86–89 *Orally:* 1–8 *Written:* 9–41 odds	Test, p. T45 (Omit items 19–24.)	Sec. **5.10**, pp. 228–229 *Orally:* 1–12 *Written:* 13–24
81	Test, p. 90 (See p. T3.) Start Review, pp. 91–92	Another Look at Chapters **3** and **4**, pp. 189–192 *Written:* 1–37 odds	Sec. **5.11**, pp. 230–233 *Orally:* 1–6 *Written:* 7–23
82	Finish Review, pp. 91–92	Another Look at Chapters **3** and **4**, pp. 189–192 *Written:* 55–95 odds	Test, p. 234 (See p. T3.) Start Review, pp. 235–236
83	Test, pp. T42–T43	Sec. **39**, pp. 524–525 *Orally:* 1–10 *Written:* 11–45 odds	Finish Review, pp. 235–236
84	Another Look at Chapters **1** and **2**, pp. 93–96 *Written:* 1–26	Sec. **40**, pp. 526–527 *Written:* 1–29 odds, 51–73 odds	Test, p. T46

Lesson	Course A	Course B	Course C
85	Another Look at Chapters 1 and 2, pp. 93–96 *Written:* 27–42	Sec. **41**, p. 528 *Written:* 1–21 odds Sec. **42**, p. 529 *Written:* 1–23 odds	Preview Test, p. 237 (See p. T2.)
86	Sec. **3.1**, pp. 100–101 *Orally:* 1–14 *Written:* 15–30	Sec. **43**, pp. 530–531 *Written:* 1–23 odds, 33–55 odds	Sec. **6.1**, pp. 240–242 *Orally:* 1–12 *Written:* 13–24 Quick Quiz, p. 242
87	Sec. **3.2**, pp. 102–103 *Orally:* 1–2 *Written:* 3–41 odds	Sec. **5.1**, pp. 196–198 *Orally:* 1–30 *Written:* 31–49 odds	Sec. **6.2**, pp. 243–246 *Orally:* 1–2 *Written:* 3–35 odds
88	Sec. **3.3**, pp. 104–105 *Orally:* 1–6 *Written:* 7–43 odds	Sec. **5.2**, pp. 200–202 *Orally:* 1–14 *Written:* 15–25	Sec. **6.3**, pp. 247–249 *Written:* 1–21 odds Quick Quiz, p. 250
89	Sec. **3.4**, pp. 107–109 *Orally:* 1–2 *Written:* 3–19 odds	Sec. **5.3**, pp. 203–205 *Orally:* 1–6 *Written:* 7–24	Sec. **6.4**, pp. 251–254 *Orally:* 1–6 *Written:* 7–37 odds
90	Sec. **3.5**, pp. 110–113 *Orally:* 1–6 *Written:* 7–43 odds	Sec. **46**, pp. 536–537 *Written:* 1–23 odds	Sec. **6.5**, pp. 255–257 *Orally:* 1–4 *Written:* 5–31 odds
91	Sec. **3.6**, pp. 114–117 *Orally:* 1–6 *Written:* 7–35 odds	Sec. **5.4**, pp. 207–208 *Orally:* 1–6 *Written:* 7–23 odds	Sec. **6.6**, pp. 259–261 *Orally:* 1–2 *Written:* 3–29 odds Quick Quiz, p. 261
92	Sec. **3.7**, pp. 118–121 *Orally:* 1–10 *Written:* 11–39 odds	Sec. **5.5**, pp. 210–213 *Orally:* 1–16 *Written:* 17–39 odds	Sec. **6.7**, pp. 262–264 *Orally:* 1–2 *Written:* 3–29 odds
93	Sec. **3.8**, pp. 123–125 *Written:* 1–39 odds	Sec. **5.6**, pp. 214–215 *Written:* 1–23 odds	Sec. **6.8**, pp. 265–267 *Orally:* 1–4 *Written:* 5–17 odds Quick Quiz: p. 267
94	Sec. **3.9**, pp. 126–129 *Orally:* 1–6 *Written:* 7–35 odds	Sec. **5.7**, pp. 216–219 *Orally:* 1–10 *Written:* 11–37 odds	Sec. **6.9**, pp. 269–271 *Orally:* 1–10 *Written:* 11–31 odds
95	Sec. **3.10**, pp. 130–134 *Orally:* 1–4 *Written:* 5–23 odds	Sec. **5.8**, pp. 220–222 *Orally:* 1–14 *Written:* 15–39 odds	A Number Called Pi, p. 274
96	Test, p. 136 (See p. T3.) Start Review, pp. 139–140	Sec. **5.9**, pp. 223–225 *Orally:* 1–8 *Written:* 9–35 odds	Sec. **6.10**, pp. 275–277 *Orally:* 1–6 *Written:* 7–33 odds
97	Finish Review, pp. 139–140	Sec. **5.10**, pp. 228–229 *Orally:* 1–12 *Written:* 13–24	Sec. **6.11**, pp. 278–280 *Orally:* 1–4 *Written:* 5–35 odds
98	Test, p. T44	Sec. **5.11**, pp. 230–233 *Orally:* 1–6 *Written:* 7–23	Test, p. 282 (See p. T3.) Start Review, pp. 283–284

Lesson	Course A	Course B	Course C
99	Sec. **4.1**, pp. 144–146 *Orally:* 1–8 *Written:* 9–27 odds	Test, p. 234 (See p. T3.) Start Review, pp. 235–236	Finish Review, pp. 283–284
100	Sec. **4.2**, pp. 147–149 *Written:* 1–43 odds	Finish Review, pp. 235–236	Test, p. T47
101	Sec. **4.3**, pp. 151–153 *Orally:* 1–4 *Written:* 5–25 odds	Test, p. T46	Another Look at Chapters 5 and 6, pp. 285–288 *Written:* 1–51 odds
102	Sec. **4.4**, pp. 154–157 *Written:* 1–23 odds	Sec. **44**, pp. 532–533 *Written:* 1–35 odds	Another Look at Chapters 5 and 6, pp. 285–288 *Written:* 53–85 odds
103	Sec. **4.5**, pp. 158–161 *Written:* 1–13 odds	Sec. **6.1**, pp. 240–242 *Orally:* 1–12 *Written:* 13–24	Preview Test, p. 289 (See p. T2.)
104	Sec. **4.6**, pp. 162–164 *Orally:* 1–2 *Written:* 3–25 odds	Sec. **6.2**, pp. 243–246 *Orally:* 1–2 *Written:* 3–35 odds	Sec. **7.1**, pp. 292–295 *Orally:* 1–18 *Written:* 19–29 odds Quick Quiz, p. 295
105	Sec. **4.7**, pp. 165–166 *Orally:* 1–2 *Written:* 3–23 odds	Sec. **6.3**, pp. 247–249 *Written:* 1–21 odds	Sec. **7.2**, pp. 296–299 *Orally:* 1–4 *Written:* 5–27 odds
106	Sec. **4.8**, pp. 170–172 *Orally:* 1–2 *Written:* 3–29 odds	Sec. **47**, pp. 538–539 *Written:* 73–95 odds	Sec. **7.3**, pp. 302–305 *Written:* 1–15 odds
107	Sec. **4.9**, pp. 173–174 *Orally:* 1–2 *Written:* 3–21 odds	Sec. **6.4**, pp. 251–254 *Orally:* 1–6 *Written:* 7–37 odds	Sec. **7.4**, pp. 306–309 *Written:* 1–17 odds
108	Sec. **4.10**, pp. 175–178 *Written:* 1–23 odds	Sec. **6.5**, pp. 255–257 *Orally:* 1–4 *Written:* 5–31 odds	Sec. **7.5**, pp. 310–313 *Orally:* 1–10 *Written:* 11–33 odds
109	Sec. **4.11**, pp. 179–181 *Written:* 1–39 odds	Sec. **6.6**, pp. 259–261 *Orally:* 1–2 *Written:* 3–29 odds	Follow the Yellow Brick Road, p. 314; Your Provincial Map, p. 314
110	Sec. **4.12**, pp. 183–185 *Written:* 1–33 odds	Sec. **6.7**, pp. 262–264 *Orally:* 1–2 *Written:* 3–29 odds	Sec. **7.6**, pp. 315–318 *Orally:* 1–14 *Written:* 15–26
111	Test, p. 186 (See p. T3.) Start Review, pp. 187–188	Sec. **6.8**, pp. 265–267 *Orally:* 1–4 *Written:* 5–17 odds	Sec. **7.7**, pp. 319–322 *Written:* 1–12 all, 13–19 odds
112	Finish Review, pp. 187–188	Sec. **6.9**, pp. 269–271 *Orally:* 1–10 *Written:* 11–31 odds	Sec. **7.8**, pp. 323–325 *Written:* 1–13 odds
113	Test, p. T45	Sec. **6.10**, pp. 275–277 *Orally:* 1–6 *Written:* 7–33 odds	Sec. **7.9**, pp. 326–329 *Written:* 1–19 odds

Lesson	Course A	Course B	Course C
114	Another Look at Chapters 3 and 4, pp. 189–192 *Written:* 1–53 odds	Sec. **6.11**, pp. 278–280 *Orally:* 1–4 *Written:* 5–35 odds	Test, p. 330 (See p. T3.) Start Review, pp. 331–332
115	Another Look at Chapters 3 and 4, pp. 189–192 *Written:* 55–109 odds	Test, p. 282 (See p. T3.) Start Review, pp. 283–284	Finish Review, pp. 331–332
116	Sec. **5.1**, pp. 196–198 *Orally:* 1–30 *Written:* 31–49 odds	Finish Review, pp. 283–284	Test, p. T48
117	Sec. **5.2**, pp. 200–202 *Orally:* 1–14 *Written:* 15–25	Test, p. T47	Preview Test, p. 333 (See p. T2.)
118	Sec. **5.3**, pp. 203-205 *Orally:* 1-6 *Written:* 7-51	Another Look at Chapters 5 and 6, pp. 285–288 *Written:* 1–51 odds	Sec. **8.1**, pp. 336–339 *Orally:* 1–4 *Written:* 5–13 odds Quick Quiz, p. 339
119	Sec. **5.4**, pp. 209-210 *Orally:* 1-6 *Written:* 7-23 odds	Another Look at Chapters 5 and 6, pp. 285–288 *Written:* 53–85 odds	Sec. **8.2**, pp. 340–343 *Orally:* 1–2 *Written:* 3–11 odds
120	Sec. **5.5**, pp. 212-213 *Orally:* 1-18 *Written:* 19-29 odds	Sec. **47**, pp. 538–539 *Written:* 1–35 odds	Sec. **8.3**, pp. 346–349 *Orally:* 1–2 *Written:* 3–10
121	Sec. **5.6**, pp. 216-217 *Written:* 1-19 odds	Sec. **45**, pp. 534–535 *Written:* 1–35 odds	Sec. **8.4**, pp. 350–353 *Orally:* 1–2 *Written:* 3–33 odds
122	Sec. **5.7**, pp. 219-220 *Orally:* 1-8 *Written:* 11-35 odds	Sec. **47**, pp. 538–539 *Written:* 74–84 evens, 97–115 odds	Sec. **8.5**, pp. 354–357 *Orally:* 1–2 *Written:* 3–9 odds
123	Sec. **5.8**, pp. 222-223 *Orally:* 1-7 *Written:* 8-14 odds	Sec. **48**, p. 540 *Orally:* 1–6 *Written:* 7–24	Sec. **8.6**, pp. 358–361 *Orally:* 1–4 *Written:* 5–11 odds
124	Sec. **5.9**, pp. 224-225 *Orally:* 1-16 *Written:* 9-21	Computing Skills Posttest, pp. T58–T59 (may require 2 days)	Sec. **8.7**, pp. 362–365 *Orally:* 1–4 *Written:* 5–19 odds Quick Quiz, p. 365
125	Sec. **5.10**, pp. 228–229 *Orally:* 1–12 *Written:* 13–24	Sec. **7.1**, pp. 292–295 *Orally:* 1–18 *Written:* 19–29 odds	Sec. **8.8**, pp. 366–369 *Orally:* 1–4 *Written:* 5–23 odds
126	Sec. **5.11**, pp. 230–233 *Orally:* 1–6 *Written:* 7–23	Sec. **7.2**, pp. 296–299 *Orally:* 1–4 *Written:* 5–27 odds	Sec. **8.9**, pp. 370–372 *Orally:* 1–10 *Written:* 11–35 odds
127	Test, p. 234 (See p. T3.) Start Review, pp. 235–236	Sec. **7.3**, pp. 302–305 *Written:* 1–15 odds	Class Projects, p. 373 (Note: Materials should be ordered at least a month in advance. These projects

Lesson	Course A	Course B	Course C
128	Finish Review, pp. 235–236	Sec. **7.4**, pp. 306–309 *Written:* 1–17 odds	can be postponed to the end of the year.)
129	Test, p. T46	Sec. **7.5**, pp. 310–313 *Orally:* 1–10 *Written:* 11–33 odds	
130	Sec. **6.1**, pp. 240–242 *Orally:* 1–12 *Written:* 13–24	Sec. **7.6**, pp. 315–318 *Orally:* 1–14 *Written:* 15–26	
131	Sec. **6.2**, pp. 243–246 *Orally:* 1–2 *Written:* 3–35 odds	Sec. **7.7**, pp. 319–322 *Written:* 1–12 all, 13–19 odds	
132	Sec. **6.3**, pp. 247–249 *Written:* 1–21 odds	Test, p. 330 (See p. T3.) Start Review, pp. 331–332	Sec. **8.10**, pp. 374–377 *Orally:* 1–6 *Written:* 7–43 odds
133	Sec. **6.4**, pp. 251–254 *Orally:* 1–6 *Written:* 7–37 odds	Finish Review, pp. 331–332	Test, p. 378 (See p. T3.) Start Review, pp. 379–380
134	Sec. **6.5**, pp. 255–257 *Orally:* 1–4 *Written:* 5–31 odds	Test, p. T48 (Omit items 15–16.)	Finish Review, pp. 379–380
135	Sec. **6.6**, pp. 259–261 *Orally:* 1–2 *Written:* 3–29 odds	Sec. **8.1**, pp. 336–339 *Orally:* 1–4 *Written:* 5–13 odds	Test, p. T49
136	Sec. **6.7**, pp. 262–264 *Orally:* 1–2 *Written:* 3–29 odds	Sec. **8.2**, pp. 340–343 *Orally:* 1–2 *Written:* 3–11 odds	Another Look at Chapters 7 and 8, pp. 381–384 *Written:* 1–25 odds
137	Sec. **6.8**, pp. 265–267 *Orally:* 1–4 *Written:* 5–17 odds	Sec. **8.3**, pp. 346–349 *Orally:* 1–2 *Written:* 3–10	Another Look at Chapters 7 and 8, pp. 381–384 *Written:* 27–36 all, 37–51 odds
138	Sec. **6.9**, pp. 269–271 *Orally:* 1–10 *Written:* 11–31 odds	Sec. **8.4**, pp. 350–353 *Orally:* 1–2 *Written:* 3–33 odds	Preview Test, p. 385 (See p. T2.)
139	Sec. **6.10**, pp. 275–277 *Orally:* 1–6 *Written:* 7–33 odds	Sec. **8.5**, pp. 354–357 *Orally:* 1–2 *Written:* 3–9 odds	Sec. **9.1**, pp. 388–391 *Orally:* 1–4 *Written:* 5–10
140	Sec. 6.11, pp. 278–280 *Orally:* 1–4 *Written:* 5–35 odds	Sec. **8.6**, pp. 358–361 *Orally:* 1–4 *Written:* 5–11 odds	Sec. **9.2**, pp. 392–394 *Orally:* 1–4 *Written:* 5–7 Quick Quiz, p. 394
141	Test, p. 282 (See p. T3.) Start Review, pp. 283–284	Sec. **8.7**, pp. 362–365 *Orally:* 1–4 *Written:* 5–19 odds	Sec. **9.3**, pp. 396–398 *Orally:* 1–8 **a** and **b**, 9 and 10 *Written:* 1–8 **c**, 11–16
142	Finish Review, pp. 283–284	Sec. **8.8**, pp. 366–369 *Orally:* 1–4 *Written:* 5–23 odds	Sec. **9.4**, pp. 399–402 *Orally:* 1–4 *Written:* 5–21 odds

Lesson	Course A	Course B	Course C
143	Test, p. T47	Test, p. 378 (See p. T3.) Start Review, pp 379–380	Sec. **9.5**, pp. 403–405 *Orally:* 1–2 *Written:* 3–19 odds
144	Another Look at Chapters 5 and 6, pp. 285–288 *Written:* 1–51 odds	Finish Review, pp. 379–380	Platonic Solids, p. 407
145	Another Look at Chapters 5 and 6, pp. 285–288 *Written:* 53–85 odds	Test, p. T49 (Omit items 19–22.)	
146	Sec. **7.1**, pp. 292–295 *Orally:* 1–18 *Written:* 19–29 odds	Another Look at Chapters 7 and 8, pp. 381–384 *Written:* 1–21 odds	Sec. **9.6**, pp. 408–411 *Orally:* 1–22 *Written:* 23–32 Quick Quiz, p. 411
147	Sec. **7.2**, pp. 296–299 *Orally:* 1–4 *Written:* 5–27 odds	Another Look at Chapters 7 and 8, pp. 381–384 *Written:* 27–36 all, 37–43 odds	Sec. **9.7**, pp. 412–414 *Orally:* 1–2 *Written:* 3–8 all, 9–15 odds
148	Sec. **7.3**, pp. 302–305 *Written:* 1–15 odds	Sec. **9.1**, pp. 388–391 *Orally:* 1–4 *Written:* 5–10	Volume Projects, p. 415
149	Sec. **7.4**, pp. 306–309 *Written:* 1–17 odds	Sec. **9.2**, pp. 392–394 *Orally:* 1–4 *Written:* 5–7	Sec. **9.8**, pp. 416–418 *Orally:* 1–8 *Written:* 9–18
150	Sec. **7.5**, pp. 310–313 *Orally:* 1–10 *Written:* 11–33 odds	Sec. **9.3**, pp. 396–398 *Orally:* 1–8 a and b, 9 and 10 *Written:* 1–8 c, 11–16	Test, p. 420 (See p. T3.) Start Review, pp. 421–422
151	Sec. **7.6**, pp. 315–318 *Orally:* 1–14 *Written:* 15–26	Sec. **9.4**, pp. 399–402 *Orally:* 1–4 *Written:* 5–21 odds	Finish Review, pp. 421–422
152	Sec. **7.7**, pp. 319–322 *Written:* 1–12 all, 13–19 odds	Sec. **9.5**, pp. 403–405 *Orally:* 1–2 *Written:* 3–19 odds	Test, p. T50
153	Sec. **7.8**, pp. 323–325 *Written:* 1–13 odds	Sec. **9.6**, pp. 408–411 *Orally:* 1–22 *Written:* 23–32	Preview Test, p. 423 (See p. T2.)
154	Sec. **7.9**, pp. 326–329 *Written:* 1–19 odds	Sec. **9.7**, pp. 412–414 *Orally:* 1–2 *Written:* 3–8 all, 9–15 odds	Sec. **10.1**, pp. 426–429 *Orally:* 1–4 *Written:* 5–14 all, 15–33 odds
155	Test, p. 330 (See p. T3.) Start Review, pp. 331–332	Test, p. 420 (See p. T3.) Start Review, pp. 421–422	Sec. **10.2**, pp. 430–431 *Written:* 1–16 Quick Quiz, p. 432
156	Finish Review, pp. 331–332	Finish Review, pp. 421–422	ESP and Probability, p. 432

Lesson	Course A	Course B	Course C
157	Test, p. T48	Test, p. T50 (Omit item 11.)	Sec. 10.3, pp. 433–435 *Orally:* 1–4 *Written:* 5–31 odds Quick Quiz, p. 435
158	Sec. 8.1, pp. 336–339 *Orally:* 1–4 *Written:* 5–13 odds	Sec. 10.1, pp. 426–429 *Orally:* 1–4 *Written:* 5–14 all, 15–33 odds	Sec. 10.4, pp. 436–438 *Orally:* 1–6 *Written:* 7–22 Quick Quiz, p. 438
159	Sec. 8.2, pp. 340–343 *Orally:* 1–2 *Written:* 3–11 odds	Sec. 10.2, pp. 430–431 *Written:* 1–16	Math Bowl, pp. 440–441
160	Sec. 8.3, pp. 346–349 *Orally:* 1–2 *Written:* 3–10	Sec. 10.3, pp. 433–435 *Orally:* 1–4 *Written:* 5–31 odds	
161	Sec. 8.4, pp. 350–353 *Orally:* 1–2 *Written:* 3–33 odds	Sec. 10.4, pp. 436–438 *Orally:* 1–6 *Written:* 7–22	Sec. 10.5, pp. 442–444 *Orally:* 1–6 *Written:* 7–12 all, 13–33 odds
162	Sec. 8.5, pp. 354–357 *Orally:* 1–2 *Written:* 3–9 odds	Sec. 10.5, pp. 442–444 *Orally:* 1–6 *Written:* 7–12 all, 13–33 odds	Sec. 10.6, pp. 445–447 *Orally:* 1–6 *Written:* 7–12 all, 13–33 odds
163	Sec. 8.6, pp. 358–361 *Orally:* 1–4 *Written:* 5–11 odds	Sec. 10.6, pp. 445–447 *Orally:* 1–6 *Written:* 7–12 all, 13–33 odds	Sec. 10.7, pp. 448–450 *Orally:* 1–6 *Written:* 7–12 all, 13–33 odds
164	Sec. 8.7, pp. 362–365 *Orally:* 1–4 *Written:* 5–19 odds	Sec. 10.7, pp. 448–450 *Orally:* 1–6 *Written:* 7–12 all, 13–33 odds	Sec. 10.8, pp. 451–453 *Orally:* 1–6 *Written:* 7–12 all, 13–37 odds
165	Sec. 8.8, pp. 366–369 *Orally:* 1–4 *Written:* 5–23 odds	Sec. 10.8, pp. 451–453 *Orally:* 1–6 *Written:* 7–12 all, 13–37 odds	Sec. 10.9, pp. 454–456 *Orally:* 1–6 *Written:* 7–12 all, 13–33 odds
166	Test, p. 378 (See p. T3.) Start Review, pp. 379–380	Test, p. 458 (See p. T3.) Start Review, pp. 459–460	Test, p. 458 (See p. T3.) Start Review, pp. 459–460
167	Finish Review, pp. 379–380	Finish Review, pp. 459–460	Finish Review, pp. 459–460
168	Test, p. T49 (Omit items 19–22.)	Test, p. T51 (Omit items 14–15.)	Test, p. T51
169	Another Look at Chapters 7 and 8, pp. 381–384 *Written:* 1–25 odds	Another Look at Chapters 9 and 10, pp. 461–464 *Written:* 1–14	Another Look at Chapters 9 and 10, pp. 461–464 *Written:* 1–16
170	Another Look at Chapters 7 and 8, pp. 381–384 *Written:* 27–36 all, 37–43 odds	Another Look at Chapters 9 and 10, pp. 461–464 *Written:* 17–39 odds	Another Look at Chapters 9 and 10, pp. 461–464 *Written:* 17–43 odds

Notes to the Teacher

Chapter 1 USING TABLES AND GRAPHS Pages 9–52

Overview: This chapter deals with basic descriptive statistical material: reading and interpreting graphs; finding the mode, median, mean, and range of a set of data; and using tables, including frequency tables. People need to know how to make sense out of the variety of graphs and tables that they are confronted with almost daily. This chapter requires students to "dig out" a lot of information from a variety of statistical material. Actual, valid data are used as examples and for exercises.

The work with the mean and with frequency tables provides a good deal of computational practice. The use of electronic calculators is particularly appropriate for this kind of computation.

The *Quick Quiz* sections in this chapter check students' skills in (a) writing words for whole numbers and decimals; (b) rounding; and (c) adding, subtracting, multiplying, and dividing whole numbers. Before doing Section 1.11, students may need to review adding decimals and dividing a decimal by a whole number in the *Computing Skills Refresher.* (See *Quick Quiz,* page 44.)

1.1 In this section and in the other sections on graphing, students are asked to interpret information presented in graphic form. This is an important life skill.

Students should have initial success in reading the bar graphs, since numbers are included on the bars.

Exercises 1–8 could be used for oral work, with the other exercises assigned as written work. The assigned exercises can be discussed at the end of the class or at the beginning of the next class. Oral discussion can help sharpen students' interpretive skills.

1.2 This section can be taught by orally doing Exercises 1–10. An alternate method is assigning Exercises 1–10 or 1–14 to be done within a specified time limit (10–15 minutes) and then discussing the answers. The examples throughout this text are written at a beginning sixth-grade level, and students should be encouraged to read the examples on their own.

Exercises 19–20 introduce the idea of spotting a trend, and Exercises 33–38 reinforce the concept.

1.3 The tables presented in this chapter are examples of tables used in everyday life. Some students may never have used a timetable. Exercises 1–12 should probably be done orally, with emphasis on Exercises 6-12. The distance charts on page 22 are two of the most common forms. Some sample problems will help students make sure that they are reading from the correct row and column.

The optional feature, *Train Time,* page 23, involves calculations with time. This previews some of the work of Chapter 5. *Cross-Number Puzzle,* page 24, provides a thorough computation review. Instruct students to first work problems that they can easily solve.

1.4 In this section the student is shown a number of different techniques used with bar and line graphs. The motor-vehicle graph introduces the notion of percent.

Look Around You, page 28, suggests a class project. The display of a collection of graphs that the students find will add to the interest in this chapter. The project can be related to *Poor Pictographs,* page 31, and *Slanting the Picture,* page 77.

Math in the Social Sciences, page 29, shows graphs that are typical of those found in high school social science texts. These graphs can easily be used for more graph-reading practice.

1.5 A pictograph is a form of a bar graph. It is frequently used in newspapers and encyclopedias. This section gives students practice in interpreting pictographs.

Poor pictographs, page 31, shows some problems of comparing quantities by using drawings of different sized objects. Contrast this example with the more appropriate use of objects on pages 28–29. Sections 1.5 and 1.6 could be combined into a single lesson.

1.6 The mode is the easiest of the three common measures of central tendency (centers or averages) to find, particularly if the data has been grouped or graphed in some way.

Look Around You, page 33, suggests a possible project. In some stores metric sizes as well as customary sizes may be available.

1.7 The median divides the distribution into two sets. These sets are equivalent if the median number is not an element of the distribution. Otherwise the sets need not be equivalent. However, it is always true that (a) at least half the items are at or above the median and (b) at least half the items are at or below the median.

Emphasize to students that in order to find the median easily, the numbers must be arranged in order of size.

Exercises 23–32 provide practice in dealing with median incomes. This is the most common way of summarizing income figures. Students will probably be interested in interpreting and discussing the answers to these exercises.

1.8 Since the mean is used so frequently, students should become familiar with this idea. There are far too many exercises for one assignment. Either assign selected exercises or spend more than one day on the section. All the exercises in this section will provide computational practice with or without electronic calculators.

1.9 The range is used to describe the spread of a distribution. Exercises 1–6 can be done orally. Exercises 7–20 can be written work and can then be discussed at the end of the class. It is important that students be able to use and recognize the two ways in which a range is commonly expressed.

It is worthwhile to note that the centers and spread can give a better picture of a large and/or jumbled set of data than a list of the data itself.

Trimming the Range, page 41, shows how to give a better picture of the spread if there are extreme values. As with all Special Topics in the book, this material is optional.

1.10 Since much data is given to consumers in the form of frequency tables, students should be able to read and construct such tables. This is the culmination of the chapter; a student mastering this section is well prepared to use descriptive statistics in everyday life.

A good lesson plan for this section might include the teacher working an example and then the students working sample problems step by step. Because of the importance and difficulty of this section, two days could well be spent working and discussing the exercises. The use of calculators would be very appropriate.

Taking the Census, page 45, shows a very practical application of statistics and provides some thought-provoking questions. These could be discussed in class.

1.11 This section uses the mode, median, mean, and range in a decimal context. There is extensive computational practice, which may be done with or without calculators. Be sure students are competent in adding decimals and in dividing a decimal by a whole number, or it will be difficult to distinguish if errors are due to weakness with decimal fractions or due to confusion over mean, median, mode, and range. Two days could be spent on this section.

What's Average?, page 49, points out differences in the ways the various averages describe a distribution. It can be used to summarize the basic ideas of the chapter.

Chapter 2 MAKING GRAPHS Pages 53–92

Overview: Chapter 2, continuing the work with graphs begun in the first chapter, contains sections on maps, coordinate graphs, statistical graphs, schematics, and nomographs. These topics are important practical skills. While in Chapter 1 students concentrated on reading and interpreting bar and line graphs, in this chapter they will be required to construct these graphs as well as read and interpret circle graphs, nomographs, and schematics.

Most exercises require a minimal amount of computation. Consequently it is possible for students who are rarely successful in mathematics to be successful in this chapter. To maintain students' proficiency with computation, *Numbers on the Map*, page 60, and the *Quick Quizzes*, pages 62 and 72, are included. The *Quick Quizzes* give students a chance to check their skills on (a) naming fractions, (b) changing fractions to decimals, (c) changing decimals to fractions, and (d) adding and subtracting fractions.

2.1 Make sure students understand the symbol-to-region correspondence that is necessary for map reading. Working Exercises 1–6 orally will help reinforce this concept. The rest of the exercises may be completed orally or as a written assignment. Using transparencies of the maps will facilitate working the exercises orally.

Numbers on the Map, page 60, provides computational practice. Instruct students that when "one" is between brackets, as in [one] 12.5 × 8 Mile House, B.C., it is not part of the name.

2.2 Most students have been exposed to coordinate graphing in earlier courses. Review the use of the coordinate axes and the "over, up" procedure for locating points.

Fishy Graphs, page 63, is a variation of the game Battleship. This activity, while optional, is fun for students and provides additional practice with graphing.

2.3 This section links the previous section, graphing points on the plane, with Section 2.4, constructing line graphs. Do Exercises 1–7 orally to emphasize that the x and y axes may have different scale values. Next, you might want to assign one of Exercises 13–18 to be done as classwork so that students understand the need for different scale values on each axis.

2.4 This section might be best approached by graphing the data on page 66. Use the three-step approach and in step 2 include deciding to "break" axes and labeling the scales. For Exercises 23–24 many logs are thinner at one end. The cross section of the prism that the log is cut into for processing is determined by the diameter of the small end of the log. In the tables for cubic metres of lumber, rounding numbers sometimes results in the same number of cubic metres for two lengths.

2.5 Most students enjoy picture graphing, and as a result, they do more intricate graphing than they might willingly do otherwise. Students should use the same scales on both axes or the pictures will not be in proportion.

Graphs and Grams, on page 73, should be an interesting change of pace.

2.6 Before beginning this section, it would probably be beneficial for the class to review Section 1.1 on bar graphs. A suggested teaching strategy would include constructing both a horizontal and a vertical bar graph for the class. Use a step-by-step approach, emphasizing the need for choosing convenient scales and labeling each bar. Then give the class a sample set of data to graph before assigning written work.

Slanting the Picture, page 77, illustrates misleading graphs. You might want to have students look for misleading or biased graphs in newspapers or magazines.

2.7 Circle graphs and other "parts of a whole" graphs are easy for students to read because the numerical information is presented along with the visual image.

The class might want to discuss the implications of the information presented in the exercise graphs. Such a discussion can help students appreciate the relevance of mathematics.

2.8 Reading schematics, an important life skill, is not included in most school courses. There will be little need to "sell" students on the value of this lesson. However, many students may find schematics difficult.

Work Exercises 1–6 orally before assigning written work. Because of the potential difficulties in reading schematics, it might be worthwhile to have students work in small groups.

If possible, generate added interest in schematics by having students assemble an item from its schematic or by having students demonstrate tying neckties from the schematic in the text.

Math in Factory Jobs, page 85, extends the work with schematics and can be used by interested students or the whole class.

2.9 Nomographs are another of the useful graphic devices that students may use on the job but will seldom encounter in school. Have students notice that on the middle scale of the nomograph on page 86 the amount of space between numbers varies.

A possible teaching method would be to use a transparency of the L/100 km nomograph for doing Exercises 1-8 and some other selected exercises orally. Exercises 9-42 ask for estimates, and not all students should have the same answer.

Chapter 3 EQUATIONS AND FORMULAS Pages 97–140

Overview: This chapter introduces the student to basic algebraic topics: simplifying phrases, solving open sentences with one variable, and using formulas to solve verbal problems. The major goal of the chapter is to prepare students to use formulas. Many of the formulas are used in actual applications. These topics also provide a context for considerable computational work with whole numbers and decimals, and some work with fractions is done toward the end of the chapter.

Quick Quiz sections provide a check on (a) adding and subtracting whole numbers and decimals, (b) multiplying and dividing whole numbers and decimals, (c) changing fractions to mixed numbers, and (d) multiplying whole numbers by fractions.

3.1 It is important for students to be able to distinguish between true, false, and open mathematical sentences. This section and the next two lay the groundwork for working with equations and formulas in the remainder of the chapter.

3.2 This section deals with the use of parentheses to indicate (1) multiplication and (2) the operation to be done first. Some students may have difficulty remembering that $5x = 5(x)$.

3.3 This section builds onto Section 3.2 and introduces the standard rules concerning order of operations. Some students may find the mnemonic "Please, my dear, Aunt Sally" helpful. If you use this mnemonic, emphasize a three-step approach. First, do the operations inside the parentheses; second, do the multiplication and division; third, do the addition and subtraction.

Ingenious Genie, page 106, is intriguing and challenging. Exercise 3 in *Puzzle = 13,628,160,* page 106, should be amusing.

3.4 This is an important section. Formulas are introduced here and appear extensively throughout the rest of the chapter. Special attention should be given to this topic. Make sure that students are solving the problems by substituting into the formula and not by just multiplying or dividing the given numbers. One possible approach would be to have selected students explain their solutions of the exercises to the class.

3.5 In this section a method of solving equations using addition or subtraction is developed using the physical analogy of a balance. The physical analogy is not used in the exercises, but some students may find this helpful and may even want to draw pictures using a balance. Exercises 1–24 involve whole numbers only; Exercises 25–30 introduce decimals.

Students should think of the check as a necessary step in solving an equation.

One possible approach to this topic would be to have the class practice solving equations at their desks or at the chalkboard. Verbal problems using formulas are included in the exercises for this section and for Sections 3.6 through 3.9. Verbal problems are troublesome for some students. Make sure that you assign these exercises according to individual or class ability.

3.6 This section can be approached in much the same way as the previous section. For this lesson students need only interpret $\frac{6}{2}$ as 6 divided by 2, not as six halves. Exercises 1–26 involve whole numbers only. Then, Exercises 27–32 introduce decimals.

3.7 This section employs the "reverse order" or "undoing" method of solving two-step equations. Exercises 1–10 focus on this technique.

Equation Cross-Number, page 122, can be used as a second day's assignment for additional drill and practice.

3.8 Although this section introduces equations with fractional solutions, extensive competency in computing with fractions is *not* needed. The *Quick Quizzes* in Chapter 3 cover the skills necessary for solving and checking these equations. If additional remedial work is necessary, use the *Computing Skills Refresher.*

3.9 The equations in this section contain fractions and decimals. If students are taught the procedure for solving equations from the examples on page 126, competency with fractions is not really needed. Checking the solutions requires multiplying a fraction and a whole number, which is covered in the *Quick Quiz* on page 122.

3.10 If possible, work sample exercises involving each formula with the class. Notice that the verbal problems from Sections 3.5 to 3.9 are similar to the verbal problems in this section. Before assigning Exercises 17–22, make sure that students understand the meaning of "t^2."

Math in Apprenticeships, page 135, shows the widespread use of formulas. The last formula is especially challenging.

Money Matters, page 137, explains the use of formulas to solve common household problems.

The formula for *The Date's Day*, page 138, is difficult. Notice the use of the greatest integer function.

Chapter 4 RATIO AND PROPORTION Pages 141–188

Overview: This chapter concentrates on the meaning of ratio, writing and solving proportions, and percent. Other important topics include similar figures, three methods of comparison shopping, and estimation. Chapter 4 contains extensive computational practice with equivalent fractions, multiplication of whole numbers and decimals, division of whole numbers (resulting in whole number and decimal quotients), and rounding numbers. Some computation involving common denominators is included (page 163). Also, realistic application problems are provided for each of the major topics. Calculators may be appropriate for solving some exercises. The *Quick Quizzes* review (a) division of whole numbers where the quotient is a decimal and (b) multiplication and division of whole numbers and decimals by 100.

4.1 The concept of ratio is developed in this section. Encourage students to use the three forms of naming a ratio. Exercises 1–18 emphasize the order of comparison in a ratio. This concept, important for working with proportions, is generally easy for students to grasp.

4.2 Some students need to see equivalent ratios and fractions in diagram form. Diagrams similar to those in Example 2 are especially helpful; such visual aids enable students to relate the concrete to the abstract. After students understand equivalent ratios, explain methods of obtaining equivalent fractions. It might be beneficial to explain one or two exercises of each type before assigning written work.

Graphing Equivalent Ratios on page 150 reviews graphing on the coordinate plane and reexamines which fractions are equivalent. You might want students to discover that the same decimal represents all equivalent fractions.

Graphing the ratio of circumference to diameter may be delayed and used in conjunction with page 274. Also, any slight error in measurement will cause the graph of $C \div d = \pi$ to be other than a line.

4.3 Solving a proportion is an important skill, necessary for successfully completing this chapter. Emphasize that the cross-products in a true proportion are equal and that we solve a proportion by finding the value that makes the cross-products equal.

You may want to require students to check their solutions. All the exercises except 15–17 have whole number solutions.

4.4 This section presents the possibility of using different pairs of equivalent ratios to solve a proportion. It is important for students to understand that the first terms of each ratio refer to the same thing, such as lengths, as do the second terms. Also, the first ratio determines how the second ratio is written. Work a sufficient number of practice exercises with the class. If students still seem to have difficulty writing the proportion, assign the exercises as classwork or group work, instead of assigning written work.

4.5 One possible approach to similar figures is to show the class a number of examples. Then discuss the definition of similarity and review the notation for lengths of line segments.

You may want to use Exercise 1 to introduce the relationship between similarity and proportions. The corresponding parts in Exercises 3–12 should be easy for students to identify. Exercises 3–5, 9, 11–14, and 16 have whole number solutions.

4.6 This section exposes students to one method of comparison shopping. At the same time students practice finding common denominators and choosing the larger of two fractions. Students should at least understand how to use common denominators for comparison shopping before using the cross-products shortcut.

Exercises 4–8 are identical to Exercises 10–14. Exercises 1–22 work with whole numbers, while Exercises 23–30 work with decimals.

4.7 These two methods of comparison shopping are practical, review solving proportions, and give students practice with division and rounding quotients. The *Quick Quiz* on page 164 covers rounding quotients in division to the nearest tenth. The greatest-amount-for-$1.00 method of comparison shopping also readies students for work with percents.

After explaining each method separately, you might want to work one example using both methods. Also, because of the extensive computation involved in the exercises, it may be appropriate to allow students to use electronic calculators for some exercises. Since comparison shopping is an important life skill, it may be desirable for students to be able to use a calculator to find the best buy.

Math in Public-Service Jobs, page 167, illustrates some practical examples of the uses of mathematics.

Proportion Rummy on page 168 is a game that gives students great practice in making true proportions. Before students begin playing the game, discuss the examples of typical strategy with them.

Shopping in the Newspaper, pages 168–169, should generate student interest in being good consumers. You might want to give the students practice in using an electronic calculator to find the best buy. Other options include substituting some or all of these exercises for the exercises on page 166 or using ads from a local newspaper to extend these exercises.

4.8 This section uses equivalent ratios to develop the meaning of percent. The examples suggest the important ideas needing emphasis. The shading diagrams in Exercises 3–8 will help students get a "feel" for percent relative to the "whole." Exercises 9–11 and 17–22 work with the more common percent-fraction equivalents.

4.9 Proportions are used in this section to find a percent of a number (the multiplication technique appears later). Stress that the ratios are written "part over whole." You may need to emphasize the subtraction necessary to find the sale price, as in Example 2. Work a sufficient number of sample exercises with the class before assigning written work. Exercises 27–28 are practical problems that should interest students.

4.10 This section covers solving the three basic types of percent problems using the proportion "part: whole = percent amount: 100." Once the students understand this proportion, they should not encounter too much difficulty in solving type-2 and type-3 problems (type-1 problems were covered in the previous section). However, have the class work practice problems before assigning written work.

Notice that Exercises 11–24 are grouped by type so that, if necessary, students will have no difficulty using the examples on page 175 as models.

4.11 Two topics are covered in this section: (1) changing percents to decimals and vice versa and (2) using multiplication to find a percent of a number. It would probably be advisable to make sure students are competent in finding equivalent decimals and percents (as in Exercises 1–24) before teaching the multiplication method for solving percent problems. Because of possible error in placing the decimal point, encourage students to ask themselves if their answers are realistic. This will also help students with estimation in the next section.

Budgets, page 182, presents information in a form often found in periodicals. The graphs should also help students "feel" percent such as the relationship between 15% and 30%. For an additional activity have students estimate the percents for an unlabeled, subdivided circle.

4.12 Estimation is an important skill, though troublesome for some students, perhaps because it has not been taught earlier. However, if you use the step-by-step approach covered in the examples on pages 183–184, there should be a minimal amount of difficulty. Have students work practice problems before beginning written work. You might want to work some of Exercises 35–38 as classwork so that students understand the use of estimation in actual situations.

Chapter 5 MEASUREMENT Pages 193–236

Overview: Chapter 5 explores measuring length, mass, capacity, temperature, time, and angles.

Measurement is an active process, and ideally, students should be measuring as well as working exercises. For explanations and class activities you will probably want to use a tape measure; a class set of metresticks; a balance for measuring in grams; a scale for measuring in kilograms; containers for measuring in litres and in millilitres; and a Celsius and a Kelvin thermometer. Also, students will need access to a dictionary, and each student will need a ruler marked in centimetres and millimetres and a protractor.

This chapter contains computational practice in multiplying whole numbers with whole numbers, fractions, mixed numbers, and decimals and in dividing where the quotients are whole numbers, fractions, mixed numbers, and decimals. The *Quick Quizzes* review (a) changing improper fractions to mixed numbers and changing mixed numbers to fractions, (b) multiplying whole numbers by fractions or mixed numbers, and (c) multiplying and dividing by 10, 100, 1000. Some practical-application exercises are included in most sections.

5.1 In this section students are asked to pick the appropriate unit of measurement or the appropriate measuring instrument for a given situation.

Exercises 31–46 provide the students with a chance to sharpen their reference skills while learning about some specialized measuring instruments. Answers to Exercises 31–40 will vary according to the students' sources of information. Also, Exercises 47–52 should be assigned to correspond with Exercises 41–46.

5.2 Exercises 1–14 will help students become familiar with the ancient units presented in this section. Have several students read their answers to Exercises 15–24. Exercise 25, while reviewing the statistics of Chapter 1, should help students realize that there is much variation from person to person in "body," or ancient, measurements. Consequently, for commerce, civilizations agreed upon standard units approximating the body units.

5.3 Measuring lengths on an overhead projector with a transparent ruler would be a helpful visual aid. To help students measure accurately, it is best to first work on measuring to the nearest centimetre, then the nearest millimetre, and so forth as in the examples on pages 203-204. This pattern is followed in Exercises 7-10. After working with the magnified part of the ruler in Exercises 28-35, students should be able to measure accurately with a ruler.

The *Measuring Activities* on page 208 will give students practical experience in estimating lengths and in measuring. You may want to have students work in pairs or in small groups. It would be best to check students as they are measuring so that you can help them correct any errors.

5.4 This section uses the proportion method to find equivalent metric lengths. For writing metric measurements of five or more digits, stress using spaces instead of commas. This style was adopted to establish international uniformity in writing metric measurements.

Math in Industry, page 211, introduces tolerance, an important concept for any student interested in industrial work. Completing the table on page 211 requires addition and subtraction of decimals and fractions.

5.5 This section covers measuring mass in the metric system. The relationship between the kilogram, the base unit of the metric system, and the units derived from it is shown in the Chart on page 212. The proportion method from previous sections is used to change from one unit of mass to another.

Another measuring activity that students enjoy is finding their height and mass in centimetres and kilograms.

5.6 Visual aids would be appropriate for this section. If metric models are not available, remember that a litre is a cube with a length of 10 cm and that a millilitre is a cube with a length of 1 cm.

Notice that the proportion method from previous sections is used to change measurements from one unit to another. Students should now be adept at this method. Exercises 5-14 review equivalent measures and may be worked by the proportion method.

5.7 Students should be familiar with the units of time mentioned in this section. The examples on pages 219-220 should be helpful. However, it may be necessary to work several examples similar to Example 5 in which one time is a.m., the other p.m.

Exercises 33-36 involve practical applications of finding a length of time. Exercise 33 should be of particular interest to students.

5.8 Some students may not be familiar with the 24-hour clock. A short discussion of the advantages of the 24-hour clock should be interesting for all students. A comparison could be made between the examples using a.m. and p.m. in the previous section on page 220 and the examples on pages 222-223.

Exercises 8-14 provide problems in a variety of situations involving the 24-hour clock. You may want to ask students to find additional examples of the 24-hour clock in use.

5.9 In sections 5.7 and 5.8, students learned to add and subtract time to find the amount of time elapsed. In this section, students apply that knowledge to calculate intervals of time and then to calculate payments based on time. Periods of time less than one hour are converted to fractions of an hour in order to calculate charges based on hourly rates.

Exercises 1-16 provide practice both in converting minutes to fractions of an hour and in converting fractions to minutes. These exercises could be done orally in class. The remaining exercises involve practical applications of calculating payments based on time.

Math in Building Trades, page 226, presents four measurement problems from the building trades.

Line Art, page 227, is fun for students. Some students may enjoy making these designs using coloured string and nails on a board.

5.10 Although the Celsius scale is used for everyday heat measurement, scientists use the Kelvin scale almost exclusively. You may want to ask students to use reference books to learn about the origins of the Kelvin scale and why it is used by scientists in preference to the Celsius scale.

William Thomson (Baron Kelvin of Largs) devised the scale named in his honor. By starting the scale at a point much lower than 0°C, the negative figures below zero on the Celsius scale are replaced by positive numbers on the Kelvin scale. Thus -20°C becomes a positive number (about 253K), on the Kelvin or Absolute scale.

However, it should be noted that a change in temperature of one degree Celsius is identical to a change of one Kelvin. Only the numerical value assigned to the temperature differs.

Measuring Activities on page 229 do not take much time and should be interesting to the class. You might want to keep a Kelvin and a Celsius thermometer in the classroom or note daily outdoor high and low temperatures.

5.11 Students may be able to see examples done with a transparent protractor on an overhead projector more easily than examples done with a large protractor on the chalkboard. The step-by-step approaches of Examples 1 and 4 will be helpful to students. Emphasize seeing if the angle is less than or greater than a right angle before deciding which scale to use.

Make sure that students are using their protractor correctly to measure and draw angles before assigning written work.

If the class does not have the same type of protractor, you might want to give a short explanation to groups of students on using each type of protractor.

Chapter 6 PLANE FIGURES Pages 237–284

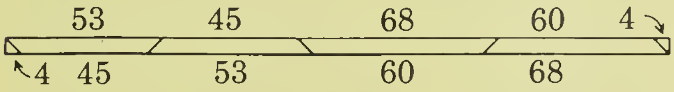
Overview: This chapter introduces students to plane figures with a "look and notice" approach. Topics covered include polygons, perimeter, area of special quadrilaterals and circles, circumference, and square roots.

The exercises will provide students with considerable computational practice in adding and multiplying whole numbers, fractions, mixed numbers, and decimals, as well as some practice in dividing, primarily with whole numbers. The *Quick Quizzes* review (a) adding mixed numbers and decimals, (b) multiplying decimals and mixed numbers, (c) finding the product of $\frac{1}{2}$ and two other numbers, (d) doing multiple operations indicated with parentheses, and (e) squaring whole numbers. Some exercises have consumer and career applications. Students will need a centimetre ruler and scissors for completing some exercises.

6.1 Polygons along with four special quadrilaterals, probably familiar to most students, are presented in this section. The exercises, especially 17–20, ask students to inspect, notice, and state properties of the polygons. Exercises 21–26 involve drawing polygons with certain properties. Students should find Exercises 27–28 interesting.

6.2 This section explains the concept of perimeter and at the same time gives students practice in adding and multiplying whole numbers, fractions, mixed numbers, and decimals. The two formulas on page 244 simplify finding the perimeter of rectangles and squares and review working with formulas and parentheses. You may need to remind students that units, such as metres, must be included in their answers.

Exercises 3–8, 21–22, and 31–32 contain whole numbers only. Exercises 37–41 are practical applications of finding perimeter. The answer given for Exercise 41 assumes that the frame is beveled and that some of it is wasted. However, if the frame is flat, it can be cut according to the diagram below.



The answer would be 226 cm.

To review measuring and to emphasize the concept of perimeter, you might want to have students find the perimeter of either the classroom or a window.

6.3 This section, an introduction to area, will help students understand that area is the measure of square units of surface, a concept that may cause some students difficulty. The exercises emphasize measuring in square units. The shortcut of multiplying the number of rows by the number of squares in each row is the basis of the formula for finding the area of a rectangle in the next section.

You may find it helpful to work some examples similar to Exercises 13–22 on an overhead projector.

Tangrams, page 250, should be challenging yet enjoyable for students. Ask students how many square units each shape would cover if the square that is the composite of pieces A–G covers one square unit. (A—$\frac{1}{4}$, B—$\frac{1}{4}$, C—$\frac{1}{16}$, D—$\frac{1}{8}$, E—$\frac{1}{16}$, F—$\frac{1}{8}$, G—$\frac{1}{8}$)

6.4 Students need to relate the word "area" and the formula $A = \ell \times w$ with finding how much surface a rectangle covers, the topic from the previous section. One strategy is to show the class examples similar to Example 1, page 251, and then find the area by the formula.

Develop the formula for the area of a square by treating the square as a special rectangle whose length and width are the same. The notation "s^2" will be new to most students.

Exercises 4–18 and 31–32 involve working with whole numbers only. The other exercises will give students practice in multiplying fractions and decimals. Exercises 41–44 are practical applications of finding area.

The Extra Square Unit, page 254, is a challenging puzzle.

6.5 The method of finding equivalent areas is the same as the method of finding equivalent units of measurement described in Chapter 5. Students should be adept at setting up the proportions, but it may be necessary to emphasize that 1 m^2 = 10 000 cm^2 and that 1 cm^2 = 100 mm^2. The two drawings, pages 255–256, are presented to help students remember these facts. Work some examples for the class, and have them work some practice exercises before assigning written work.

Exercises 21-23 are interesting application problems.

Patterns with Squares, page 258, is an activity that involves area relationships. It may be helpful for students to make the patterns by tracing a 1 cm cardboard square.

6.6 Finding the area of a parallelogram is very similar to finding the area of a rectangle. You may need to emphasize that the height is the length of any segment forming right angles with both bases, as shown in the figures on page 259. It may be advisable to encourage students to check their answers by finding the area of the rectangle that can be formed from each of the parallelograms.

Exercises 7–10 are designed so that only the base and its corresponding height can be measured as whole numbers on the grid. Exercises 15–24 and 27–29 give students practice in computing with fractions and decimals along with finding the areas of parallelograms.

The *Quick Quiz* on page 261 reviews the computation skills necessary for finding the areas of triangles and trapezoids.

6.7 Visual aids are helpful for showing that any parallelogram can be divided into two identical triangles and that any two identical triangles can be combined into a parallelogram. Exercise 2 would be a good in-class activity. After students can visualize the relationship between a parallelogram and two identical triangles, slowly develop the area formula for a triangle.

You might want to use Exercises 3–8 as classwork. Exercises 4, 6–10, 12, and 26–29 contain only whole numbers and have whole number answers.

6.8 Most students should be able to follow the development of the formula for the area of a trapezoid and will, consequently, find remembering the formula easier. Make sure students know how to recognize the height and bases.

You may want to have the class do some practice exercises before assigning written work. A few students may need to review using parentheses. Exercises 2, 3, 5–7, 13, and 15–18 involve no computation with decimals or fractions, other than finding $\frac{1}{2}$ of an even number.

Area or Perimeter?, page 268, points out the need in five different occupations for deciding when to use area or perimeter.

6.9 Most students have not previously been exposed to square roots. The physical model of the area of a square representing a number and the length of a side representing the square root should be helpful. Make the transition from the model to the "$\sqrt{}$" symbol slowly.

After students realize that not all numbers are perfect squares, explain finding square roots and squares using the table on page 272.

Exercises 1–10 will reinforce the concept of square root. Exercises 29–31 illustrate using square roots to solve practical application problems.

A Bargain? on page 273 is a good example of why checking an advertisement to see if it is really a bargain is necessary. Notice that the activities involve both measuring length and finding area.

A Number Called Pi, page 274, is an activity for finding the value of pi. It may be combined with the activity on page 150. These activities are a good introduction to working with circles.

6.10 Students will probably need some examples before believing that circumference ÷ diameter = π. You might want to measure the circumference and diameter of several objects in the class; or divide the class into groups and have each group measure the circumference and diameter of an object. Once students understand that $C \div d = \pi$, it will naturally follow that $C = \pi \times d$. You might want to talk about the need for approximating π as $\frac{22}{7}$ or 3.14.

In the exercises students are asked to find the circumference using $\frac{22}{7}$ or 3.14 for π, given either the radius or the diameter.

The exercises involve multiplying decimals with whole numbers, fractions, and other decimals.

6.11 Approach the formula for the area of the circle slowly; students should be able to understand why $3r^2$ is a reasonable approximation for the area of a circle.

Exercises 13–20, 27–30, and 38 involve multiplying decimals with decimals or fractions. Exercises 31–36 require students to find the area given either a radius or a diameter. Exercises 39–40 are interesting application problems.

For Pizza Freaks Only, page 281, is both amusing and practical.

Chapter 7 SCALE DRAWINGS Pages 289–332

Overview: This chapter contains various approaches to scale drawings: enlargement and reduction ratios, two methods of drawing figures according to these ratios, map reading, similar figures, constructing triangles from measurements, and indirect measurement.

The exercises give students additional practice with measurement, ratios, equivalent fractions, and solving proportions. The *Quick Quiz* reviews multiplying whole numbers by decimals or mixed numbers. For completing the exercises, students will need a centimetre ruler, a protractor, and paper with grids of 1, 2, and 3 squares to the centimetre.

7.1 Two terms, "enlargement by a factor of" and "ratio of enlargement," are introduced in this section and used throughout the chapter. Stress that both are the ratio of the new figure to the original figure. One approach to this section would be to use a grid on an overhead projector to draw enlargements and reductions.

Exercises 1–18 done orally will help students get a feel for ratios of enlargements and reductions. Exercises 19–30 help relate the ratios to the actual changes in size of the figures.

7.2 This section explores the practical application of enlargement and reduction ratios to scale drawings. Again, emphasize that the ratios are "new:original" and explain the use of "=" to indicate the scale rather than equality. You will probably find it helpful to work some examples similar to Examples 2 and 3, pages 296–297, emphasizing the two-step approach—measure and multiply.

Exercises 5–10 help reinforce the scale as the ratio of length of the new figure to the length of the original figure. Exercises 11–26 give students practice measuring lengths and multiplying fractions, decimals, and mixed numbers.

Scale Drawings at Work . . ., page 300, *. . . and at Home*, page 301, show the relevance of mathematics before, during, and after the construction of an apartment. The questions give students more practice with scale drawings.

7.3 Working square by square, students should be able to draw the enlargements or reductions without any difficulty. Although most exercises do not require students to choose the grid size, it might be advisable to explain how the size of the grid is determined. You might want to explain the proportion in the examples as "the ratio of enlargement (reduction) = squares to the centimetre (original grid): squares to the centimetre (new grid)."

7.4 The point method of making scale drawings involves measuring lengths and multiplying those lengths by the ratio of enlargement or reduction. Make sure students use point P as one endpoint when measuring the new length. Notice that the point method gives students practice measuring lengths and multiplying fractions and mixed numbers. You might want to assign several problems, including a curve, as classwork before assigning written work.

7.5 The necessity of being able to read road maps should be evident to most students. Some students may be familiar with map reading, while others may not. Finding the straight-line distance between cities gives students more practice multiplying mixed numbers and decimals and measuring lengths.

In measuring map distances discrepancies arise, depending on whether the shortest distance or the distance between approximate centers of towns is found. You may wish to allow some variation in the answers or to adopt the convention of measuring between approximate centers. The latter convention was used to determine the answers for the Teacher's Edition.

Follow the Yellow Brick Road, page 314, should be fun for students while giving them added practice in map reading.

Your Provincial Map, page 314, contains exercises that will increase the students' map-reading skill and will familiarize them with the map of their province.

7.6 Two topics are covered in this section: (1) two figures with the same shape are similar even if their sizes or orientations differ; (2) a similarity statement names the corresponding parts of two similar figures. An overhead projector would be a useful visual aid for covering both these topics.

7.7 You might find it helpful to use Exercises 1–10 as an introduction to similar figures. Use the answers to the exercises as a basis for discussing the ratio of similarity, the measures of the corresponding angles, and the use of proportions to find the lengths of the missing sides.

The exercises give students additional work measuring lengths and angles, finding equivalent fractions, and solving proportions.

7.8 One possible teaching strategy would be to construct a triangle from measurements for the class. Then have the class construct a triangle from measurements as you demonstrate the process. Finally, have the students construct several triangles from measurements as classwork. You might also want to have the class do an example similar to Example 3, page 324, before assigning written work.

7.9 Indirect measurement requires using scale drawings, similar figures, and proportions. It would be advisable to work, step by step, several examples similar to Example 2 on page 326. Point out to students that although these problems involve several steps, they are already familiar with each step. You might want to assign several exercises as classwork before assigning written work.

Exercises 7–10 involve only setting up and solving the proportion. Exercises 11–18 require making a scale drawing as well as setting up and solving the proportion. Advise students that the drawings in the exercises are not accurate, because they have been drawn to account for the third dimension represented in each photo (and to require student drawings).

Chapter 8 CONSUMER SKILLS Pages 333–380

Overview: The goal of this chapter is to develop the skills necessary for being a wise consumer. Students are exposed to budgets and management of personal finances, comparison shopping, chequing accounts, and the cost of credit purchases, as well as to taxes and time measurements. Student interest should be high in this chapter because the topics are obviously relevant and beneficial. A good project would be having the students monitor their income and expenses for two months in preparation for making a personal budget.

The *Quick Quizzes* review (a) reducing fractions to lowest terms and (b) dividing whole numbers. Because many of the exercises deal with money, students will get extensive computational practice with decimals. The exercises also include work with proportions, division, percents, and formulas. Many of the exercises require computation that may be difficult for some students, and the use of calculators is suggested. The ability to work consumer oriented problems with a calculator can be a beneficial skill.

Talk It Over exercises are included after several sections. The questions posed are fundamental to good personal financial management.

8.1 Keeping track of expenses and income is basic to good money management. Students should readily understand how income and expenses affect cash on hand.

Exercises 9–14 require students to find the monthly expenditures for specific items and the relationship of total expenses to total income.

Talk It Over, page 339, introduces students to fixed and variable expenses and the problem of income not meeting expenses.

8.2 It might be necessary to remind students that the division of expenses into categories is dependent on the individual situation. Net income is used to figure the percent of the total spent on each category. You might want the class to work an example similar to the one on pages 340–341. Encourage students to check their work by making sure they have a total of 100%. The shortcut for solving proportions should be helpful.

Talk It Over, page 343, covers keeping written records and a regular savings plan.

That'll Be 5 Clams, page 344, is a brief history of the development from a system of bartering to an economic system using money. The list of primitive and current monetary units should interest students.

Math in Summer Jobs, page 345, gives students some practical advice for finding summer jobs. The exercises provide more practice solving verbal problems containing percents and fractions.

8.3 Making a workable budget requires a good deal of thought; the questions on page 346 explore topics that need to be considered. Remind students that the basis of a budget is past financial history and anticipated needs. Again, the amount allotted to each category is a percent of the net income. It might be necessary to review both the proportion method and the multiplication method for finding a percent of a number.

Talk It Over, page 349, poses questions relating to budgets. You might want to have students discuss these questions with their families.

8.4 This section covers the unit-cost method of comparison shopping (also presented in Section 4.7). A shortcut involving division for finding unit cost is presented. Because it is necessary to find unit cost quickly in grocery stores, you might want students to become adept at finding unit cost with a calculator. Point out that even if stores list unit prices, it is still desirable to be able to check them quickly and accurately.

Exercises 3–30 give students practice in comparison shopping, and Exercises 31–34 make evident the financial benefits of comparison shopping.

Talk It Over, page 353, asks some important questions concerning comparison shopping.

8.5 Students should be able to use a

chequing account, an important practical skill. You might find it helpful to duplicate blank cheques and cheque records for the students. It may be appropriate to allow students to use a calculator for keeping the cheque record balance.

8.6 One suggested strategy would be to thoroughly discuss all the information found on a bank statement. It is important that students understand the necessity of reconciling a bank statement.

As an optional activity you might want to have students contact banks to inquire about charges for chequing accounts and overdrawing an account.

Exercises 1–4 review the important terms and procedures for reconciling a bank statement. Exercises 5–10 give students practice working with the figures found on a bank statement. Exercises 11–12 require students to reconcile a bank statement with a cheque record from a previous section.

8.7 An understanding of the cost of credit and knowing how it is determined are important consumer skills. You might want to work several examples using the formula. Point out that the cost of credit increases as the amount of money borrowed or the length of time taken to repay the loan increases. Also, make sure students understand the note, page 363, stating that the dollar cost of credit *must* be stated on all installment contracts.

Exercises 5–16 are in table form, which makes it easier for students to apply the formula. Exercises 17–22 require students to set up the formula, as in a real-life situation, in order to find the credit cost.

8.8 Few students will have prior knowledge or understanding of the annual percentage rate of interest on credit purchases. The explanation on page 366 should be helpful. Make sure students understand each term in the formula for the annual percentage rate of interest. Calculators may be appropriate for the exercises.

Exercises 5–22 are in table form to make it easier for students to apply the formulas. For Exercises 23–26, students must figure the cost of credit and the annual percentage rate of interest as they would for an actual purchase.

An optional activity would be having students contact stores, automobile dealers, credit-card companies, and banks to find out the cost of credit and the annual percentage rate of interest for borrowing various amounts of money for varying lengths of time. This would develop the idea that a person can shop for credit to get the *best* buy.

8.9 This section introduces students to reading a tax schedule. Work some examples similar to the example on page 371. The exercises afford students additional practice finding a percent of a number. The tax unit from Revenue Canada, Taxation is helpful for teaching students how to fill out a tax return.

Class Projects, page 373, lists units, available without charge, on taxes, consumer finance, insurance, and investments that should be interesting and informative.

8.10 This section develops the three possible ways of working with time. The examples on pages 374–375 should be helpful. If you have covered Sections 5.7 to 5.9 on measuring time, you may want to cover this section quickly.

Exercises 7–30 concentrate on adding and subtracting times or lengths of time. Exercises 43–48 are relevant time-application problems.

Chapter 9 SOLIDS Pages 385–422

Overview: This chapter concentrates mainly on finding the volumes of common geometric solids, such as rectangular solids, right prisms, cylinders, and spheres; one section covers surface area. Most sections of the chapter contain only one topic, short enough to be easily mastered by students in one lesson. The first three sections will familiarize students with solid figures and give students an initial feeling of success with solids while preparing a basis for the volume formulas. Drawings in both the exercises and examples will help students visualize important concepts.

The exercises for finding volume and surface area give students practice solving proportions and computing with whole numbers, fractions, and decimals. The *Quick Quizzes* review multiplying three, four, or more factors. Application exercises are included for most sections.

9.1 All students will be familiar with rectangular solids in the form of boxes. Exercises 1–4 review the properties of rectangular solids. Exercises 5–10 emphasize the characteristics of rectangular solids by having students draw or make them. You might find it helpful to have students keep, for use in Sections 9.3 and 9.6, the rectangular solids that they make.

Patterns with Cubes, page 391, requires students to use spatial relationships to find patterns for a cube.

9.2 Make sure students understand the properties of right prisms; they should readily be able to identify some solids as right prisms.

Exercises 5–7 involve having the students make right prisms; caution students not to use tracing paper.

Slicing Cubes, page 395, gives students additional practice visualizing spatial relationships.

9.3 Measuring with cubes establishes a basis for introducing the volume formulas. The *Class Activity* on page 398. provides a concrete example of measuring with cubes.

Exercises 1–10 lay the groundwork for the formula for finding the volume of a rectangular solid; the drawings will help students visualize filling the solid with cubes. Exercises 11–14 require finding the number of cubes using the method developed in Exercise 10.

9.4 This section introduces cubic units for measuring volume. Measuring with cubes, from the previous section, is used to develop the formula for the volume of a right prism or rectangular solid, $V = Bh$. Stress that B represents the number of cubes in one layer or the area of the base. The formula, $V = lwh$, for the volume of a rectangular solid is then easily developed.

Exercises 5–16 and 20–22 involve working with whole numbers only. Exercises 15–26 are interesting application problems. Remind students that the proper cubic units need to be included in their answers.

9.5 Make sure students understand that $1\ \text{m}^3 = 1\ 000\ 000\ \text{cm}^3$ before covering equivalent volumes. The proportion method, developed in Chapter 5 for finding equivalent measurements, is used to find equivalent volumes.

Exercises 3-6 involve whole numbers only.

Math in Paraprofessions, page 406,

exposes students to more examples of mathematics, specifically finding volume, being used in various occupations.

Platonic Solids, page 407, should arouse the students' interest. You might find it helpful to have students make an equilateral triangle, a square, and a regular pentagon out of cardboard and then trace the patterns from these figures.

9.6 Use a visual aid so that students can see which faces have equal area and that surface area is the sum of the areas of all six sides.

Exercises 1–22 will help students distinguish which faces and edges have the same measure. Exercises 23–26 involve whole numbers only. Exercises 29–38 are application problems. The answer to Exercise 34 in the Teacher's Edition assumes that gold leaf is applied to all sides; however, it would not be incorrect for students to assume that the gold leaf should cover only five sides. For Exercises 37–38, you might want to ask students which box would be the best in terms of cost and shape for packaging a specific product.

9.7 Before developing the formula for the volume of a cylinder, review the formula for the volume of a right prism and the formula for the area of a circle.

Both $\frac{22}{7}$ and 3.14 are used as approximations for π in the exercises. The figures for Exercises 3–8 should be helpful. Exercises 3–6 and 13–16 involve whole numbers only. Exercise 17 is an application problem that you might want to work with the class.

Volume Projects, page 415, will probably have to be seen to be believed. The relationship between right pyramids and right prisms is used to justify the formula for the volume of a sphere in Section 9.8.

9.8 The justification of the formula for the volume of a sphere is similar to the justification of the formula for the area of a circle. Visual aids will be helpful in developing the formula, and students should be able to understand the justification if it is approached slowly. It may be helpful to work some sample exercises to make sure students can apply the formula. Some students may not be familiar with cubing numbers.

Exercises 5–8 are designed to make it easy for students to initially apply the formula for the volume of a sphere. Exercises 9–12 involve relatively simple computation. Exercise 20 might be an interesting application exercise to work in class.

Packaging Can Fool You, page 419, is an informative consumer education lesson. You might want to have students survey stores and note packaging that is deceiving to the buyer.

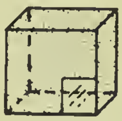
Chapter 10 CHANCE AND ESTIMATES Pages 423–460

Overview: This chapter covers two topics, probability and estimation. Students are exposed to experimental probability, mathematical probability, mutually exclusive events, and independent events. Throughout the probability sections it is assumed that dice and coins are "fair." This is stated on the opening page of the chapter (page 425). Estimation is used in computational situations; the emphasis is on estimating a sum (difference, product, or quotient) to check a calculation or as an accurate-enough rough answer. Calculators are useful in obtaining the exact answers for checking the estimates.

The chapter contains computational practice with equivalent fractions, decimals, and percents; adding and multiplying fractions; rounding numbers; and adding, subtracting, multiplying, and dividing whole numbers and decimals. The *Quick Quizzes* cover (a) adding fractions, (b) multiplying fractions, (c) rounding numbers, and (d) computing with multiples of 10, 100, and 1000.

10.1 This section introduces students to experimental probability and mathematical probability. Be sure students understand the distinction between them and their different abbreviations. After working some examples similar to those on pages 426–427, stress that the experimental probability of an outcome is very close to the mathematical probability if the number of trials is large enough. (Thus, an experimental probability can be used as an estimate of a mathematical probability, especially in cases where the number of possible outcomes is not evident or is very large.) Some examples will be helpful for explaining impossible and sure-to-happen events.

10.2 For one of the experiments, you might want to record all the results on the chalkboard so the students have an example of the experimental probability being close to the mathematical probability if the number of trials is large. For Experiment 3 you might have the students make a "shaker" using a small box. Insert a cellophane window in one corner as shown below.



In this way the marbles can be shaken without being removed from the box. Each time one marble will fall into the corner where it will be visible through the window.

If time permits, you might have students perform another experiment involving two coins. When the two coins, such as a penny and a nickel, are tossed, four equally likely outcomes are possible. Have the students find the experimental probability of each outcome. Be sure to point out that one head and one tail can occur in two ways.

ESP and Probability, page 432, should be interesting for students while giving them more of a "feel" for probability.

10.3 The explanations and examples on page 433 are helpful for explaining mutually exclusive events. Point out that the probability of either one of two outcomes occurring should be greater than the probability of either outcome occurring by itself. Make sure students are able to find both the experimental and mathematical probabilities of mutually exclusive events.

Exercises 1–4 amplify the definition of mutually exclusive events, while Exercises 5–8 give students practice finding pr (A or B). The other exercises require students to find both the experimental and mathematical probabilities of the same outcomes as percents and fractions.

10.4 The concept of independent events is extremely important in probability. Emphasize that independent events may or may not happen at the same time, but that the first outcome must be independent of the second outcome. Point out that the probability of two outcomes occurring should, reasonably, be less than the probability of either outcome occurring.

Exercises 1–6 concentrate on the meaning of independent events, and Exercises 21–22 are interesting application problems. Exercise 21 involves experimental probability. However, independent events are defined for mathematical probabilities. Consequently, the experimental probability $\frac{33}{50}$ must be used as an estimate for the mathematical probability.

What Are the Chances?, page 439, contains some real-life applications of probability.

Math Bowl, page 440, will be an enjoyable learning experience. Besides being a novelty, this game provides a good drill in addition of whole numbers.

The questions in *Math Bowl Chances*, page 441, require students to decide whether to find the probability of mutually exclusive events or of independent events.

10.5 Sections 10.5 through 10.9 follow a similar pattern. The students are asked to estimate a calculation by first rounding to a preselected place and then carrying out the calculations with the rounded numbers.

Three types of exercises are included in each section: selecting the best estimate from three choices, estimating and checking the answer by finding the exact calculation, and application exercises. If students have access to calculators, all the exercises may be assigned. Otherwise, select exercises carefully to avoid overly long and tedious assignments.

Skill in estimating, both to get an approximate answer and to determine if a calculated answer is reasonable, is extremely important. Encourage students to use estimation as a tool in all computational work.

Estimating sums is covered in this section.

10.6 This section covers estimating differences.

10.7 This section covers estimating products.

10.8 This section covers estimating quotients.

10.9 Section 10.9 combines estimating sums, differences, products, and quotients. In the verbal exercises students are asked to choose the correct operation.

Sports and Sales, page 457, requires students to apply the mathematics of this chapter, probability and estimation, to solve four real-life problems.

COMPUTING SKILLS REFRESHER Pages 466–540

Overview: The Computing Skills Refresher covers addition, subtraction, multiplication, and division with whole numbers and decimals in Part I, and with fractions and mixed numbers in Part II. The topics are divided into sections that can usually be covered in one lesson.

Each section contains numerous exercises, and care should be taken to make sure that assignments are not overly long or tedious. Encourage students to get in the habit of checking their work. Some of the exercises in Part I can be used to practice working with calculators.

The table on page T39 keys the items in the Computing Skills Pretests and Posttests on pages T52–T59 to the appropriate sections of the Computing Skills Refresher in the text.

1–3 Emphasize flexibility in reading the numbers. The object is for the students to be able to communicate names for numbers.

Stress 1.6 = 1.60 = 1.600. Answers in the Teacher's Edition do not have final zeros.

Some towns employ numerical names for

streets as numerals, while other towns use words.

4 The final zero(s) following a decimal point are kept in the rounding-off process to show what place the number has been rounded to. For example, 10.00 shows rounding to the nearest hundredth.

5–9 These sections cover adding and subtracting whole numbers and decimals. Work with students until they can line up the decimal points properly when performing additions and subtractions.

10–14 It is possible to work with students until the majority have the multiplication facts memorized. However, you may wish to allow those students who are having great difficulty in memorizing the multiplication facts to use a multiplication table.

Multiplying by multiples of 10 is an important topic, especially for estimation. When multiplying whole numbers and decimals or decimals and decimals, remind students that final zeros following a decimal point may be dropped (except in rounding) or may be kept.

15 Again, it may be possible for most students to memorize the division facts, but it might be helpful to allow certain students to use a table.

16–18 These sections cover division of whole numbers by whole numbers with remainders possible. Encourage students to decide if their answer is reasonable, along with checking their work.

19–20 Dividing decimals by whole numbers readily follows from the previous section because the only new procedure is aligning the decimal points.

Exercises 29–56, page 493, require students to add following zeros. The exercises on pages 487, 489, and 491 can also be used as practice for giving decimal answers.

21–22 Students may find both dividing by decimals and rounding quotients troublesome. One possible approach would be to work examples and sample problems before assigning written work.

23 This section provides a good review of multiplying and dividing whole numbers and decimals.

24 This section, an introduction to fractions, uses diagrams to emphasize the meaning of fractions.

25 The cross-product method of finding equivalent fractions is the same method used to solve proportions in Chapter 4.

26–27 These sections cover changing proper and improper fractions to decimals and introduce properties of zero and one in relation to fractions.

28 Students are asked to state all fractional answers in lowest terms throughout the rest of the *Computing Skills Refresher*. Most students will probably do best with the informal method used in Examples 1 and 2. Some students might be able to use the method of Example 3. The use of prime factorization is not recommended.

29 It may be helpful to review reading decimals before covering this section.

30 The exercises on page 509 review the work with fractions so far.

31–32 The diagrams on page 510 should be helpful for explaining addition of fractions. Exercises 35–40, page 511, help prepare students for subtracting fractions.

33–36 Fractions with unequal denominators and adding and subtracting these fractions are topics covered in these sections. You may wish to point out that unlike fractions can be changed to like fractions with any common denominator. One common denominator is found by multiplying the denominators of the unlike fractions.

Section 35 covers more properties of zero and one with regard to fractions. The exercises reinforce these properties and provide more practice adding and subtracting unlike fractions.

37–38 Diagrams similar to the one on page 520 should be helpful for explaining multiplication of fractions. Exercises 43–62, page 521, give students practice with reciprocals and multiplication by a fractional form of one.

Exercises 31–39, page 523, are challenging. Deciding what numbers will *not* permit cancellation will be helpful in working the problems.

39 Dividing fractions will be easier for students if they understand the reason for multiplying by the reciprocal of the divisor. The explanation presented on the top of page 524 should be helpful.

40 Additional practice multiplying and dividing fractions is provided in this section.

41–43 Understanding mixed numbers in relation to whole numbers, fractions, decimals, and improper fractions is the aim of these sections.

44–45 Two methods of adding and subtracting mixed numbers are described in these sections. Encourage students to use the method that they find easier.

46 It might be helpful to point out to students that multiplying and dividing mixed numbers requires no new learning, only combining previously learned skills.

47 This section reviews adding, subtracting, multiplying, and dividing mixed numbers.

48 Working with exponents may be new to many students. You may want to cover this topic before starting Chapter 6, the first chapter containing exponents (except for two formulas with squared terms in Section 3.10).

KEY FOR CORRELATING PRETEST OR POSTTEST, PAGES T52–T55, TO COMPUTING SKILLS REFRESHER

Section	1	2	3	4	5	6	7	8	10	11
Test Items	1–4, 9–12	5–8, 13–16	17–24	25–30	31–40	41–46	47–54	55–65	66–72	73–75

Section	12	13	14	15	16	17	18	19	20	21	22
Test Items	76–83	84–87	88–91	92–95	96–97	98–102	103–106	107–110	111–113	114–116	117–118

KEY FOR CORRELATING PRETEST OR POSTTEST, PAGES T56–T59, TO COMPUTING SKILLS REFRESHER

Section	24	25	26	27	28	29	31	32	33	34	35	37
Test Items	1–10	11–15	16–19	20–22	23–25	26–28	29–33	34–38	39–43	44–48	49–58	59–64

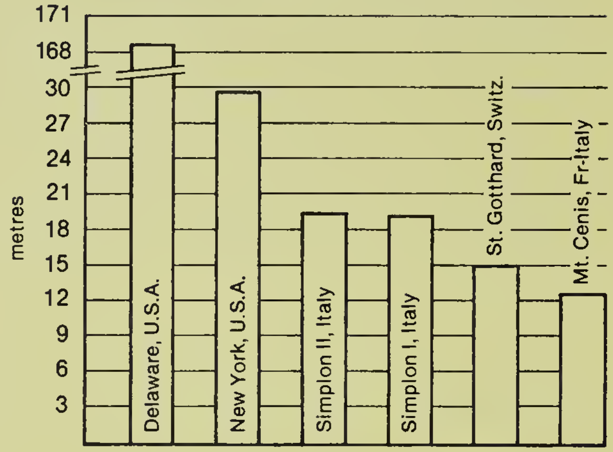
Section	38	39	41	42	43	44	45	46	48
Test Items	65–67	68–73	74–81	82–85, 90–91	86–89, 92–94	95–101	102–107	108–113	114–118

Test for Chapter 1 Using Tables and Graphs

Refer to the bar graph.

1. In 1976, where were the two longest tunnels in the world located?
2. About how long was the longest tunnel in the world in 1976?
3. What is the name and location of the shortest tunnel shown in the graph?
4. About how much longer is the Simplon II than the St. Gotthard?

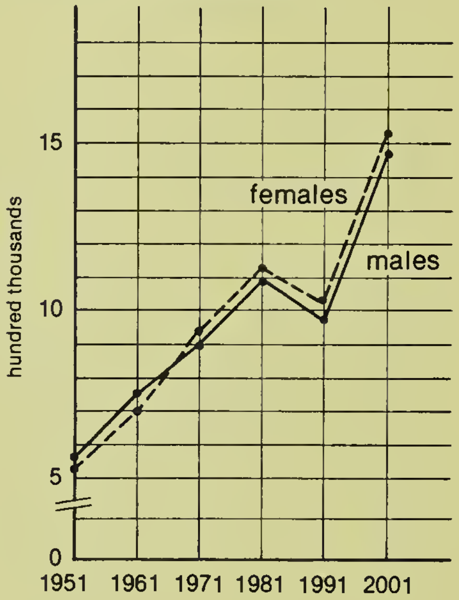
THE WORLD'S LONGEST TUNNELS 1976



Refer to the line graph.

5. Were there more males or more females fifteen to nineteen years old in the population in 1971?
6. The biggest increase in the number of people in this age group is in which ten-year period?
7. Over which ten-year period is the number of males and females in this age group expected to decrease?
8. About how many more females are expected in this age group in 1981 than there are expected to be in 1991?

15–19 YEAR OLDS



Refer to the table. Which activity (or activities) scores highest in promoting the health factor listed?

9. muscle tone
10. weight control
11. balance
12. sleep
13. muscle strength
14. flexibility

Ratings of Selected Activities for Fitness

		Jogging	Bicycling	Swimming	Skating	Tennis	Walking
Physical Fitness	Stamina	21	19	21	18	16	13
	Muscular endurance	20	18	20	17	16	14
	Muscle strength	17	16	14	15	14	11
	Flexibility	9	9	15	13	14	7
	Balance	17	18	12	20	16	8
Well-being	Weight control	21	20	15	17	16	13
	Muscle tone	14	15	14	14	13	11
	Digestion	13	12	13	11	12	11
	Sleep	16	15	16	15	11	14

Refer to the line graph for Exercises **5–8**. Which of these does that graph have?

15. a percent scale

16. a break in the vertical scale

17. different vertical scale values for each line

18. an informed guess at the future

In a pictograph, the symbol stands for 50 people. How many symbols would be needed to stand for

19. 200 people?

20. 75 people?

Use these sets of numbers to find each mode, median, mean, and range.

A: 3, 7, 8, 7, 4, 9, 5, 4, 6, 7, 6

B: 10, 4, 6, 10, 8, 10, 12, 10, 6, 4

21. the mode for set A

22. the mode for set B

23. the median for set A

24. the median for set B

25. the mean for set A

26. the mean for set B

27. the range for set A

28. the range for set B

29. Make a frequency table for set A above.

30. Make a frequency table for set B above.

Find the range, mode, median, and mean.

31.

Score	0	5	10	15	20	25
Number of students	2	2	1	3	7	2

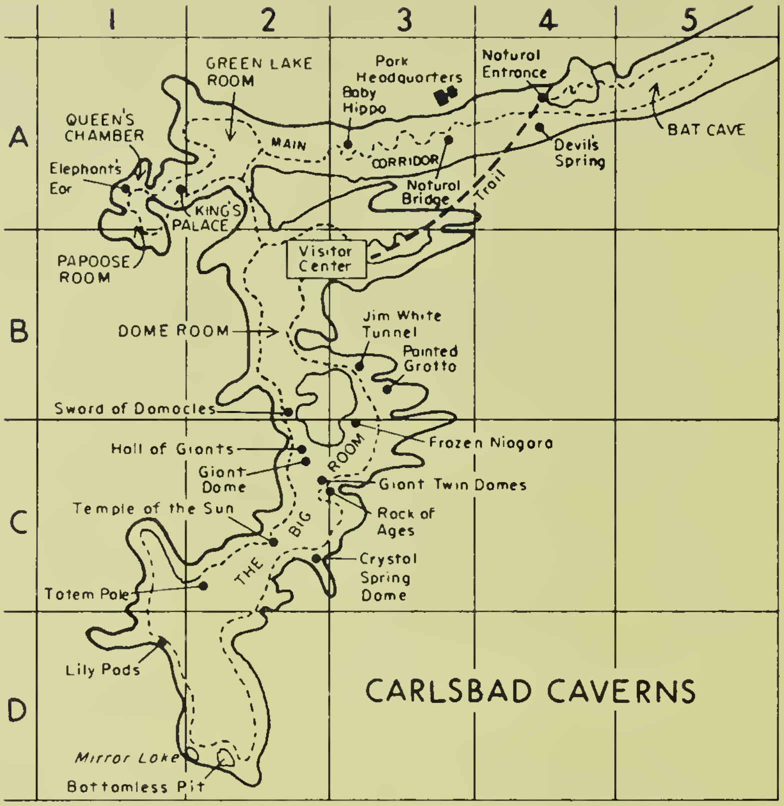
32. 2.2, 1.3, .7, 1.9, 2.5, 1.3, .8, 1.3

Test for Chapter 2 Making Graphs

Refer to the map.

1. In what region is each of these?
   a. Park Headquarters
   b. Totem Pole
   c. Bottomless Pit
   d. Green Lake Room
   e. Painted Grotto

2. In what regions is the Visitor Center?


1
2
3
4
5
A
B
C
D
GREEN LAKE ROOM
Park Headquarters
Baby Hippo
Natural Entrance
QUEEN'S CHAMBER
MAIN
CORRIDOR
Devils Spring
BAT CAVE
Elephants Ear
KING'S PALACE
Natural Bridge
Trail
PAPOOSE ROOM
Visitor Center
DOME ROOM
Jim White Tunnel
Painted Grotto
Sword of Damocles
Frozen Niagara
Hall of Giants
Giant Dome
ROOM
Giant Twin Domes
Temple of the Sun
BIG
Rock of Ages
Crystal Spring Dome
THE
Totem Pole
Lily Pads
Mirror Lake
Bottomless Pit
CARLSBAD CAVERNS

Graph and label each point.

3. (5, 2), (4, 6), (3, 3), (4, 0)

4.

x	5	0	1½	3	4	2
y	6	4	5	2½	4	0

5. Make a line graph from this table showing how temperature changes with altitude.

100 m above sea level	0	15	30	45	60	75	90
Temperature (°C)	15°	5°	-5°	-15°	-10°	8°	21°

6. Graph the points and join them in order.

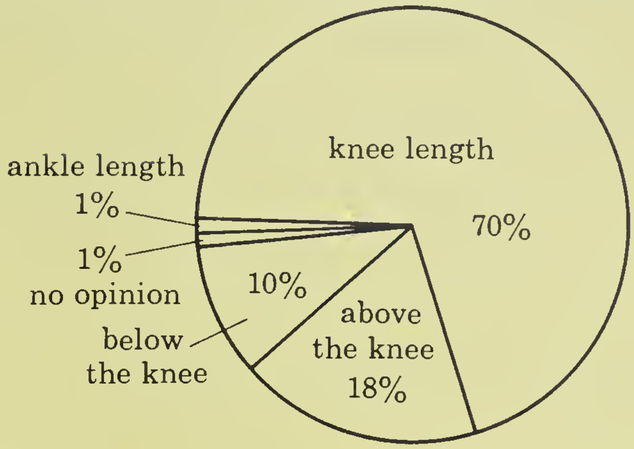
x	7	4	4	3	3	2	1	½	1½	2	2	2½	4	9	9½	9	9	8½	7½	8	7
y	1	1	0	0	1½	2½	1½	3	4	4½	5½	4½	3½	3½	4	3	1	0	0	½	1½

7. Make a bar graph to show the given information.

SPEEDS OF ANIMALS IN KILOMETRES PER HOUR

Quarter horse	80	Domestic cat	50
Greyhound	65.7	Human	46.5
Rabbit	58.3	Pig	18.3

Skirt Lengths Women Prefer

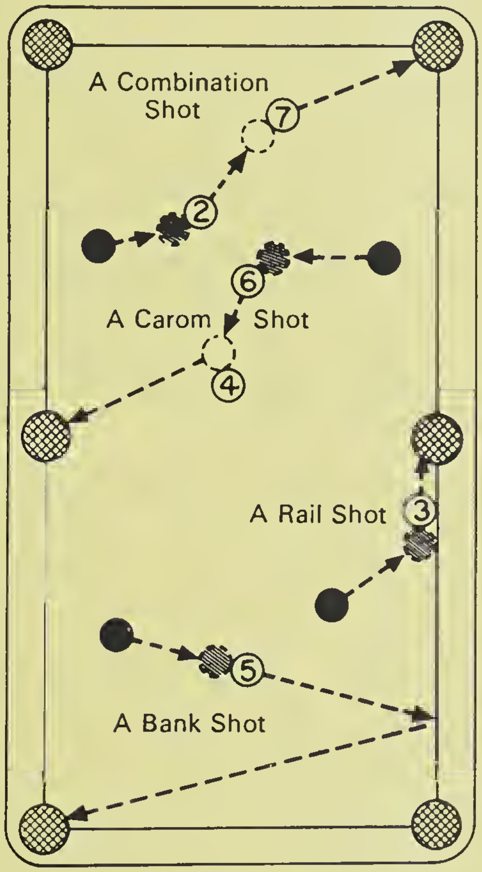


Refer to the circle graph showing the results of a survey taken to find what skirt length most women prefer.

8. Out of every 100 women, how many prefer skirts above the knee?

9. Out of every 100 women, how many did *not* say they prefer knee-length skirts?

The schematic shows four ways in which the cue ball (shown in black) can be used in pocket billiards (pool) to knock the numbered balls into pockets at the side of the pool table. (The cue ball itself must not go into a pocket.) Refer to the schematic to answer the questions below.


A Combination
Shot
7
2
6
A Carom Shot
4
A Rail Shot
3
5
A Bank Shot

10. How is the path of a ball shown?

11. How is the second position of the cue ball shown?

12. In a bank shot how many numbered balls are involved?

13. In which shots are two numbered balls involved?

14. In a combination shot, does the cue ball strike the ball that is pocketed?

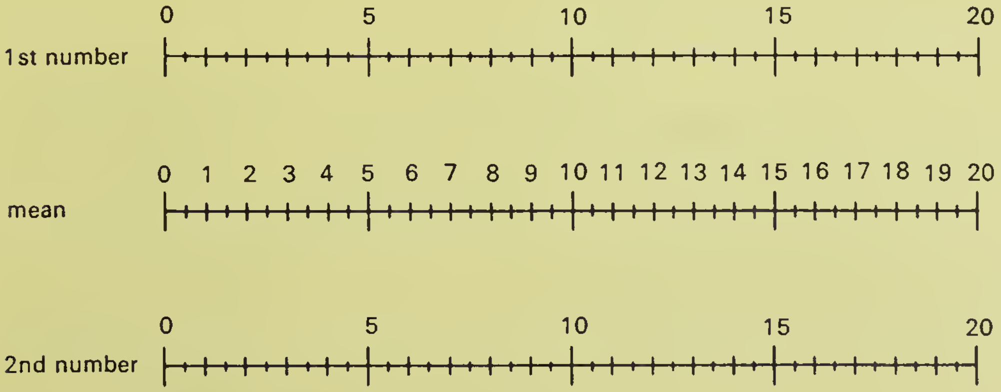
15. In which shots is the side of the table involved?

Use the nomograph to estimate the mean.

16. 5, 9

17. 16, 5

18. $5\frac{1}{2}$, $2\frac{1}{2}$


0
5
10
15
20
1st number
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
mean
0
5
10
15
20
2nd number

Test for Chapter 3 Equations and Formulas

Tell if each sentence is true, false, or open.

1. $8 \times 2 = 12 + 6$

2. $7 + x = 10 - 2$

3. $46 = 2 \times m$

4. $8 + 10 = 9 \times 2$

Find each answer when y is replaced with 5.

5. $2y$

6. $3(y + 1)$

7. $y(8 - 3)$

Find each answer.

8. $9 - 6 + 2$

9. $4 + 14 \div 2$

10. $2(7) - 3(2)$

Use the given formula to solve.

11. A car travels 95 km/h. How far will it go in 3.4 h?

$d = rt$

d: distance
r: km/h
t: number of hours

12. Find the blood pressure of a person who is 18 years old.

$P = \frac{a}{2} + 110$

P: blood pressure
a: age

Solve and check.

13. $x - 12 = 25$

14. $32 = b + 17$

15. $6n = 78$

16. $9 = \frac{x}{12}$

17. $5a + 6 = 26$

18. $4 = \frac{c}{3} - 10$

19. $5m = 14$

20. $22 = 6b - 5$

21. $\frac{2}{3}z = 30$

22. $10 = \frac{7}{4}w - 4$

Use the given formula.

23. A room has a perimeter of 22 m. If the width is 5 m, find the length.

$P = 2\ell + 2w$

P: perimeter
ℓ: length
w: width

24. A movie theater charges \$2 per ticket. The daily expenses are \$260. How many tickets must be sold each day in order to make a daily profit of \$300?

$P = tn - e$

P: profit
t: price per ticket
n: number of tickets
e: expenses

Test for Chapter 4 Ratio and Proportion

Give the ratio of circles to squares.

1. ○○○□□□□□   2. ○□□□○□□

Give each ratio in lowest terms.

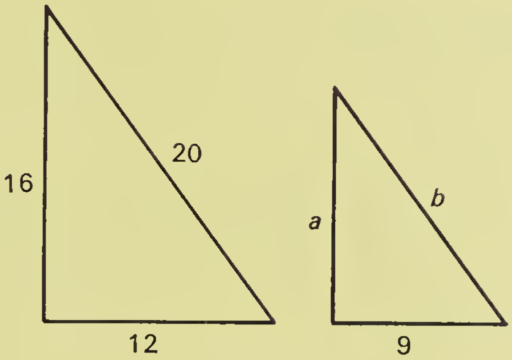
3. $\frac{9}{15}$ 4. $\frac{4}{14}$ 5. $\frac{18}{45}$

Solve each proportion.

6. $\frac{4}{6} = \frac{10}{x}$ 7. $\frac{14}{8} = \frac{n}{12}$ 8. $\frac{18}{a} = \frac{6}{11}$

Solve with a proportion.

9. A 10 kg mass stretches a spring 24 cm. How far will a 15 kg mass stretch the spring?



Use the similar figures at the right.

10. Find the values of a and b.

Use cross-products to find the better buy.

11. 100 g for 47¢, 175 g for 81¢   12. 275 g for 63¢, 500 g for 84¢

Find the best buy by finding how much of each you get for $1.00.

13. 175 g for 25¢, 475 g for 50¢, 625 g for 92¢

Find the best buy by finding the cost of 100 mL of each.

14. 75 mL for 45¢, 150 mL for 80¢, 175 mL for 98¢

Change to a percent.

15. $\frac{6}{15}$ 16. 9 out of 12

Find each number.

17. 60% of 40   18. 35% of 80

Solve each problem by using a proportion.

19. How much money would you save if you bought an $80 bicycle at 16% off?

20. What would your percent be if you got 18 of the 24 questions on this test correct?

Find by multiplying.

21. 65% of 20   22. 23% of 60

Estimate, using $\frac{1}{10}$.

23. 9% of 61   24. 21% of 29.95

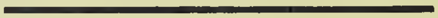
Test for Chapter 5 Measurement

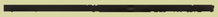
Multiple Choice.

1. The (jar, litre) is not a standard unit of measure.

2. The (cubit, thumbwidth) is an ancient unit that is about 2.5 cm.

Measure each segment to the nearest millimetre.

3. 

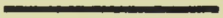
4. 

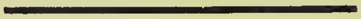
Complete the following.

5. 850 cm is _____ m.

6. 30 km is _____ m.

Measure each segment to the nearest centimetre.

7. 

8. 

Complete the following.

9. 71 cm is _____ mm.

10. 2800 m is _____ km.

11. 1250 mg is _____ g.

12. 3.8 kg is _____ g.

13. 800 L is _____ kL.

14. 1400 mL is _____ L.

Add or subtract.

15.
28 min 14 s
+ 7 min 51 s

16.
7:20 p.m.
−2:35 p.m.

17.
2:40 p.m.
−10:00 a.m.

18. What is the freezing point of water on the Celsius scale?

Use a protractor to draw an angle having each measure.

19. 32°

20. 118°

Test for Chapter 6 Plane Figures

1. How many sides does an octagon have?

2. How many angles does a trapezoid have?

Find the perimeter of each polygon.

3. rectangle with length 37 cm and width 16 cm

4. square with sides 3.5 m

How many square units are in each figure?

5. rectangle with 6 rows of 9 squares each

6. square with 8 rows of 8 squares each

Find the area of each polygon.

7. rectangle with length 1.7 m and width 1.2 m

8. square with sides 3.5 m

Change each measurement as follows.

9. 5 m^2 to cm^2

10. 350 mm^2 to cm^2

Find the area of each parallelogram.

11. base, 18 cm; height, 7 cm

12. base, 43 mm; height, 22 mm

Find the area of each triangle.

13. base, 16 m; height, 12 m

14. base, 74 cm; height, 60 cm

15. Find the area of a trapezoid with a height of 14 m and bases of 22 m and 18 m.

Find each square root.

16. $\sqrt{64}$

17. $\sqrt{25}$

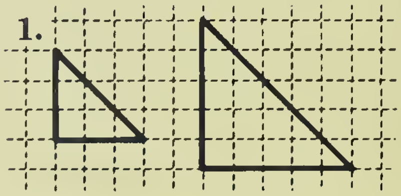
18. Find the circumference of a circle with a diameter of 49 cm. Use $\frac{22}{7}$ for π.

19. Find the area of a circle with a radius of 12 cm. Use 3.14 for π.

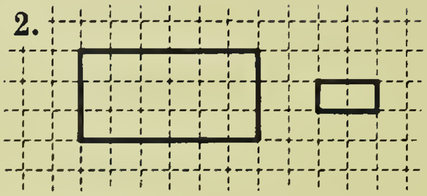
Test for Chapter 7 Scale Drawings

Give each ratio new figure: original figure.

1.

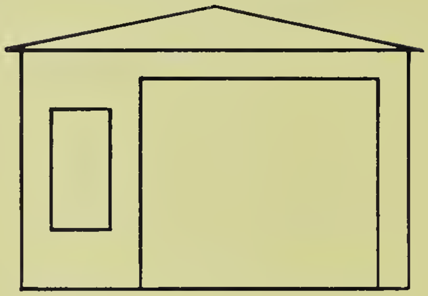


2.



Find each measurement of the real garage.

Scale: 1 cm = 0.9 m

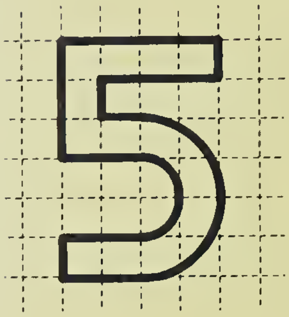


3. width of garage

4. width of door

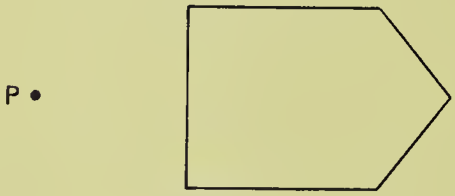
5. height of door

6. height of window



7. Use the grid method to make a 3:2 enlargement of the figure at the left. (Use a grid with 2 squares per centimetre.)

8. Use the point method to make a 3:1 enlargement of the figure at the right.



A map has a scale of 1 cm = 30 km. Find the real distance for each map distance.

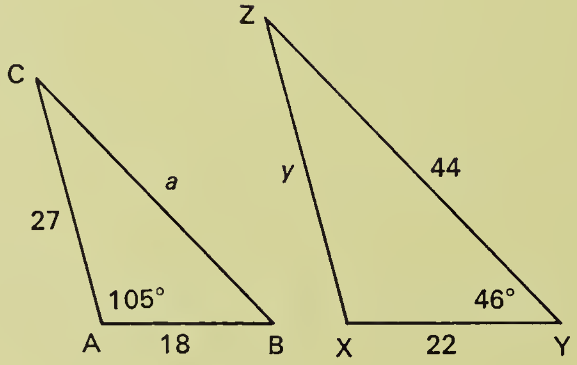
9. 3 cm

10. 2.5 cm

11. 3.4 cm

12. 1.6 cm

Triangle *ABC* is similar to triangle *XYZ*.



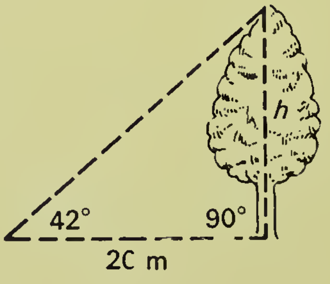
13. Give all 6 pairs of corresponding parts.

14. Find all five missing parts.

Construct triangle *DEF* from measurements.

15. DE is 7 cm, $\angle D$ is 55°, and $\angle E$ is 35°.

Make a scale drawing from the measurements at the right. Then measure, and use a proportion.



16. How tall is the tree?

Test for Chapter 8 Consumer Skills

Find each cash-on-hand amount.

Income	Expense	Cash on hand
		58.20
	24.75	**1.**
	8.27	**2.**
20.00		**3.**
	18.43	**4.**

Find each percent.

Amount	Percent
$20	**5.**
$12	**6.**
$4	**7.**
$44	**8.** ____
$80	100%

Find each amount.

budget category	Percent	Amount
food	22%	**9.**
clothing	14%	**10.**
savings	8%	**11.**
other	56%	**12.** ____
totals	100%	$150

Using unit costs, find the better buys.

13. 150 g for 75¢
175 g for 98¢

14. 100 mL for 64¢
250 mL for $1.53

15. Make a cheque record (see form at right) and fill it in. Starting balance: $94.75
Cheque No. 124: Lang's Market, $14.60, *groceries*
125: A & B Garage, $32.18, *car repair*
Deposit: $125.00
126: S. Miller, $105.00, *rent*

Cheque No.	Description	Amount	Deposit	Bal.
	To For			Bal.
	To For			Bal.
	To For			Bal.
	To For			Bal.

16. Make a form (see right) to reconcile the account. Bank balance: $142.68; cheque record balance: $116.25; outstanding cheques: $62.93; outstanding deposit: $35.00; service charge: $1.50

	Bank balance		
−	Oustanding cheques		Cheque record balance
		−	Service charges
+	Outstanding deposits		Corrected balance
	Corrected balance		

17. For a $300 credit purchase, there are 15 payments of $23 each. Find the credit cost.

$C = na - b$

C: credit cost
n: number of payments
a: amount of each payment
b: amount borrowed

18. An item is bought on credit for $200. The credit cost is $20, and there are 15 monthly payments. Find the annual percentage rate.

$R = \dfrac{24C}{b(n + 1)}$

R: annual percentage rate
C: credit cost
b: amount borrowed
n: number of payments

19. Find the tax on $11,240 if the Tax Rate Schedule reads $2,090 + 27% of the amount over $10,000.

Add or subtract the times.

20. $\begin{array}{r} 6{:}36 \\ +2{:}42 \\ \hline \end{array}$

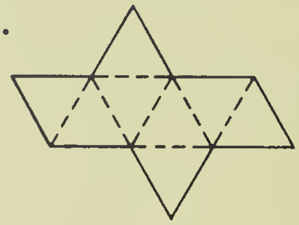
21. $\begin{array}{r} 8{:}15 \\ -6{:}28 \\ \hline \end{array}$

22. $\begin{array}{r} 4{:}12 \text{ p.m.} \\ -8{:}35 \text{ a.m.} \\ \hline \end{array}$

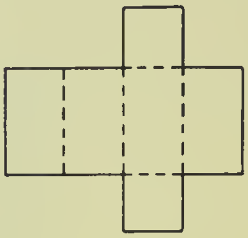
Test for Chapter 9 Solids

Refer to the patterns below to answer questions **1** and **2**.

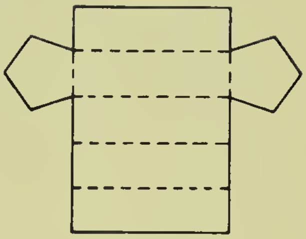
a.



b.



c.

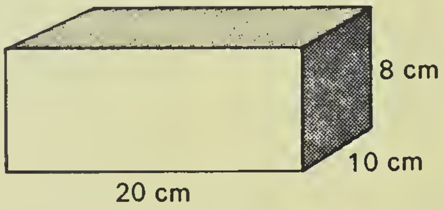


1. Which pattern could be folded to form a rectangular solid?

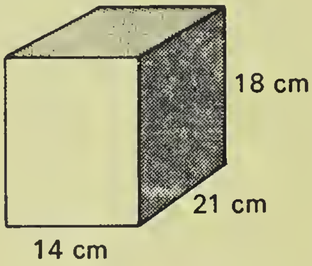
2. Which patterns could be folded to form a right prism?

How many 1 cm cubes would each box hold?

3.

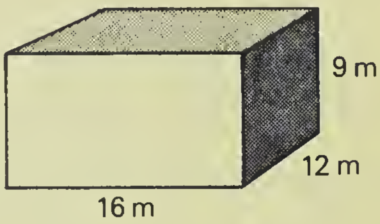


4.

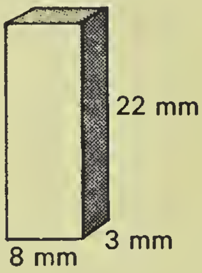


What is the volume of each rectangular solid?

5.



6.



Change each volume.

7. 0.36 m^3 to cm^3

8. 13 850 cm^3 to mL

9. Find the surface area of the solid in Exercise **5.**

10. Find the volume of a cylinder whose base has a radius of 13 cm. The height is 27 cm. Use 3.14 for π.

11. Find the volume of a sphere whose radius is 21 cm. Use $\frac{22}{7}$ for π.

Test for Chapter 10 Chance and Estimates

1. In an experiment, a single marble was drawn from a bag containing 20 marbles—13 blue and 7 red. After each draw, the marble was replaced. What is the experimental probability of drawing a blue marble if blue occurred 61 times out of 100? Express it as a fraction, a decimal, and a percent.

2. In Exercise **1**, what is the mathematical probability of drawing a blue marble? Express it as a fraction, a decimal, and a percent.

3. In an experiment, a single coin is to be tossed 100 times. In how many tosses would you expect tails to occur?

4. A bag contains 10 marbles—2 white, 3 black, and 5 yellow. If a single marble is drawn, what is the mathematical probability of drawing a white or a black marble? Express it as a fraction.

5. A single die is tossed, and a coin is flipped at the same time. What is the mathematical probability of getting 3 on the die and heads on the coin? Express it as a fraction.

6–9. Estimate each answer by rounding each number to the nearest ten. Then find the exact answer.

6. 38 + 41

7. 72 + 22 + 29

8. 77 − 27

9. 133 − 51

10–13. Estimate each answer by rounding each number to the nearest hundred. Then find the exact answer.

10. 205 × 197

11. 511 × 302

12. 1025 ÷ 205

13. 1379 ÷ 197

14. Record albums sell for $4.89 apiece. Estimate the cost of four albums. Round $4.89 to the nearest dollar.

15. On a calculator, you got the following result: 7122 − 4981 = 2141. Check your result by estimating. Round each number to the nearest hundred.

Computing Skills Pretest: Whole Numbers and Decimals

What is the value of each underlined digit? (Example: 5$\underline{6}$4 Answer: 6 tens)

1. 72$\underline{3}$ **2.** 4$\underline{0}$31 **3.** $\underline{7}$51 643 **4.** 1 $\underline{2}$34 567 890

5. 8.$\underline{2}$ **6.** 6.34$\underline{8}$ **7.** 0.6$\underline{5}$4 **8.** 1.038$\underline{2}$

Express in words.

9. 68 **10.** 3146 **11.** 8 702 600 **12.** 8 000 231 467

13. 9.3 **14.** 48.96 **15.** 13.082 **16.** 5.900

17. 8:21 (time of day) **18.** $6.83 **19.** Jan. 27, 1982 **20.** 832-9982 (phone)

Write in symbols.

21. six hours ten minutes **22.** September fifteenth, sixteen thirty-two

23. thirteen fifty (dollars) **24.** sixteen twenty-one, Fifteenth Street

Round as indicated.

25. 2.86 (nearest tenth) **26.** 1.364 (nearest hundredth) **27.** 85 (nearest ten)

28. 1492 (nearest hundred) **29.** 0.6873 (nearest thousandth) **30.** 6.99 (nearest tenth)

Add.

31. $\begin{array}{r} 57 \\ +21 \\ \hline \end{array}$ **32.** $\begin{array}{r} 35 \\ +28 \\ \hline \end{array}$ **33.** $\begin{array}{r} 72 \\ +43 \\ \hline \end{array}$ **34.** $\begin{array}{r} 86 \\ +57 \\ \hline \end{array}$

35. $\begin{array}{r} 406 \\ +371 \\ \hline \end{array}$ **36.** $\begin{array}{r} 320 \\ +\ 73 \\ \hline \end{array}$ **37.** $\begin{array}{r} 8566 \\ +\ 439 \\ \hline \end{array}$ **38.** $\begin{array}{r} 1324 \\ +\ 869 \\ \hline \end{array}$

39. 641 + 732 + 159 + 924 **40.** 1320 + 27 + 6 + 592

41. $\begin{array}{r} 31.24 \\ +16.53 \\ \hline \end{array}$ **42.** $\begin{array}{r} 2.54 \\ +7.28 \\ \hline \end{array}$ **43.** $\begin{array}{r} 13.7 \\ +22.5 \\ \hline \end{array}$ **44.** $\begin{array}{r} 16.839 \\ +\ 8.601 \\ \hline \end{array}$

45. 6.93 + 10.72 + 154.27 **46.** 8.5 + 6.93 + 6 + 0.314

Subtract.

47. $\begin{array}{r} 785 \\ -573 \\ \hline \end{array}$ **48.** $\begin{array}{r} 956 \\ -\ 34 \\ \hline \end{array}$ **49.** $\begin{array}{r} 572 \\ -438 \\ \hline \end{array}$ **50.** $\begin{array}{r} 613 \\ -252 \\ \hline \end{array}$ **51.** $\begin{array}{r} 816 \\ -\ 37 \\ \hline \end{array}$

52. $\begin{array}{r} 169 \\ -\ 30 \\ \hline \end{array}$ **53.** $\begin{array}{r} 2900 \\ -1337 \\ \hline \end{array}$ **54.** $\begin{array}{r} 7932 \\ -5927 \\ \hline \end{array}$ **55.** $\begin{array}{r} 89.384 \\ -72.163 \\ \hline \end{array}$ **56.** $\begin{array}{r} 9.483 \\ -7.005 \\ \hline \end{array}$

57. $\begin{array}{r} 327.47 \\ -215.96 \\ \hline \end{array}$ **58.** $\begin{array}{r} 54.86 \\ -36.97 \\ \hline \end{array}$ **59.** $\begin{array}{r} 4.7364 \\ -4.7358 \\ \hline \end{array}$ **60.** $\begin{array}{r} 12.001 \\ -\ 9.232 \\ \hline \end{array}$ **61.** $\begin{array}{r} 46.23 \\ -\ 0.79 \\ \hline \end{array}$

62. $141.6 - 1.832$ **63.** $175 - 0.83$ **64.** $14.87 - 3$ **65.** $6.53 - 6$

Multiply.

66. 4×7 **67.** $(6)(8)$ **68.** $5(9)$ **69.** 8×3 **70.** 1×9

71. $(7)(7)$ **72.** $(7)(0)$ **73.** 10×72 **74.** 570×100 **75.** 30×40

76. $\begin{array}{r} 72 \\ \times\ 4 \\ \hline \end{array}$ **77.** $\begin{array}{r} 32 \\ \times 13 \\ \hline \end{array}$ **78.** $\begin{array}{r} 57 \\ \times\ 4 \\ \hline \end{array}$ **79.** $\begin{array}{r} 48 \\ \times 73 \\ \hline \end{array}$ **80.** $\begin{array}{r} 473 \\ \times 215 \\ \hline \end{array}$

81. 327×405 **82.** $2 \times 9 \times 5$ **83.** $8 \times 2 \times 6 \times 5$ **84.** 1.75×10

85. $\begin{array}{r} 28 \\ \times 0.2 \\ \hline \end{array}$ **86.** $\begin{array}{r} 534 \\ \times 0.23 \\ \hline \end{array}$ **87.** $\begin{array}{r} 7.53 \\ \times\ 17 \\ \hline \end{array}$ **88.** $\begin{array}{r} 8.34 \\ \times\ 0.2 \\ \hline \end{array}$ **89.** $\begin{array}{r} 23.43 \\ \times 0.212 \\ \hline \end{array}$

90. 72.53×5.26 **91.** $(9.2)(3)(6.43)$

Divide and give the remainder, if any.

92. $8\overline{)56}$ **93.** $54 \div 9$ **94.** $\frac{40}{5}$ **95.** $64 \div 8$ **96.** $7\overline{)20}$

97. $73 \div 9$ **98.** $3\overline{)93}$ **99.** $6\overline{)726}$ **100.** $5\overline{)560}$ **101.** $7\overline{)497}$

102. $8\overline{)4988}$ **103.** $32\overline{)736}$ **104.** $43\overline{)649}$ **105.** $27\overline{)986}$ **106.** $324\overline{)65\,772}$

Divide. Express the result as a decimal.

107. $4\overline{)0.68}$ **108.** $0.32 \div 8$ **109.** $45\overline{)496.8}$ **110.** $34 \div 4$ **111.** $923 \div 100$

112. $0.76 \div 10$ **113.** $572.3 \div 100$ **114.** $26.04 \div .06$ **115.** $0.32\overline{)2.5024}$ **116.** $27 \div .03$

117. $4.325 \div 0.7$ (round to nearest hundredth) **118.** $49 \div 4$ (round to nearest tenth)

Computing Skills Posttest: Whole Numbers and Decimals

What is the value of each underlined digit? (Example: $5\underline{6}4$ Answer: 6 tens)

1. $68\underline{5}$

2. $2\underline{7}08$

3. $\underline{4}32\ 875$

4. $4\ \underline{3}21\ 987\ 605$

5. $13.\underline{7}$

6. $9.06\underline{3}$

7. $2.0\underline{9}1$

8. $0.821\underline{4}$

Express in words.

9. 59

10. 62 324

11. 630 020

12. 934 623 251

13. 6.4

14. 12.27

15. 734.125

16. 8.30

17. $12.50

18. 7:32 (time of day)

19. 981-3232 (phone)

20. 13 Dec. 1932

Write in symbols.

21. nine hours three minutes

22. eighty ten, Ninety-seventh Street

23. seven and a half dollars

24. the sixth of May, nineteen seven

Round as indicated.

25. 31.74 (nearest tenth)

26. 4.308 (nearest hundredth)

27. 91.86 (nearest ten)

28. 1999 (nearest hundred)

29. 0.1353 (nearest thousandth)

30. 13.58 (nearest tenth)

Add.

31. $\begin{array}{r} 34 \\ +63 \\ \hline \end{array}$

32. $\begin{array}{r} 67 \\ +28 \\ \hline \end{array}$

33. $\begin{array}{r} 81 \\ +75 \\ \hline \end{array}$

34. $\begin{array}{r} 99 \\ +36 \\ \hline \end{array}$

35. $\begin{array}{r} 782 \\ +106 \\ \hline \end{array}$

36. $\begin{array}{r} 1462 \\ +\ \ 316 \\ \hline \end{array}$

37. $\begin{array}{r} 236 \\ +\ \ 98 \\ \hline \end{array}$

38. $\begin{array}{r} 2381 \\ +\ \ 419 \\ \hline \end{array}$

39. 1320 + 1461 + 2372 + 8484

40. 8 + 364 + 92 + 1574

41. $\begin{array}{r} 75.35 \\ +13.62 \\ \hline \end{array}$

42. $\begin{array}{r} 4.35 \\ +3.29 \\ \hline \end{array}$

43. $\begin{array}{r} 36.3 \\ +12.9 \\ \hline \end{array}$

44. $\begin{array}{r} 27.927 \\ +\ \ 9.506 \\ \hline \end{array}$

45. 131.82 + 9.35 + 21.46

46. 13 + 4.86 + 0.99 + 5.5

Subtract.

47. $\begin{array}{r} 3477 \\ -1254 \\ \hline \end{array}$ **48.** $\begin{array}{r} 437 \\ -\ \ 26 \\ \hline \end{array}$ **49.** $\begin{array}{r} 783 \\ -267 \\ \hline \end{array}$ **50.** $\begin{array}{r} 925 \\ -642 \\ \hline \end{array}$ **51.** $\begin{array}{r} 521 \\ -\ \ 43 \\ \hline \end{array}$

52. $\begin{array}{r} 2571 \\ -\ \ 140 \\ \hline \end{array}$ **53.** $\begin{array}{r} 3050 \\ -1316 \\ \hline \end{array}$ **54.** $\begin{array}{r} 5213 \\ -4208 \\ \hline \end{array}$ **55.** $\begin{array}{r} 45.738 \\ -33.517 \\ \hline \end{array}$ **56.** $\begin{array}{r} 6.391 \\ -4.008 \\ \hline \end{array}$

57. $\begin{array}{r} 536.28 \\ -314.73 \\ \hline \end{array}$ **58.** $\begin{array}{r} 63.45 \\ -44.86 \\ \hline \end{array}$ **59.** $\begin{array}{r} 13.5624 \\ -13.5618 \\ \hline \end{array}$ **60.** $\begin{array}{r} 28.003 \\ -\ \ 7.469 \\ \hline \end{array}$ **61.** $\begin{array}{r} 93.72 \\ -\ \ 0.84 \\ \hline \end{array}$

62. $356.8 - 9.052$ **63.** $231 - 0.74$ **64.** $27.43 - 4$ **65.** $8.025 - 8$

Multiply.

66. 8×5 **67.** $(7)(9)$ **68.** $3(6)$ **69.** 4×9 **70.** 7×1

71. $(8)(8)$ **72.** $(0)(6)$ **73.** 38×10 **74.** 100×430 **75.** 60×40

76. $\begin{array}{r} 63 \\ \times\ \ 3 \\ \hline \end{array}$ **77.** $\begin{array}{r} 43 \\ \times 21 \\ \hline \end{array}$ **78.** $\begin{array}{r} 67 \\ \times\ \ 5 \\ \hline \end{array}$ **79.** $\begin{array}{r} 36 \\ \times 82 \\ \hline \end{array}$ **80.** $\begin{array}{r} 584 \\ \times 136 \\ \hline \end{array}$

81. 278×504 **82.** $5 \times 7 \times 2$ **83.** $2 \times 7 \times 8 \times 5$ **84.** 3.62×10

85. $\begin{array}{r} 34 \\ \times 0.2 \\ \hline \end{array}$ **86.** $\begin{array}{r} 462 \\ \times 0.34 \\ \hline \end{array}$ **87.** $\begin{array}{r} 8.67 \\ \times\ \ 13 \\ \hline \end{array}$ **88.** $\begin{array}{r} 7.23 \\ \times\ \ 0.3 \\ \hline \end{array}$ **89.** $\begin{array}{r} 31.21 \\ \times 0.233 \\ \hline \end{array}$

90. 84.62×6.27 **91.** $(3.6)(4)(2.15)$

Divide and give the remainder, if any.

92. $7\overline{)63}$ **93.** $72 \div 9$ **94.** $\frac{42}{7}$ **95.** $36 \div 6$ **96.** $8\overline{)49}$

97. $69 \div 7$ **98.** $4\overline{)48}$ **99.** $5\overline{)605}$ **100.** $6\overline{)690}$ **101.** $4\overline{)248}$

102. $7\overline{)1648}$ **103.** $42\overline{)756}$ **104.** $34\overline{)792}$ **105.** $26\overline{)786}$ **106.** $421\overline{)97\,672}$

Divide. Express the result as a decimal.

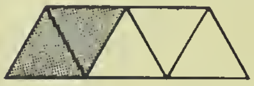
107. $6\overline{)0.84}$ **108.** $0.56 \div 7$ **109.** $25\overline{)325.5}$ **110.** $27 \div 5$ **111.** $1321 \div 100$

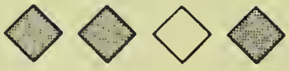
112. $0.37 \div 10$ **113.** $965.7 \div 100$ **114.** $1.74 \div 0.04$ **115.** $0.46\overline{)2.8658}$ **116.** $63 \div 0.07$

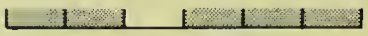
117. $5.367 \div 0.9$ (round to nearest hundredth) **118.** $67 \div 6$ (round to nearest tenth)

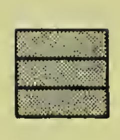
Computing Skills Pretest: Fractions and Mixed Numbers

Use a fraction to tell how much is shaded.

1. 

2. 

3. 

4. 

Express in words.

5. $\frac{1}{7}$ 6. $\frac{4}{9}$ 7. $\frac{11}{100}$ 8. $\frac{4}{4}$ 9. $\frac{15}{1000}$ 10. $\frac{0}{6}$

Tell what number should replace the ? to make the fractions equivalent.

11. $\frac{2}{3} = \frac{?}{12}$ 12. $\frac{3}{5} = \frac{6}{?}$ 13. $\frac{6}{6} = \frac{?}{5}$ 14. $\frac{18}{24} = \frac{?}{8}$ 15. $\frac{6}{8} = \frac{?}{20}$

Change to a whole number or a decimal.

16. $\frac{4}{5}$ 17. $\frac{5}{16}$ 18. $\frac{5}{9}$ 19. $\frac{0}{6}$ 20. $\frac{24}{8}$ 21. $\frac{17}{4}$ 22. $\frac{3}{3}$

Change to a fraction in lowest terms.

23. $\frac{10}{15}$ 24. $\frac{36}{48}$ 25. $\frac{28}{100}$ 26. 0.7 27. 0.009 28. 0.35

Add or subtract. Write the result as a whole number or as a fraction in lowest terms.

29. $\frac{3}{7} + \frac{2}{7}$ 30. $\frac{7}{11} + \frac{4}{11}$ 31. $\frac{4}{5} + \frac{2}{5}$ 32. $\frac{1}{6} + \frac{2}{6}$ 33. $\frac{1}{9} + \frac{5}{9} + \frac{8}{9}$

34. $\frac{9}{10} - \frac{6}{10}$ 35. $\frac{15}{11} - \frac{7}{11}$ 36. $\frac{13}{8} - \frac{5}{8}$ 37. $\frac{9}{12} - \frac{5}{12}$ 38. $\frac{20}{9} - \frac{5}{9}$

Change to like fractions with the least common denominator.

39. $\frac{1}{4}, \frac{1}{12}$ 40. $\frac{1}{3}, \frac{1}{7}$ 41. $\frac{1}{6}, \frac{1}{8}$ 42. $\frac{2}{3}, \frac{1}{4}$ 43. $\frac{2}{3}, \frac{4}{5}, \frac{1}{6}$

Add or subtract. Write the result as a whole number or as a fraction in lowest terms.

44. $\begin{array}{r} \frac{1}{4} \\ +\frac{1}{6} \\ \hline \end{array}$ 45. $\begin{array}{r} \frac{2}{3} \\ +\frac{3}{5} \\ \hline \end{array}$ 46. $\begin{array}{r} \frac{7}{15} \\ \frac{1}{3} \\ +\ \frac{2}{5} \\ \hline \end{array}$ 47. $\begin{array}{r} \frac{9}{7} \\ -\frac{2}{3} \\ \hline \end{array}$ 48. $\begin{array}{r} \frac{4}{5} \\ -\frac{12}{15} \\ \hline \end{array}$

49. $\frac{3}{8} + \frac{0}{3}$ 50. $1 + \frac{4}{7}$ 51. $\frac{3}{11} + \frac{8}{11}$ 52. $2 + \frac{7}{8}$ 53. $\frac{13}{8} - \frac{5}{8}$

54. $\frac{4}{9} - \frac{0}{6}$ 55. $\frac{14}{9} - 1$ 56. $1 - \frac{4}{7}$ 57. $\frac{13}{3} - 2$ 58. $2 - \frac{3}{7}$

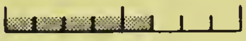
Multiply or divide. Write the result as a whole number or as a fraction in lowest terms.

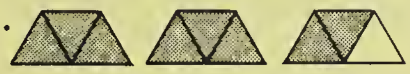
59. $\frac{3}{5} \times \frac{2}{7}$
60. $\frac{2}{5} \times \frac{4}{3} \times \frac{1}{3}$
61. $\frac{2}{3} \times 7$
62. $\frac{5}{6} \times \frac{0}{3}$
63. $\frac{5}{9} \times \frac{4}{4}$

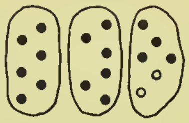
64. $\frac{3}{7} \times \frac{7}{3}$
65. $\frac{3}{7} \times \frac{14}{13}$
66. $\frac{3}{4} \times \frac{8}{9}$
67. $\frac{1}{3} \times \frac{6}{7} \times \frac{21}{4}$
68. $\frac{5}{7} \div \frac{2}{5}$

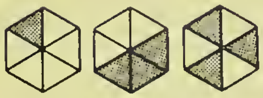
69. $\frac{2}{15} \div \frac{4}{5}$
70. $8 \div \frac{2}{5}$
71. $\dfrac{\frac{12}{7}}{4}$
72. $\frac{5}{13} \div \frac{3}{3}$
73. $\frac{0}{9} \div \frac{2}{3}$

Use a mixed number to tell how much is shaded.

74.


75.


76.


77.


Write as a mixed number; then express in words.

78. $5 + \frac{4}{5}$
79. $\frac{5}{6} + 4$
80. $\frac{7}{1} + \frac{3}{4}$
81. $\frac{5}{5} + \frac{5}{9}$

Change to a decimal.

82. $9\frac{1}{2}$
83. $4\frac{1}{4}$
84. $2\frac{5}{8}$
85. $7\frac{1}{6}$

Change to a whole number or a fraction.

86. $3\frac{1}{4}$
87. $4\frac{2}{3}$
88. $5\frac{2}{2}$
89. $0\frac{7}{10}$

Change to a whole number or to a mixed number with the fraction in lowest terms.

90. 4.3
91. 6.75
92. $\frac{17}{4}$
93. $\frac{30}{6}$
94. $\frac{12}{8}$

Find each result as a whole number, a mixed number, or a proper fraction in lowest terms.

95. $3\frac{3}{7} + 4\frac{2}{7}$
96. $9\frac{3}{8} + 7\frac{1}{8}$
97. $5\frac{4}{9} + 3\frac{5}{9}$
98. $3\frac{3}{4} + 2\frac{1}{5}$
99. $7 + 3\frac{3}{5}$

100. $\begin{array}{r} 5\frac{7}{8} \\ +2\frac{5}{16} \\ \hline \end{array}$
101. $\begin{array}{r} 7\frac{1}{6} \\ +\ \frac{3}{8} \\ \hline \end{array}$
102. $\begin{array}{r} 8\frac{6}{7} \\ -3\frac{4}{7} \\ \hline \end{array}$
103. $\begin{array}{r} 12\frac{3}{8} \\ -\ 3 \\ \hline \end{array}$
104. $\begin{array}{r} 15 \\ -\ 3\frac{4}{5} \\ \hline \end{array}$

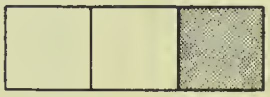
105. $9\frac{5}{16} - 2\frac{1}{8}$
106. $5\frac{1}{4} - 3\frac{1}{2}$
107. $6 - \frac{5}{7}$
108. $1\frac{1}{3} \times 2\frac{4}{7}$
109. $4\frac{3}{4} \times \frac{4}{5}$

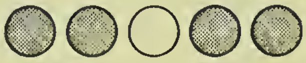
110. $2\frac{2}{3} \times 9$
111. $2\frac{4}{5} \div 1\frac{1}{10}$
112. $\frac{5}{9} \div 2\frac{1}{3}$
113. $12 \div 2\frac{2}{3}$
114. 2^4

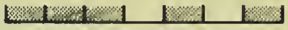
115. $(\frac{2}{5})^2$
116. 10^5
117. 0.5^2
118. $3^2 \times 2^3 \times 4$

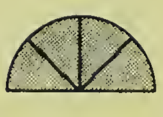
Computing Skills Posttest: Fractions and Mixed Numbers

Use a fraction to tell how much is shaded.

1. 

2. 

3. 

4. 

Express in words.

5. $\frac{1}{5}$
6. $\frac{5}{8}$
7. $\frac{13}{100}$
8. $\frac{6}{6}$
9. $\frac{27}{1000}$
10. $\frac{0}{4}$

Tell what number should replace the ? to make the fractions equivalent.

11. $\frac{3}{4} = \frac{?}{8}$
12. $\frac{2}{7} = \frac{4}{?}$
13. $\frac{4}{4} = \frac{?}{7}$
14. $\frac{16}{36} = \frac{?}{9}$
15. $\frac{4}{6} = \frac{?}{15}$

Change to a whole number or a decimal.

16. $\frac{3}{5}$
17. $\frac{3}{16}$
18. $\frac{4}{9}$
19. $\frac{0}{7}$
20. $\frac{24}{6}$
21. $\frac{13}{4}$
22. $\frac{5}{5}$

Change to a fraction in lowest terms.

23. $\frac{9}{12}$
24. $\frac{24}{36}$
25. $\frac{16}{100}$
26. 0.3
27. 0.007
28. 0.15

Add or subtract. Write the result as a whole number or as a fraction in lowest terms.

29. $\frac{2}{9} + \frac{5}{9}$
30. $\frac{7}{12} + \frac{5}{12}$
31. $\frac{5}{7} + \frac{6}{7}$
32. $\frac{3}{8} + \frac{1}{8}$
33. $\frac{1}{5} + \frac{3}{5} + \frac{4}{5}$
34. $\frac{9}{10} - \frac{6}{10}$
35. $\frac{18}{13} - \frac{9}{13}$
36. $\frac{15}{9} - \frac{6}{9}$
37. $\frac{7}{12} - \frac{3}{12}$
38. $\frac{21}{8} - \frac{7}{8}$

Change to like fractions with the least common denominator.

39. $\frac{1}{3}, \frac{1}{9}$
40. $\frac{1}{7}, \frac{1}{5}$
41. $\frac{1}{10}, \frac{1}{4}$
42. $\frac{2}{3}, \frac{3}{5}$
43. $\frac{1}{2}, \frac{3}{5}, \frac{5}{4}$

Add or subtract. Write the result as a whole number or as a fraction in lowest terms.

44. $\begin{array}{r} \frac{1}{3} \\ +\frac{1}{5} \\ \hline \end{array}$
45. $\begin{array}{r} \frac{5}{6} \\ +\frac{3}{4} \\ \hline \end{array}$
46. $\begin{array}{r} \frac{5}{12} \\ \frac{3}{4} \\ +\ \frac{1}{3} \\ \hline \end{array}$
47. $\begin{array}{r} \frac{7}{5} \\ -\frac{3}{4} \\ \hline \end{array}$
48. $\begin{array}{r} \frac{3}{2} \\ -\frac{6}{4} \\ \hline \end{array}$
49. $\frac{7}{9} + \frac{0}{4}$
50. $1 + \frac{2}{3}$
51. $\frac{5}{9} + \frac{4}{9}$
52. $2 + \frac{3}{5}$
53. $\frac{11}{6} - \frac{5}{6}$
54. $\frac{9}{10} - \frac{0}{5}$
55. $\frac{13}{7} - 1$
56. $1 - \frac{5}{6}$
57. $\frac{7}{2} - 2$
58. $2 - \frac{3}{5}$

Multiply or divide. Write the result as a whole number or as a fraction in lowest terms.

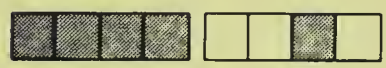
59. $\frac{2}{5} \times \frac{3}{7}$ **60.** $\frac{2}{3} \times \frac{1}{3} \times \frac{4}{5}$ **61.** $\frac{3}{4} \times 5$ **62.** $\frac{5}{8} \times \frac{0}{2}$ **63.** $\frac{7}{10} \times \frac{3}{3}$

64. $\frac{4}{9} \times \frac{9}{4}$ **65.** $\frac{3}{2} \times \frac{4}{11}$ **66.** $\frac{2}{3} \times \frac{9}{16}$ **67.** $\frac{1}{2} \times \frac{4}{5} \times \frac{15}{8}$ **68.** $\frac{3}{5} \div \frac{4}{3}$

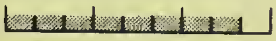
69. $\frac{3}{16} \div \frac{9}{4}$ **70.** $5 \div \frac{2}{3}$ **71.** $\dfrac{\frac{10}{7}}{2}$ **72.** $\frac{8}{11} \div \frac{4}{4}$ **73.** $\frac{0}{8} \div \frac{3}{4}$

Use a mixed number to tell how much is shaded.

74.



75.



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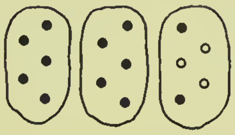

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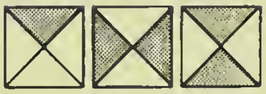

Write as a mixed number; then express in words.

78. $6 + \frac{5}{8}$ **79.** $\frac{4}{5} + 3$ **80.** $\frac{4}{1} + \frac{2}{3}$ **81.** $\frac{3}{3} + \frac{3}{7}$

Change to a decimal.

82. $3\frac{1}{2}$ **83.** $2\frac{3}{4}$ **84.** $5\frac{3}{8}$ **85.** $8\frac{2}{3}$

Change to a whole number or a fraction.

86. $4\frac{1}{2}$ **87.** $2\frac{2}{5}$ **88.** $3\frac{4}{4}$ **89.** $0\frac{7}{8}$

Change to a whole number or to a mixed number with the fraction in lowest terms.

90. 6.7 **91.** 5.25 **92.** $\frac{16}{3}$ **93.** $\frac{25}{5}$ **94.** $\frac{10}{4}$

Find each result as a whole number, a mixed number, or a proper fraction in lowest terms.

95. $2\frac{1}{5} + 5\frac{3}{5}$ **96.** $8\frac{1}{4} + 3\frac{1}{4}$ **97.** $4\frac{4}{7} + 5\frac{3}{7}$ **98.** $2\frac{3}{5} + 5\frac{1}{4}$ **99.** $8 + 2\frac{7}{8}$

100. $\begin{array}{r} 3\frac{4}{5} \\ +2\frac{7}{10} \\ \hline \end{array}$ **101.** $\begin{array}{r} 4\frac{2}{3} \\ +\ \frac{5}{8} \\ \hline \end{array}$ **102.** $\begin{array}{r} 9\frac{7}{9} \\ -3\frac{5}{9} \\ \hline \end{array}$ **103.** $\begin{array}{r} 10\frac{3}{5} \\ -\ 2 \\ \hline \end{array}$ **104.** $\begin{array}{r} 12 \\ -\ 5\frac{2}{3} \\ \hline \end{array}$

105. $7\frac{3}{8} - 4\frac{1}{4}$ **106.** $11\frac{1}{2} - 2\frac{3}{4}$ **107.** $5 - \frac{3}{8}$ **108.** $3\frac{1}{2} \times 2\frac{2}{3}$ **109.** $7\frac{1}{4} \times \frac{2}{3}$

110. $3\frac{2}{5} \times 10$ **111.** $4\frac{1}{3} \div 1\frac{1}{6}$ **112.** $\frac{5}{8} \div 2\frac{1}{4}$ **113.** $8 \div 1\frac{1}{3}$ **114.** 2^3

115. $(\frac{2}{3})^2$ **116.** 10^4 **117.** 0.3^2 **118.** $2^2 \times 3^2 \times 5$

Answers to Tests

Chapter 1

1. U.S.A.   2. 168.9 m   3. Mont Cenis, Fr-Italy   4. 5 m

5. Females   6. 1991-2001   7. 1981-91   8. 500 000   9. Bicycling

10. Jogging   11. Skating   12. Jogging, swimming   13. Jogging   14. Swimming

15. No   16. Yes   17. No   18. Yes   19. 4   20. $1\frac{1}{2}$ 21. 7

22. 10   23. 6   24. 9   25. 6   26. 8   27. 3–9 or 6   28. 4–12 or 8

29.

3	4	5	6	7	8	9
1	2	1	2	3	1	1

30.

4	6	8	10	12
2	2	1	4	1

	range	mode	median	mean
31.	0–25 or 25	20	20	15
32.	0.7-2.5 or 1.8	1.3	1.3	1.5

Chapter 2

1. **a.** A-3   **b.** C-2   **c.** D-2   **d.** A-2   **e.** B-3   2. B-2, B-3   3–5. *Graphs*   6. *Dog*

7. *Graph*   8. 18   9. 30   10. Arrows   11. Shading   12. 1

13. Combination, carom   14. No   15. Rail, bank   16. 7   17. $10\frac{1}{2}$ 18. 4

Chapter 3

1. False   2. Open   3. Open   4. True   5. 10   6. 18

7. 25   8. 5   9. 11   10. 8   11. 323 km   12. 119

13. 37   14. 15   15. 13   16. 108   17. 4   18. 42

19. $\frac{14}{5}$ 20. $\frac{9}{2}$ 21. 45   22. 8   23. 6 m   24. 280

Chapter 4

1. $\frac{3}{5}$ 2. $\frac{4}{3}$ 3. $\frac{3}{5}$ 4. $\frac{2}{7}$ 5. $\frac{2}{5}$ 6. 15

7. 21   8. 33   9. 36 cm   10. $a = 12$, $b = 15$

Ex. 11–14: Answers given in order; arrow indicates better buy.

11. 8100, 8225 ←   12. 23 100, 31 500 ←   13. 700, 950 ←, 679.3   14. 0.6, 0.53 ←, 0.56

15. 40%   16. 75%   17. 24   18. 28   19. $12.80   20. 75%

21. 13   22. 13.8   *Ex. 23–24: Estimates*   23. 6   24. 6

Chapter 5

1. jar   2. thumbwidth   3. 45 mm   4. 22 mm   5. 7

6. 30 000   7. 2 cm   8. 4 cm   9. 710   10. 1.25   11. 9

12. 3800
13. 0.8
14. 1.4
15. 36 min 5 s
16. 4:45
17. 4:40
18. 0°C
19–20. *Student drawings*

Chapter 6

1. 8
2. 4
3. 106 cm
4. 14 m
5. 54
6. 64
7. 2.04 m^2
8. 12.25 m^2
9. 50 000 cm^2
10. 3.5 cm^2
11. 126 cm^2
12. 946 mm^2
13. 96 m^2
14. 2220 cm^2
15. 280 m^2
16. 8
17. 5
18. 154 cm
19. 452.16 cm^2

Chapter 7

1. 5:3
2. 1:3
3. 4.6 m
4. 2.3 m
5. 2 m
6. 1.2 m
7–8. *Drawings*
9. 90 km
10. 75 km
11. 102 km
12. 48 km
13. $\angle A \leftrightarrow \angle X$, $\angle B \leftrightarrow \angle Y$, $\angle C \leftrightarrow \angle Z$
$\overline{AB} \leftrightarrow \overline{XY}$, $\overline{BC} \leftrightarrow \overline{YZ}$, $\overline{AC} \leftrightarrow \overline{XZ}$
14. $\angle A = 105°$, $\angle C = 29°$, $\angle Y = 46°$,
$a = 36$, $y = 33$
15. *Drawing*
16. About 18 m

Chapter 8

1. 33.45
2. 25.18
3. 45.18
4. 26.75
5. 25%
6. 15%
7. 5%
8. 55%
9. \$33
10. \$21
11. \$12
12. \$84
13. 2
1.8 ←
14. 1.56 ←
1.63
15. 80.15
47.97
172.97
67.97
16.

$$\begin{array}{rr} 142.68 & \\ -\ 62.93 & 116.25 \\ 79.75 & -\ \ 1.50 \\ +\ 35.00 & 114.75 \\ 114.75 & \end{array}$$

17. \$45
18. 15%
19. \$2424.80
20. 9:18
21. 1:47
22. 7:37 or 7 h 37 min

Chapter 9

1. b
2. b, c
3. 1600
4. 5292
5. 1728 m^3
6. 528 mm^3
7. 360 000 cm^3
8. 13 850 L
9. 888 m^2
10. 14 327.82 cm^3
11. 38 808 m^3

Chapter 10

1. $\frac{61}{100}$, 0.61, 61%
2. $\frac{13}{20}$, 0.65, 65%
3. 50
4. $\frac{1}{2}$
5. $\frac{1}{12}$
6. 80, 79
7. 120, 123
8. 50, 50
9. 80, 82
10. 40 000; 40 385
11. 150 000; 154 322
12. 5, 5
13. 7, 7
14. \$20.00
15. 2100

Computing Skills Pretest: Whole Numbers and Decimals

1. 3 ones **2.** 0 hundreds **3.** 7 hundred thousands **4.** 2 hundred millions

5. 2 tenths **6.** 8 thousandths **7.** 5 hundredths **8.** 2 ten-thousandths

9. Sixty-eight **10.** Three thousand one hundred forty-six

11. Eight million seven hundred two thousand six hundred **12.** Eight billion two hundred thirty-one thousand four hundred sixty-seven

13. Nine and three-tenths **14.** Forty-eight and ninety-six hundredths

15. Thirteen and eighty-two thousandths **16.** Five and nine-hundred thousandths

17. Eight twenty-one **18.** Six dollars and eighty-three cents

19. January twenty-seventh, nineteen eighty-two **20.** Eight three two, nine nine eight two **21.** 6:10

22. Sept. 15, 1632 or 15 Sept. 1632 **23.** $13.50 **24.** 1621 15th Street

25. 2.9 **26.** 1.36 **27.** 90 **28.** 1500 **29.** 0.687 **30.** 7.0 **31.** 78

32. 63 **33.** 115 **34.** 143 **35.** 777 **36.** 393 **37.** 9005 **38.** 2193

39. 2456 **40.** 1945 **41.** 47.77 **42.** 9.82 **43.** 36.2 **44.** 25.440 **45.** 171.92

46. 21.744 **47.** 212 **48.** 922 **49.** 134 **50.** 361 **51.** 779 **52.** 139

53. 1563 **54.** 2005 **55.** 17.221 **56.** 2.478 **57.** 111.51 **58.** 17.89 **59.** 0.0006

60. 2.769 **61.** 45.44 **62.** 139.768 **63.** 174.17 **64.** 11.87 **65.** 0.53 **66.** 28

67. 48 **68.** 45 **69.** 24 **70.** 9 **71.** 49 **72.** 0 **73.** 720

74. 57 000 **75.** 1200 **76.** 288 **77.** 416 **78.** 228 **79.** 3504 **80.** 101 695

81. 132 435 **82.** 90 **83.** 480 **84.** 17.5 **85.** 5.6 **86.** 122.82 **87.** 128.01

88. 1.668 **89.** 4.967 16 **90.** 381.5078 **91.** 177.468 **92.** 7 **93.** 6 **94.** 8

95. 8 **96.** 2R6 **97.** 8R1 **98.** 31 **99.** 121 **100.** 112 **101.** 71

102. 623R4 **103.** 23 **104.** 15R4 **105.** 36R14 **106.** 203 **107.** 0.17 **108.** 0.04

109. 11.04 **110.** 8.5 **111.** 9.23 **112.** 0.076 **113.** 5.723 **114.** 434 **115.** 7.82

116. 900 **117.** 6.18 **118.** 12.3

Computing Skills Posttest: Whole Numbers and Decimals

1. 5 ones **2.** 7 hundreds **3.** 4 hundred thousands **4.** 3 hundred millions

5. 7 tenths **6.** 3 thousandths **7.** 9 hundredths **8.** 4 ten-thousandths

9. Fifty-nine **10.** Sixty-two thousand three hundred twenty-four

11. Six hundred thirty thousand twenty **12.** Nine hundred thirty-four million six hundred twenty-three thousand two hundred fifty-one

13. Six and four-tenths **14.** Twelve and twenty-seven hundredths

15. Seven hundred thirty-four and one hundred twenty-five thousandths

16. Eight and thirty hundredths **17.** Twelve dollars and fifty cents

18. Seven thirty-two **19.** Nine eight one, three two three two

20. December thirteenth, nineteen thirty-two **21.** 9:03

22. 8010 97th Street **23.** $7.50 **24.** May 6, 1907 or 6 May 1907

25. 31.7 **26.** 4.31 **27.** 91.9 **28.** 2000 **29.** 0.135 **30.** 13.6 **31.** 97

32. 95 **33.** 156 **34.** 135 **35.** 888 **36.** 1778 **37.** 334 **38.** 2800

39. 13 637 **40.** 2038 **41.** 88.97 **42.** 7.64 **43.** 49.2 **44.** 37.433 **45.** 162.63

46. 24.35 **47.** 2223 **48.** 411 **49.** 516 **50.** 283 **51.** 478 **52.** 2431

53. 1734 **54.** 1005 **55.** 12.221 **56.** 2.383 **57.** 221.55 **58.** 18.59 **59.** 0.0006

60. 20.534 **61.** 92.88 **62.** 347.748 **63.** 230.26 **64.** 23.43 **65.** 0.025 **66.** 40

67. 63 **68.** 18 **69.** 36 **70.** 7 **71.** 64 **72.** 0 **73.** 380

74. 43 000 **75.** 2400 **76.** 189 **77.** 903 **78.** 335 **79.** 2952 **80.** 79 424

81. 140 112 **82.** 70 **83.** 560 **84.** 36.2 **85.** 6.8 **86.** 157.08 **87.** 112.71

88. 2.169 **89.** 7.271 93 **90.** 530.5674 **91.** 30.96 **92.** 9 **93.** 8 **94.** 6

95. 6 **96.** 6R1 **97.** 9R6 **98.** 12 **99.** 121 **100.** 115 **101.** 62

102. 235R3 **103.** 18 **104.** 23R10 **105.** 30R6 **106.** 232 **107.** 0.14 **108.** 0.08

109. 13.02 **110.** 5.4 **111.** 13.21 **112.** 0.037 **113.** 9.657 **114.** 43.5 **115.** 6.23

116. 900 **117.** 5.96 **118.** 11.2

Computing Skills Pretest: Fractions and Mixed Numbers

1. $\frac{2}{5}$ **2.** $\frac{3}{4}$ **3.** $\frac{5}{6}$ **4.** $\frac{3}{3}$ **5.** One-seventh **6.** Four-ninths **7.** Eleven-hundredths

8. Four-fourths **9.** Fifteen-thousandths **10.** Zero-sixths

11. 8 **12.** 10 **13.** 5 **14.** 6 **15.** 15 **16.** .8 **17.** .3125

18. $.\overline{5}$ **19.** 0 **20.** 3 **21.** 4.25 **22.** 1 **23.** $\frac{2}{3}$ **24.** $\frac{3}{4}$

25. $\frac{7}{25}$ **26.** $\frac{7}{10}$ **27.** $\frac{9}{1000}$ **28.** $\frac{7}{20}$ **29.** $\frac{5}{7}$ **30.** 1 **31.** $\frac{6}{5}$

32. $\frac{1}{2}$ **33.** $\frac{14}{9}$ **34.** $\frac{3}{10}$ **35.** $\frac{8}{11}$ **36.** 1 **37.** $\frac{1}{3}$ **38.** $\frac{5}{3}$

39. $\frac{3}{12}, \frac{1}{12}$ **40.** $\frac{7}{21}, \frac{3}{21}$ **41.** $\frac{4}{24}, \frac{3}{24}$ **42.** $\frac{8}{12}, \frac{3}{12}$ **43.** $\frac{20}{30}, \frac{24}{30}, \frac{5}{30}$ **44.** $\frac{5}{12}$

45. $\frac{19}{15}$ **46.** $\frac{6}{5}$ **47.** $\frac{13}{21}$ **48.** 0 **49.** $\frac{3}{8}$ **50.** $\frac{11}{7}$ **51.** 1

52. $\frac{23}{8}$ **53.** 1 **54.** $\frac{4}{9}$ **55.** $\frac{5}{9}$ **56.** $\frac{3}{7}$ **57.** $\frac{7}{3}$ **58.** $\frac{11}{7}$

59. $\frac{6}{35}$ **60.** $\frac{8}{45}$ **61.** $\frac{14}{3}$ **62.** 0 **63.** $\frac{5}{9}$ **64.** 1 **65.** $\frac{6}{13}$

66. $\frac{2}{3}$ **67.** $\frac{3}{2}$ **68.** $\frac{25}{14}$ **69.** $\frac{1}{6}$ **70.** 20 **71.** $\frac{3}{7}$ **72.** $\frac{5}{13}$

73. 0 **74.** $1\frac{1}{4}$ **75.** $2\frac{2}{3}$ **76.** $2\frac{4}{6}$ **77.** $1\frac{1}{6}$

78. $5\frac{4}{5}$; five and four-fifths **79.** $4\frac{5}{6}$; four and five-sixths

80. $7\frac{3}{4}$; seven and three-fourths **81.** $1\frac{5}{9}$; one and five-ninths

82. 9.5 **83.** 4.25 **84.** 2.625 **85.** $7.1\overline{6}$ **86.** $\frac{13}{4}$ **87.** $\frac{14}{3}$ **88.** 6

89. $\frac{7}{10}$ **90.** $4\frac{3}{10}$ **91.** $6\frac{3}{4}$ **92.** $4\frac{1}{4}$ **93.** 5 **94.** $1\frac{1}{2}$ **95.** $7\frac{5}{7}$

96. $16\frac{1}{2}$ **97.** 9 **98.** $5\frac{19}{20}$ **99.** $10\frac{3}{5}$ **100.** $9\frac{3}{16}$ **101.** $7\frac{13}{24}$ **102.** $5\frac{2}{7}$

103. $9\frac{3}{8}$ **104.** $11\frac{1}{5}$ **105.** $7\frac{3}{16}$ **106.** $1\frac{3}{4}$ **107.** $5\frac{2}{7}$ **108.** $3\frac{3}{7}$ **109.** $3\frac{4}{5}$

110. 24 **111.** $2\frac{6}{11}$ **112.** $\frac{5}{21}$ **113.** $4\frac{1}{2}$ **114.** 16 **115.** $\frac{4}{25}$ **116.** 100 000

117. 0.25 **118.** 288

Computing Skills Posttest: Fractions and Mixed Numbers

1. $\frac{1}{3}$ **2.** $\frac{4}{5}$ **3.** $\frac{5}{7}$ **4.** $\frac{4}{4}$ **5.** One-fifth **6.** Five-eighths **7.** Thirteen-hundredths

8. Six-sixths **9.** Twenty-seven thousandths **10.** Zero-fourths

11. 6 **12.** 14 **13.** 7 **14.** 4 **15.** 10 **16.** 0.6 **17.** 0.1875

18. $.\overline{4}$ **19.** 0 **20.** 4 **21.** 3.25 **22.** 1 **23.** $\frac{3}{4}$ **24.** $\frac{2}{3}$

25. $\frac{4}{25}$ **26.** $\frac{9}{10}$ **27.** $\frac{3}{1000}$ **28.** $\frac{3}{20}$ **29.** $\frac{7}{9}$ **30.** 1 **31.** $\frac{11}{7}$

32. $\frac{1}{2}$ **33.** $\frac{8}{5}$ **34.** $\frac{3}{10}$ **35.** $\frac{9}{13}$ **36.** 1 **37.** $\frac{1}{3}$ **38.** $\frac{7}{4}$

39. $\frac{3}{9}$, $\frac{1}{9}$ **40.** $\frac{5}{35}$, $\frac{7}{35}$ **41.** $\frac{2}{20}$, $\frac{5}{20}$ **42.** $\frac{10}{15}$, $\frac{9}{15}$ **43.** $\frac{10}{20}$, $\frac{12}{20}$, $\frac{25}{20}$ **44.** $\frac{8}{15}$

45. $\frac{19}{12}$ **46.** $\frac{3}{2}$ **47.** $\frac{13}{20}$ **48.** 0 **49.** $\frac{7}{9}$ **50.** $\frac{5}{3}$ **51.** 1

52. $\frac{13}{5}$ **53.** 1 **54.** $\frac{9}{10}$ **55.** $\frac{6}{7}$ **56.** $\frac{1}{6}$ **57.** $\frac{3}{2}$ **58.** $\frac{7}{5}$

59. $\frac{6}{35}$ **60.** $\frac{8}{45}$ **61.** $\frac{15}{4}$ **62.** 0 **63.** $\frac{7}{10}$ **64.** 1 **65.** $\frac{6}{11}$

66. $\frac{3}{8}$ **67.** $\frac{3}{4}$ **68.** $\frac{9}{20}$ **69.** $\frac{1}{12}$ **70.** $\frac{15}{2}$ **71.** $\frac{5}{7}$ **72.** $\frac{8}{11}$

73. 0 **74.** $1\frac{1}{4}$ **75.** $2\frac{2}{3}$ **76.** $2\frac{2}{5}$ **77.** $1\frac{1}{4}$

78. $6\frac{5}{8}$; six and five-eighths **79.** $3\frac{4}{5}$; three and four-fifths

80. $4\frac{2}{3}$; four and two-thirds **81.** $1\frac{3}{7}$; one and three-sevenths

82. 3.5 **83.** 2.75 **84.** 5.375 **85.** $8.\overline{6}$ **86.** $\frac{9}{2}$ **87.** $\frac{12}{5}$ **88.** 4

89. $\frac{7}{8}$ **90.** $6\frac{7}{10}$ **91.** $5\frac{1}{4}$ **92.** $5\frac{1}{3}$ **93.** 5 **94.** $2\frac{1}{2}$ **95.** $7\frac{4}{5}$

96. $11\frac{1}{2}$ **97.** 10 **98.** $7\frac{17}{20}$ **99.** $10\frac{7}{8}$ **100.** $6\frac{1}{2}$ **101.** $5\frac{7}{24}$ **102.** $6\frac{2}{9}$

103. $8\frac{3}{5}$ **104.** $6\frac{1}{3}$ **105.** $3\frac{1}{8}$ **106.** $8\frac{3}{4}$ **107.** $4\frac{5}{8}$ **108.** $9\frac{1}{3}$ **109.** $4\frac{5}{6}$

110. 34 **111.** $3\frac{5}{7}$ **112.** $\frac{5}{18}$ **113.** 6 **114.** 8 **115.** $\frac{4}{9}$ **116.** 10 000

117. 0.09 **118.** 180

DATE DUE SLIP

EDUC APR 19 '83
RETURN APR 15 '83
DUE EDUC OCT 5 '83
RETURN OCT 5 '83
DUE EDUC OCT 23 '83
RETURN OCT 23 '83
DUE EDUC DEC 4 '83
DUE EDUC DEC 11 '83
DEC 11 RETURN
DUE EDUC MAR 8 '84
MAR 7 RETURN
DUE EDUC OCT 20 '84
OCT 15 RETURN
DUE EDUC NOV 09 '85
DUE EDUC NOV 15 '85
F. 255

NOV 1 RETURN
DUE EDUC APR 13 '86
APR 17 RETURN
DUE EDUC JUL 22 '86
JUL 22 RETURN
DUE EDUC MAR 23 '87
MAR 22 RETURN
DUE EDUC MAR 14 '88
MAR 09 RETURN
DUE EDUC NOV 23 '88
NOV 23 RETURN
DEC 16 '94
RETURN DEC 15 '94
DUE EDUC FEB 21 '95
RETURN FEB 17 '95
RETURN AUG 06 '97

QA 39.2 W45 1981 TCH.ED.
WELLS DAVID W
MATHEMATICS FOR DAILY USE/

39585951 CURR

000011575552

QA 39.2 W45 1981 tch.ed.
Wells, David W.
Mathematics for daily use /

39585951 CURR